# Non relativistic SUSY in variants of the planar Lévy-Leblond equation

(Exotic symmetries and spacetimes, Tours, November 2022)

László Palla

Institute for Theoretical Physics Eötvös University

based on
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My encounters with Peter

- KÖMAL math. competitions    undergraduate years

- monopole era mid'70 - mid'80

- Tours 1994-96

2+1 dim. Chern-Simons collaboration with C. Duval

several visits to Tours

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# Outline

- Introduction
- The free planar LLE and its bosonic symmetries
- Supercharge candidates and the weak identification
- $N = 2$ SUSY for the free LLE
- SUSY for the gauged LLE
- SUSY for the Chern-Simons coupled LLE
- Summary

## Non relativistic SUSY in planar Lévy-Leblond equation

non rel. SUSY: extend Schrödinger symmetry by anticommuting generators
(Duval, Horváthy) in $d > 2$ unique $N = 1$ extension
in $d = 2$ two $N = 1$ combining into $N = 2$

Lévy-Leblond equation (LLE): in $3+1$
"non relativistic Dirac eq." for $s = \frac{1}{2}$ particle $\begin{pmatrix} -i\vec{\sigma}\vec{\partial} & -2im \\ \partial_t & i\vec{\sigma}\vec{\partial} \end{pmatrix} \begin{pmatrix} \Phi \\ \chi \end{pmatrix} = 0$
its square Pauli equation $N = 1$ extension (Aizawa et al.)
here planar LLE its Schrödinger symmetry known (Duval, Horváthy, Palla)

describe non rel. $2+1$ space $R$ by Kaluza-Klein reduction:
$(M, g)$ $3+1$ dim. Lorentz $\xi$ cov. const. lightlike $R$ quotient of $M$
coordinates on $M$ $(t, x^i, s)$ $i = 1, 2$ $\xi \equiv \partial_s$ metric $\sum (dx^i)^2 + 2dtds$
non rel. symmetries: higher dim. ones leaving $\xi$ invariant

Free LLE and its bosonic symmetries massles 4d Dirac eq. $\nabla\!\!\!/\psi = 0$

$$\{\gamma^\mu, \gamma^\nu\} = -2g^{\mu\nu} \quad \gamma^t = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \quad \gamma^i = \begin{pmatrix} -i\sigma^i & 0 \\ 0 & i\sigma^i \end{pmatrix} \quad i = 1, 2 \quad \gamma^s = \begin{pmatrix} 0 & -2 \\ 0 & 0 \end{pmatrix}$$

equivariance condition $\nabla_\xi \psi = im\psi$ and Ansatz $\psi = e^{ims}\begin{pmatrix} \Phi(t, x^i) \\ \chi(t, x^i) \end{pmatrix}$

$$\begin{pmatrix} -i\sigma^j\partial_j & -2im \\ \partial_t & i\sigma^j\partial_j \end{pmatrix}\begin{pmatrix} \Phi \\ \chi \end{pmatrix} = 0 \qquad \Gamma = -\frac{\sqrt{-g}}{4!}\epsilon_{\mu\nu\rho\sigma}\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma = \begin{pmatrix} -i\sigma^3 & 0 \\ 0 & i\sigma^3 \end{pmatrix}$$

$\Gamma$ splits into two for 2 component $\psi_\epsilon$ $\Gamma\psi_\epsilon = -i\epsilon\psi_\epsilon$ $\epsilon = \pm$

$$\psi_+ = e^{ims}\begin{pmatrix} \phi_+ \\ 0 \\ 0 \\ \chi_+ \end{pmatrix} \quad \begin{pmatrix} -i(\partial_1 + \epsilon i\partial_2) & -2im \\ \partial_t & i(\partial_1 - \epsilon i\partial_2) \end{pmatrix}\begin{pmatrix} \phi_\epsilon \\ \chi_\epsilon \end{pmatrix} = 0 \quad \psi_- = e^{ims}\begin{pmatrix} 0 \\ \phi_- \\ \chi_- \\ 0 \end{pmatrix}$$

in 2d there are two LLE-s (!) since $\nabla\!\!\!/\nabla\!\!\!/ = (-2im\partial_t - \partial_k\partial_k)\mathbf{1}_4$ their "squares"

$$i\partial_t\begin{pmatrix} \Phi \\ \chi \end{pmatrix} = H\begin{pmatrix} \Phi \\ \chi \end{pmatrix} \qquad H = -\frac{1}{2m}\partial_k\partial_k\mathbf{1}_4$$

bosonic symmetries: $\xi (\equiv \partial_s)$ preserving 4d conformal transformations

$$[\mathcal{B}, \nabla\!\!\!/] = \Sigma_{\mathcal{B}} \nabla\!\!\!/ \qquad \Sigma_{\mathcal{B}} = -\frac{1}{4} \nabla_\mu X^\mu_{\mathcal{B}}$$

explicitely ($\partial_t$ and $\partial_j$ are trivial)

$$d = \begin{pmatrix} 2t\partial_t + x^k\partial_k + 1 & 0 \\ 0 & 2t\partial_t + x^k\partial_k + 2 \end{pmatrix} \quad b^j = \begin{pmatrix} t\partial_j - imx^j & 0 \\ \frac{i}{2}\sigma^j & t\partial_j - imx^j \end{pmatrix}$$

$$K = \begin{pmatrix} t^2\partial_t + tx^k\partial_k + t - \frac{im}{2}r^2 & 0 \\ \frac{i}{2}\sigma^j x_j & t^2\partial_t + tx^k\partial_k + 2t - \frac{im}{2}r^2 \end{pmatrix} \qquad r^2 = x^k x^k$$

$$J = -\begin{pmatrix} x_1\partial_2 - x_2\partial_1 + \frac{i\sigma^3}{2} & 0 \\ 0 & x_1\partial_2 - x_2\partial_1 + \frac{i\sigma^3}{2} \end{pmatrix}$$

Schrödinger symmetry algebra $sch(2) = (sl_2 \oplus so(2)) \circledS h(2)$

Heisenberg algebra $h(2)$ $[i\partial_j, b^k] = m\delta^{jk} 1_4$ $J$ forms $so(2)$

$(i\partial_t, d, K)$ form $sl_2$ $[d, i\partial_t] = -2i\partial_t, \quad [i\partial_t, K] = i \cdot d, \quad [d, K] = 2K$

all $\mathcal{B}$ $[\mathcal{B}, \Gamma] = 0$ for later reference $[d, H] = -2H$

Supercharge candidates fermionic extensions $\mathcal{F}$ $\{\mathcal{F}, \not\nabla\} = \Sigma_{\mathcal{F}} \not\nabla$
(trivial sol. $\mathcal{F} = 1_4$, $\mathcal{F} = \Gamma$ generate chiral rotation) non-trivial solutions

$$\mathcal{F} = \tilde{Q} = \frac{1}{\sqrt{2m}} \begin{pmatrix} -i\epsilon_{kl}\sigma^k\partial_l & 0 \\ 0 & i\epsilon_{kl}\sigma^k\partial_l \end{pmatrix} = \frac{1}{\sqrt{2m}}\gamma^k\epsilon_{kl}\partial_l, \qquad (\Sigma_{\tilde{Q}} = 0)$$

$$\mathcal{F} = \Lambda = \begin{pmatrix} 0 & \beta \\ \alpha\partial_t & 0 \end{pmatrix} = \alpha\gamma^t\partial_t - \frac{\beta}{2}\gamma^s \qquad (\Sigma_\Lambda = 0) \quad \text{if} \quad \beta - 2im\alpha = 0$$

normalizations $\tilde{Q}\tilde{Q} = H$ $\Lambda\Lambda = \alpha\beta\partial_t 1_4$ commute with $\xi$
$\{\Gamma, \tilde{Q}\} = 0,$ $\{\Gamma, \Lambda\} = 0,$ $\{\Lambda, \tilde{Q}\} = 0$
$\tilde{Q}\psi_\epsilon/\Lambda\psi_\epsilon$ opposite chirality to $\psi_\epsilon$ $\rightarrow$ $\tilde{Q}\psi_+/\Lambda\psi_+$ solve LLE for $\psi_-$ (!)
we need both $\psi_+$ and $\psi_-$ to represent $\tilde{Q}/\Lambda$

$\tilde{Q}$ and $\Lambda$ commute with translations and rotation supercharge candidates

choose $\alpha\beta = i$ $\Lambda = \begin{pmatrix} 0 & i\sqrt{2m} \\ \frac{\partial_t}{\sqrt{2m}} & 0 \end{pmatrix}$ for **solutions** of LLE

$$\Lambda\Lambda = i\partial_t 1_4 = H = \tilde{Q}\tilde{Q}$$

for solutions of LLE ("weakly") identify $i\partial_t$ and $-\frac{1}{2m}\partial_k\partial_k$

check weak identification in bosonic algebra

$[H, i\partial_j] = 0$ $[H, J] = 0$ and $[H, b^j] = i\partial_j = [i\partial_t, b^j]$ look whether $(H, d, K)$ also form $sl_2$ algebra

$$[H, K] = \begin{pmatrix} 2t(-\frac{1}{2m}\partial_k\partial_k) + ix^k\partial_k + i & 0 \\ -\frac{i}{2m}\sigma^k\partial_k & 2t(-\frac{1}{2m}\partial_k\partial_k) + ix^k\partial_k + i \end{pmatrix} =$$

$$i \cdot d + \frac{1}{2m}\begin{pmatrix} 0 & 0 \\ -i\sigma^k\partial_k & -2mi \end{pmatrix} = i \cdot d + \frac{1}{2m}\gamma^t \not\nabla$$

last term vanishes for **sol.s** of LLE (!) **weakly** $sl_2$ algebra OK

Jacobi identity for $[\mathcal{B}, [H, K]]$ ?

## Fermionic extension of the Schrödinger symmetry

new fermionic generators: commute $\Lambda/\tilde{Q}$ with $\mathcal{B}$ start with $\Lambda$

$$[\Lambda, b^j] := Z^j, \quad j = 1, 2 \quad \{Z^j, Z^k\} = m\delta^{jk}1_4 \quad \{\Lambda, Z^j\} = i\partial_j 1_4$$

$[\Lambda, K] := \widehat{S}$ $\widehat{S}\widehat{S} = iK$ conformal supercharge with $\tilde{Q}$ analogously

$$[\tilde{Q}, b^j] := \tilde{Z}^j \quad \tilde{Z}^j = -\epsilon_{jk} Z^k \quad \{\tilde{Q}, \tilde{Z}^j\} = i\partial_j 1_4 \quad \{\tilde{Z}^j, \tilde{Z}^k\} = m\delta^{jk} 1_4$$

$[\tilde{Q}, K] := \tilde{S} \qquad \tilde{S}\tilde{S} = iK + \frac{t}{2m}\gamma^t \not\nabla$ weakly also conf. supercharge

$(\Lambda, Z^j, \widehat{S})$ or $(\tilde{Q}, \tilde{Z}^j, \tilde{S})$ give two $N = 1$ extensions of $sch(2)$

Q.: can we have them simultaneously? (anti)commutators between different sets

$\{\Lambda, \tilde{S}\} = iJ + Y \qquad Y = \frac{-i}{2m}\begin{pmatrix} im\sigma^3 & 0 \\ \epsilon_{pq}\sigma^p\partial_q & im\sigma^3 \end{pmatrix}$ $Y$ new bosonic generator ?

$[\mathcal{B}, Y] = 0 \qquad \mathcal{B} = i\partial_t,\ i\partial_j,\ b^j,\ d,\ K,\ J$

$[\Lambda, Y] = i\tilde{Q} \qquad [\tilde{Q}, Y] = -i\Lambda - \frac{i}{\sqrt{2m}}\not\nabla \qquad Y: \quad \Lambda \leftrightarrow \tilde{Q}$ weakly

similarly $Y: \quad Z^j \leftrightarrow \tilde{Z}^j \qquad Y: \quad \widehat{S} \leftrightarrow \tilde{S} \qquad Y$ can be included

$$\mathcal{B} = i\partial_t,\ i\partial_j,\ b^j,\ d,\ K,\ J,\ Y \qquad \mathcal{F} = \Lambda,\ Z^j,\ \widehat{S},\ \tilde{Q},\ \tilde{Z}^j,\ \tilde{S}$$

$N = 2$ extension of Schrödinger symmetry described by C. Duval and Peter

checked generalized Jacobi identities hold weakly

Remarks, discussion

- $[\mathcal{B}, \Gamma] = 0, \qquad \{\mathcal{F}, \Gamma\} = 0$ we need both $\psi_+$ and $\psi_-$

- this $N = 2$ extension same as the one in planar Pauli eq. (Duval, Horváthy)

generators are related non trivially $Q_{\rm 2d} \sim \sigma^i \partial_i$ "twisted" $Q_{\rm 2d} \sim \epsilon_{ij} \sigma^i \partial_j$

$\tilde{Q}$ 4d version of the "twisted" $Q_{\rm 2d}$ but $\Lambda$ is tricky 4d generalization of $Q_{\rm 2d}$

$Q = \frac{1}{\sqrt{2m}} \begin{pmatrix} -i\sigma^k \partial_k & 0 \\ 0 & -i\sigma^k \partial_k \end{pmatrix}$ **commutes** with the Dirac op. $[Q, \not\nabla] = 0$

$$Q = \Lambda + \frac{1}{\sqrt{2m}} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \not\nabla$$ weakly they coincide

- in $d$ (space) $+ 1$ (time) dim. just one LLE when $d$ odd and **two** when $d$ even

free Dirac eq. on $M_{D=d+2}$ Dirac spinor $2^{[D/2]}$ dim.

irred. for $D/d$ odd, no chirality $\Gamma$

splits into two $2^{[d/2]}$ dim. Weyl for $D/d$ even, $\exists \quad \Gamma$

gen. of $\Lambda$ for any $d$ for even $d$ $\{\Lambda, \Gamma\} = 0$ $N = 1$ extension for any $d$

$N = 2$ extension for $d = 2$ only

## SUSY in gauged LLE

reduction from gauged Dirac eq. $\not{D}\psi = 0$ $D_\mu = \nabla_\mu - iea_\mu$ with $a_\mu(x)$
$U(1)$ gauge field $f_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu$ closed 2 form no e.o.m.

$f_{\mu\nu}\xi^\nu = 0$ guarantees $f_{\mu\nu}$ lift of closed $F_{\alpha\beta}$ $\alpha\ \beta = t, 1, 2$
$a_\mu$ lift of vector potential $A_\alpha = (A_t, A_j)$ for $F_{\alpha\beta}$
gauged LLE by substitution $\partial_j \to D_j \equiv \partial_j - ieA_j$ $\partial_t \to D_t \equiv \partial_j - ieA_t$

$$\begin{pmatrix} -i\sigma^j D_j & -2im \\ D_t & i\sigma^j D_j \end{pmatrix} \begin{pmatrix} \Phi \\ \chi \end{pmatrix} = 0$$

crutial difference to free LLE $\partial_j\ \partial_t$ commute $D_j\ D_t$ do not

consider s.p.m. gauge fields $A_t = 0$ $\partial_t A_j = 0$ $(F_{tk} \equiv 0$ $D_t = \partial_t)$
"square" of gauged LLE

$$i\partial_t \begin{pmatrix} \Phi \\ \chi \end{pmatrix} = H_e \begin{pmatrix} \Phi \\ \chi \end{pmatrix} \qquad H_e = -\frac{1}{2m} \begin{pmatrix} D_j^2 + e\sigma^3\epsilon_{kl}\partial_k A_l & 0 \\ 0 & D_j^2 + e\sigma^3\epsilon_{kl}\partial_k A_l \end{pmatrix}$$

bosonic symmetries of gauged LLE [DHP] more complicated

$[\mathcal{B}, \not{D}] = -ie\gamma^\mu (L_{X_\mathcal{B}} a)_\mu - \frac{1}{4}\nabla_\mu X^\mu_\mathcal{B} \not{D}$ $(L_{X_\mathcal{B}} a)_\mu$ Lie derivative of $a_\mu$

if $\psi$ solves gauged LLE, then $\mathcal{B}\psi$ solves rather $\not{D}(\mathcal{B}\psi) - ie\gamma^\mu (L_{X_\mathcal{B}} a)_\mu \psi = 0$

check fermionic extension start with $(\Lambda,\ Z^j,\ \widehat{S})$

$\{\Lambda, \not{D}\} = 0$ if $A_t = 0$ $\partial_t A_j = 0$ i.e. if gauge field s.p.m.

using simple identities and case-by case analysis: this $N = 1$ extension survives

story of $(\tilde{Q},\ \tilde{Z}^j,\ \tilde{S})$ is different $\{\tilde{Q}, \not{D}\} \neq 0$ and $\tilde{Q}\tilde{Q} = H \neq H_e$

can define $\tilde{Q}_e = \frac{1}{\sqrt{2m}} \begin{pmatrix} -i\epsilon_{kl}\sigma^k D_l & 0 \\ 0 & i\epsilon_{kl}\sigma^k D_l \end{pmatrix}$ $\{\tilde{Q}_e, \not{D}\} = 0$ for s.p.m.

but cannot be supercharge as

$$[i\partial_j, \tilde{Q}_e] = \frac{e}{\sqrt{2m}} \gamma^k \epsilon_{kl} \partial_j A_l \neq 0$$

SUSY in LLE coupled to Chern-Simons th. (CS) 4d form of CS

$f_{\mu\nu} = \frac{e}{\kappa}\sqrt{-g}\epsilon_{\mu\nu\rho\sigma}\xi^\rho j^\sigma$ $\quad f_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu$ $\quad j^\mu$ 4d current

$f_{\mu\nu}$ lift of a closed $F_{\alpha\beta}$ $\quad j^\mu$ projects to a 3 current $J^\alpha = (\rho, J^k)$ $\alpha = t, 1, 2$

CS descends as $F_{\alpha\beta} = -\frac{e}{\kappa}\sqrt{-g}\varepsilon_{\alpha\beta\gamma}J^\gamma$ with our metric

$$B \equiv \epsilon_{ij}\partial_i A_j = -\frac{e}{\kappa}\rho \qquad E^j \equiv F_{jt} = \frac{e}{\kappa}\epsilon_{jk}J^k \qquad \partial_t J^t + \partial_j J^j = 0$$

couple CS to LLE
$$\begin{pmatrix} -i\sigma^j D_j & -2im \\ D_t & i\sigma^j D_j \end{pmatrix} \begin{pmatrix} \Phi \\ \chi \end{pmatrix} = 0$$

by identifying CS curent and natural LLE current

Dirac adjoint $\quad \bar\psi = \psi^\dagger G \quad \bar\gamma_\mu := G^{-1}\gamma_\mu^\dagger G = \gamma_\mu \quad G^\dagger = G \quad G = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$$\nabla_\mu(\bar\psi\gamma^\mu\psi) = 0 \qquad j^\mu = \bar\psi\gamma^\mu\psi$$ with our Dirac matrices

$$\rho \equiv J^t = |\Phi|^2 = |\phi_+|^2 + |\phi_-|^2 \qquad J^j = i(\Phi^\dagger\sigma^j\chi - \chi^\dagger\sigma^j\Phi)$$

In [DHP] $\xi$ preserving conformal transformations act as symmetries on sol.s

what about $N = 1$ extension by $(\Lambda,\ Z^j,\ \widehat{S})$ ?

$(\Lambda,\ Z^j,\ \hat{S})$ symmetry of gauged LLE if $A_t \equiv 0$ $\partial_t A_j = 0$
here imply static sol.s with **definite chirality** spinors only

$F_{jt} \equiv 0 \longrightarrow J^j = 0$ and also $\partial_t \rho = 0 \longrightarrow \chi \equiv 0,$ $\partial_t \Phi = 0$
$\chi_+ \equiv 0$ $\chi_- \equiv 0$ and static $\phi_\pm$ satisfy

$$(D_1 + iD_2)\phi_+ = 0 \qquad (D_1 - iD_2)\phi_- = 0 \qquad \epsilon_{ij}\partial_i A_j = -\frac{e}{\kappa}(|\phi_+|^2 + |\phi_-|^2)$$

normalizable sol. when only one of $\phi_\pm \neq 0$

since $\{\Gamma, \mathcal{F}\} = 0$ under any $\mathcal{F}$

$$\begin{pmatrix} \Phi \\ \chi \end{pmatrix} = \begin{pmatrix} \phi_+ \\ 0 \\ 0 \\ 0 \end{pmatrix} \qquad \begin{pmatrix} \tilde{\Phi} \\ \tilde{\chi} \end{pmatrix} = \mathcal{F} \begin{pmatrix} \Phi \\ \chi \end{pmatrix} = \begin{pmatrix} 0 \\ \tilde{\phi}_- \\ \tilde{\chi}_- \\ 0 \end{pmatrix}$$ has opposite chirality

$\rho$ and $J^j$ stay invariant CS eq.s preserve their form under any $\mathcal{F}$
$N = 1$ extension survives

## Summary

- showed $N = 2$ extension of Schrödinger sym. for the free **planar** LLE

$\exists$ two LLE-s for $\psi_{\pm}$ we need both

extension exists **weakly** for solutions of LLE

- $N = 1$ part survives for **gauged** LLE when gauge field is s.p.m.

- the same $N = 1$ extension is a symmetry of the solution space of the coupled Chern-Simons - LLE system