A FOUR-FIELD MODEL FOR THREE-PHASE FLOWS

CONTEXT

Some nuclear accidental scenarii assume that very hot liquid metal might interact with quiet liquid water. Steam explosion might then occur : liquid water heated by metal suddendly changes into steam and a steam layer appears around metal particles; heat transfer is thus inhibited until this layer becomes unstable, which may lead to steam explosion.

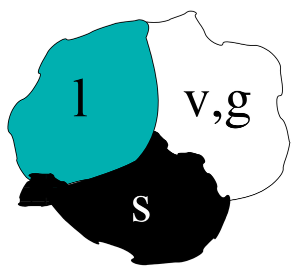
We propose here a dedicated model composed of four fields to describe this multiphase system, with a similar approach to [3, 4, 5, 6]. Three phases are interacting : a solid phase (s), a liquid water phase (l) and a gaseous phase as a miscible mixture of steam water (v) and ambient air or fission gases (g).

Steam explosion occuring when hot lava enters in seawater.

THE FOUR-FIELD MODEL

We define : $\mathbb{K} := \{s, l, v, g\}$. Steam (v) and air (g) are miscible whereas liquid (l), solid (s) and the gases (v+g) are immiscible :

$$\alpha_v = \alpha_g \quad ; \quad \alpha_l + \alpha_s + \alpha_v = 1.$$



3-phase system.

The state variable is :
$\mathbf{Y} = (\alpha_s, \alpha_l, m_k, m_k u_k, \alpha_k E_k)^t \in \mathbb{R}^{14}$ with $k \in \mathbb{K}$.

System of Partial Differential Equations :

$$\begin{cases} \partial_t \alpha_k + v_I(\mathbf{Y}) \partial_x \alpha_k = \Phi_k(\mathbf{Y}), & k \in \mathbb{K}, \\ \partial_t m_k + \partial_x (m_k u_k) = \Gamma_k(\mathbf{Y}), & k \in \mathbb{K}, \\ \partial_t (m_k u_k) + \partial_x (m_k u_k^2 + \alpha_k p_k) + \sum_{l \neq k} \pi_{kl}(\mathbf{Y}) \partial_x \alpha_l = \mathcal{S}_{Q_k}(\mathbf{Y}), & k \in \mathbb{K}, \\ \partial_t (m_k E_k) + \partial_x (u_k (m_k E_k + \alpha_k p_k)) - \sum_{l \neq k} \pi_{kl}(\mathbf{Y}) \partial_t \alpha_l = \mathcal{S}_{E_k}(\mathbf{Y}), & k \in \mathbb{K}. \end{cases}$$

where $E_k = u_k^2/2 + \varepsilon_k(p_k, \rho_k)$ so that :

$$\rho_k c_k^2 = \left(\frac{\partial \varepsilon_k}{\partial p_k}\right)^{-1} \left(\frac{p_k}{\rho_k} - \rho_k \frac{\partial \varepsilon_k}{\partial \rho_k}\right) \text{ and } \frac{1}{T_k} = \frac{\partial s_k}{\partial p_k} \left(\frac{\partial \varepsilon_k}{\partial p_k}\right)^{-1} > 0.$$

Constraints :

$$\Phi_v = \Phi_g, \quad \Phi_s + \Phi_l + \Phi_v = 0, \quad \sum_{k \in \mathbb{K}} \sum_{l \neq k} \pi_{kl}(\mathbf{Y}) \partial_x \alpha_l = 0,$$

$$\sum_{k \in \mathbb{K}} \Gamma_k(\mathbf{Y}) = 0, \quad \sum_{k \in \mathbb{K}} \mathcal{S}_{Q_k}(\mathbf{Y}) = 0, \quad \sum_{k \in \mathbb{K}} \mathcal{S}_{E_k}(\mathbf{Y}) = 0.$$

Admissible source terms The case $v_I(\mathbf{Y}) = u_s$ is now considered. $\forall (k, l) \in \mathbb{K}^2, l \neq k$, $\pi_{kl}(\mathbf{Y})$ fulfilling the minimal dissipation entropy constraint are uniquely defined. Moreover, for the following closures :

$$\Gamma_k = \sum_{l \neq k} \Gamma_{kl}(\mathbf{Y}), \quad S_{Q_k} = \sum_{l \neq k} D_{kl}(\mathbf{Y}) + \sum_{l \neq k} \mathcal{V}_{kl} \Gamma_{kl}(\mathbf{Y}),$$

$$S_{E_k} = \sum_{l \neq k} \psi_{kl}(\mathbf{Y}) + \sum_{l \neq k} \mathcal{V}_{kl} D_{kl}(\mathbf{Y}) + \sum_{l \neq k} H_{kl} \Gamma_{kl}(\mathbf{Y}),$$

with the following requirements :

$$\mathcal{V}_{kl} = (u_k + u_l)/2, \quad H_{kl} = u_k u_l / 2,$$

$$\begin{cases} \Gamma_{kl}(T_l \mu_k - T_k \mu_l) \geq 0, \\ D_{kl}(u_l - u_k) \geq 0, \\ \psi_{kl}(T_l - T_k) \geq 0, \end{cases} \quad \text{and} \quad \begin{pmatrix} \Phi_s \\ \Phi_l \end{pmatrix} = \mathbb{D} \begin{pmatrix} p_s - (p_v + p_g) \\ p_l - (p_v + p_g) \end{pmatrix}, \quad \mathbb{D} \text{ symmetric positive definite.}$$

any regular solution of the system complies with the entropy inequality :

$$\partial_t \eta(\mathbf{Y}) + \partial_x f_\eta(\mathbf{Y}) \geq 0, \quad \text{with} \quad \eta = \sum_{k \in \mathbb{K}} m_k s_k \; ; \; f_\eta = \sum_{k \in \mathbb{K}} m_k s_k u_k.$$

Properties of the model :

- The system is hyperbolic with real eigenvalues u_k, $u_k \pm c_k$, $k \in \mathbb{K}$. Resonance occurs if :

$$(u_l - u_s)^2 - c_l^2 = 0, \text{ or } (u_v - u_s)^2 - c_v^2 = 0, \text{ or } (u_g - u_s)^2 - c_g^2 = 0.$$

- The coupling wave associated with $v_I = u_s$ is linearly degenerated and admits the following Riemann invariants :

$$\begin{array}{l} u_s, \quad s_k, \quad m_k(u_k - u_s), \\ \epsilon_k + p_k/\rho_k + (u_k - u_s)^2/2, \\ \sum_{k \in \mathbb{K}} \alpha_k p_k + \sum_{k \in \mathbb{K} \setminus \{s\}} m_k (u_k - u_s)^2. \end{array} \qquad \text{where } k \in \mathbb{K} \setminus \{s\};$$

- Unique jump conditions hold field by field.

CONCLUSIONS AND PERSPECTIVES

Symmetrization : The proof can be directly adapted to the four-field case from [4].

Relaxation effects : Inner pressure-velocity relaxation effects of the model have to be studied i.e. the time evolution of the relative velocities $\Delta u_{ks} = u_k - u_s$ and pressures $\Delta p_{ks} = p_k - p_s$, with $k \in \mathbb{K} \setminus \{s\}$ without convection. Complex behavior can already occur in simpler models :

- when focusing on Baer-Nunziato type system, a threshold effect is observed in the non-barotropic case; a few initial conditions do not enable the pressure relaxation towards equilibrium.
- with a three-field model with three immiscible phases, the two relative pressures decrease but not uniformly, which leads in practice to pressure oscillations [1].

Numerical simulation : Some difficulties are encountered for the numerical simulation of the model :

- Relaxation effects : pressure relaxation terms not only require physically relevant time scales [2], but also suitable algorithms in order to obtain numerical approximation in the physical domain .
- Convection effects : this kind of system requires accurate numerical schemes such as the one introduced in [7].

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