

# Journées de Mathématiques Appliquées

Conférence en l'honneur de **Laurence Halpern**

20, 21 et 22 janvier 2015

Institut Henri Poincaré

11, rue Pierre et Marie Curie – Paris 5<sup>ème</sup>



## Conférenciers

Mohamed Amara  
Abderrahmane Bendali  
Yann Brenier  
Oana Ciobanu  
Georges-Henri Cottet  
Pierre Degond  
Björn Engquist  
Jean-Claude Guillot

l'algorithme Patrick Joly  
Jean-Jacques Marigo  
Frédéric Nataf  $\phi(\omega) = u^0$   
Jean-Claude Nedelec  $\frac{\partial u}{\partial n_j} = \frac{\partial u}{\partial n_j} + p_j u$   
Jeffrey Rauch  
Marc Schoenquer  $\frac{\partial u^k}{\partial y_j} = -\frac{\partial u_j}{\partial n_j} + p_j u_j^k$   
Jérémi Szeftel  
Nick Trefethen  $g^k$  converge vers la solution

**Organisateurs** : Filipa Caetano, Martin Gander, Florence Hubert, Caroline Japhet, Stéphane Labbé, Olivier Lafitte, Véronique Martin, Juliette Ryan, Kevin Santugini, Jérémie Szeftel



Aix-Marseille  
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## Mardi 20 janvier 2015

### Matin

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8h30–9h00 : Enregistrement

9h00–9h15 : M. J. GANDER, C. JAPHET - Ouverture du congrès

**Chair :** M. J. GANDER

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9h15–10h00 : J. RAUCH

The Artificial Beach ; Absorbing Boundary Layers for Water Waves

10h00–10h30 : Pause café

**Chair :** M. J. GANDER

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10h30–11h15 : J.-C. NÉDÉLEC

About some operators over a unit disc related to the Laplace equation

11h15–12h00 : N. TREFETHEN

Mathematics of the Faraday cage

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### Après-Midi

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**Chair :** O. LAFITTE

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14h30–15h15 : J.-C. GUILLOT

Mathematical Models for the Weak Interactions in Quantum Field Theory

15h15–16h00 : J. SZEFTTEL

The resolution of the bounded L2 curvature conjecture in general relativity

16h00–16h30 : Pause café

**Chair :** J. SZEFTTEL

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16h30–17h15 : Y. BRENIER

Topology-preserving diffusion of magnetic fields

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## Mercredi 21 janvier 2015

### Matin

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**Chair :** K. SANTUGINI

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9h15–10h00 : P. JOLY

Periodic topographic open wave guides : theoretical and computational aspects

10h00–10h30 : Pause café

**Chair :** C. JAPHET

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10h30–11h15 : F. NATAF

Time Reversed Absorbed Conditions (TRAC) and applications to inverse problems

11h15–12h00 : A. BENDALI

Multiple scattering by small obstacles - Improved Foldy's model

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### Après-Midi

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**Chair :** J. RYAN

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14h30–15h15 : M. SCHOENAUER

Numbers don't count

15h15–16h00 : J.-J. MARIGO

A two-scale approach for the issue of crack nucleation in fracture mechanics

16h00–16h30 : Pause café

**Chair :** V. MARTIN

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16h30–17h15 : O. A. CIOBANU

Adaptive time stepping and Schwarz Waveform Relaxation (SWR) Method for Compressible Navier–Stokes Equations

17h15–18h00 : G.-H. COTTET

Turbulent transport : the interplay of Physics, Mathematics and High Performance Computing

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18h00–19h00 : **Poster Session**

**Following poster session : cocktail from 19h00 to 21h00**

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## Poster Session

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**Marc BAKRI** : Analyse d'erreur a posteriori et raffinement auto-adaptatif pour les éléments finis de frontière en acoustique

**Zakaria BELHAJ** : The variational spline method for solving Troesch's problem

**Faycal CHAOUQUI** : Comparison of Neuman-Neuman and Optimal Schwarz Methods with Many Subdomains in one Spatial Dimensions

**Soheil HAJIAN** : A new approach for preconditioning discontinuous Galerkin discretizations

**Imen HASSAIRI** : D-solutions for doubly reflected BSDEs

**Caroline JAPHET** et **Michel KERN** : Space-time domain decomposition methods for mixed formulations of flow and transport problems in porous media

**Asma TOUMI** : Méthodes asynchrones de haute précision pour la modélisation de phénomènes multi-échelles

**Shu-Lin WU** : Wave-Ray Multigrid Method for The 1D Helmholtz Equation - A precise mathematical formulation and first analysis

**Hui ZHANG** : Factorizations, Sweeping, Source Transfer, Potentials and Schwarz : One Algorithm

## Jeudi 22 janvier 2015

### Matin

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**Chair :** F. HUBERT

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9h15–10h00 : P. DEGOND  
Collective dynamics in living and social systems  
10h00–10h30 : Pause café

**Chair :** S. LABBÉ

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10h30–11h15 : M. AMARA  
A Local Wave Tracking Strategy for Solving High-Frequency Helmholtz Problems  
11h15–12h00 : B. ENGQUIST  
On seismic imaging

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# **Conférences Plénières**

## The Artificial Beach ; Absorbing Boundary Layers for Water Waves

Jeffrey RAUCH

Reports on the construction of absorbing layers for the one dimensional linearized water wave equation. The equation is nonlocal forcing it immediately out of the realm of standard ideas. A key and simple idea is one way water wave equations related to D'Alembert's method. Joint work with Izbicki, Karni, Carney, Abgrall, and Prigge (in chronological order).

## About some operators over a unit disc related to the Laplace equation

Jean-Claude NÉDELEC

We introduce four integral operators closely related to the Laplace equation in three-dimensions on the circular unit disc. Two of them are closed to the simple layer on the disc and the other two are related to the hyper singular operator. Contrary to the case of a closed domain, these operators no longer map fractional Sobolev spaces in a dual fashion but degenerate into different subspaces depending on their extensibility by zero. We establish their variational formulations and the coercivity properties in some Sobolev spaces. They are also linked to the Laplace operator on the disc.

These results are a tentative extension to  $\mathbb{R}^3$  of previous results in  $\mathbb{R}^2$ , contains in a common work with Carlos Jerez-Hanckes that we present in the beginning of the talk. We have introduced explicit and exact variational formulations for some weakly and hyper-singular operators associated to the Log operator over an open flat slit as well as for their corresponding inverses.

## Mathematics of the Faraday cage

Nick TREFETHEN

Everybody has heard of the Faraday cage effect, in which a wire mesh does a good job of blocking electric fields. Surely the mathematics of such a famous and useful phenomenon has been long ago worked out and written up in the textbooks ?

It seems to be not so. One reason may be that that the effect is not as simple as one might expect : it depends on the wires having finite radius. Nor is it as strong as one might imagine : the shielding improves only linearly as the wire spacing decreases.

This talk will present results by Jon Chapman, Dave Hewett and myself including (a) numerical simulations, (b) a theorem proved by conformal mapping, (c) a continuous model derived by multiple scales analysis, (d) a discrete model derived by energy minimization, (e) a connection with the periodic trapezoidal rule for analytic integrands, and (f) a physical explanation.

## Mathematical models for the Weak Interactions in Quantum Field Theory

Jean-Claude GUILLOT

We consider Hamiltonians with cutoffs which are self-adjoint operators in appropriate Fock spaces with a unique ground state. A limiting Absorbion Principle is proved

## The resolution of the bounded L2 curvature conjecture in general relativity

Jérémie SZFTEL

In order to control locally a space-time which satisfies the Einstein equations, what are the minimal assumptions one should make on its curvature tensor? The bounded L2 curvature conjecture roughly asserts that one should only need L2 bounds of the curvature tensor on a given space-like hypersurface. This conjecture has its roots in the remarkable developments of the last twenty years centered around the issue of optimal well-posedness for nonlinear wave equations. In this context, a corresponding conjecture for nonlinear wave equations cannot hold, unless the nonlinearity has a very special nonlinear structure. I will present the proof of this conjecture, which sheds light on the specific null structure of the Einstein equations. This talk is intended for a general audience and will require no specific background. This is joint work with Sergiu Klainerman and Igor Rodnianski.

## Topology-preserving diffusion of magnetic fields

Yann BRENIER

In the 1990, Moffatt discussed a dissipative model of Magneto-hydrodynamics (that he called “magnetic relaxation” but could also be called “Darcy” or “Stokes” MHD), in order to get stationary solutions of the Euler equations with prescribed topology. We will discuss the corresponding PDEs and some concepts of generalized solutions related both to P.-L. Lions’ dissipative solutions to the Euler equations and to the recent approach by Ambrosio, Gigli and Savare of the heat equation in general metric spaces.

## Periodic topographic open wave guides : theoretical and computational aspects

Patrick JOLY

We consider the propagation of waves in a periodic structure that can be represented as a infinite thick graph. We show that, provided that adequate boundary conditions are satisfied, the introduction of a lineic geometric perurbation of this reference structure can create the apparition of guided waves associated to frequencies inside any band gap of the periodic medium. The proof is based on an asymptotic analysis with respect to the thickness of the graph. We also explain how to compute such waves. The method ins based on specific transparent conditions for periodic media.

## Time Reversed Absorbed Conditions (TRAC) and applications to inverse problems

Frédéric NATAF

We introduce the time reversed absorbing conditions (TRAC) in time reversal methods. They enable to “recreate the past” without knowing the source which has emitted the signals that are back-propagated. The method is very insensitive to noise in the data. Applications to coefficients reconstruction and source identification are given.

## Multiple scattering by small obstacles - Improved Foldy's model

Abderrahmanne BENDALI

Usual Foldy's model is used to approximate a multiple scattering problem involving small scatterers by monopole scatterers. Using the method of matched asymptotic expansions, with P. H. Cocquet and S. Tordeux, we have first proved that the scattered field can be approximated at any order of accuracy by multipoles. This first provides a mathematical justification for the Foldy model and next makes it possible to improve it by adding some self-interaction terms and adequately choosing the centers of phase of the effective monopoles.

## Numbers don't count

Marc SCHOENAUER

Optimisation is concerned with finding the arguments that result in the largest (or lowest) value of some objective function. However, whereas the range and scale of possible values of the objective function is often arbitrary, the performance of optimisation algorithms very often heavily depends on the chosen coordinates. Comparison-based methods can hence get an edge over value-dependent approaches in many situations. Three examples (time permitting) will illustrate such situations : stochastic black-box optimisation with surrogate models ; algorithm selection using Collaborative Filtering ; 'Programming by Feedback', or how possibly-dummy users could actually improve their pet robot's behavior.

## A two-scale approach for the issue of crack nucleation in fracture mechanic

Jean-Jacques MARIGO

It is well-accepted that Griffith-like models are appropriate for crack propagation at the scale of a structure, but inadequate for the modeling of crack nucleation in brittle materials. Arguably, finer models, where a microscopic (material) length scale plays a fundamental role, are necessary to determine the critical load and crack geometry at the onset. The consistent combined modeling and numerical simulation of crack nucleation and propagation from the material to the structural length-scale is a challenging and largely open issue.

In this talk we will consider gradient damage models and cohesive force models which contain both the concepts of critical stress and fracture energy and hence are good candidate to include in a unique setting the issues of nucleation and propagation of cracks. By considering fundamental problems as those of a thermal shock or of a notched body, we will show in particular that the nucleation of cracks predicted by such models is in fact an instability process. Theoretical arguments coupled with numerical tests will support this claim.

## Adaptive time stepping and Schwarz Waveform Relaxation (SWR) Method for Compressible Navier–Stokes Equations

Oana Alexandra CIOBANU

A space-time domain decomposition algorithm for the compressible Navier–Stokes problem has been designed, with the aim of implementing it in three dimensions, in an industrial code. We improve the SWR method adding an adaptive time stepping inside each time window and compare its performances for different second order explicit/implicit algorithms, on complexe cases.

## Turbulent transport : the interplay of Physics, Mathematics and High Performance Computing

Georges-Henry COTTET

In this talk we show how numerical models and algorithms can be tailored to HPC platforms to address multiscale problems occurring in turbulent transport.

## Collective dynamics in living and social systems

Pierre DEGOND

Collective dynamics refers to the ability of motile agents to achieve large-scale coordination through purely local interactions. Systems exhibiting collective dynamics can be found in the living world (motor proteins, cells, birds, pedestrians) as well as in the social world (opinion, wealth). Collective dynamics challenges the existing theories relating microscopic to macroscopic dynamics. In this talk, we will review some of these mathematical challenges.

## A Local Wave Tracking Strategy for Solving High-Frequency Helmholtz Problems

Mohamed AMARA

The presentation deals about a procedure for selecting basis function orientation to improve the efficiency of solution methodologies that employ local plane-wave approximations. The proposed adaptive approach consists of a local wave tracking strategy. Each plane-wave basis set within considered elements of the mesh partition is individually or collectively rotated to best align one function of the set with the local propagation direction of the field.

Systematic determination of the approximated local direction of the field inside the computational domain is formulated as a minimization problem.

To illustrate the salient features and evaluate the performance of the proposed wave tracking approach, error estimates as well as numerical results are presented for the case of a least-squares method (LSM) using plane-wave based approach. The numerical results obtained for the case of a two-dimensional rigid scattering problem indicate that (a) convergence was achievable to a prescribed level of accuracy, even upon initial application of the tracking wave strategy outside the pre-asymptotic convergence region, (b) the proposed approach reduced the size of the resulting system by up to two orders of magnitude, depending on the frequency range, with respect to the size of the standard LSM system.

## On seismic imaging

Björn ENGQUIST

There are several computational steps in seismic imaging. We will discuss a few, including algorithms for fast high frequency wave propagation and the application of Wasserstein metric to full waveform inversion.

# **Présentations Murales**

## Analyse d'erreur a posteriori et raffinement auto-adaptatif pour les éléments finis de frontière en acoustique

Marc BAKRI

Nous présenterons plusieurs indicateurs d'erreur /a posteriori /adaptés aux méthodes d'éléments finis utilisées pour discrétiser les équations intégrales en acoustique 2D. En particulier, nous introduirons une nouvelle classe d'estimateurs fiables et efficaces dont la construction est basée sur une nouvelle technique de localisation des normes de Sobolev fractionnaires.

## The variational spline method for solving Troesch's problem

Zakaria BELHAJ

Troesch's problem arises in the investigation of the confinement of a plasma column by radiation pressure. Recently, this problem has been studied extensively. We present a variational approximation method for solving Troesch's problem. The existence and the uniqueness of this problem are shown. Moreover, we construct a sequence of solutions of the problem from the number of knots in the partition of the domain. Such sequence convergence to the exact solution of the problem. Finally, we analyze some numerical and graphical examples in order to show the efficiency of our method.

## Comparison of Neuman-Neuman and Optimal Schwarz Methods with Many Subdomains in one Spatial Dimensions

Faycal CHAOUQUI

Optimal Schwarz methods and Neuman-Neuman methods have for two subdomains both the interesting property that they can lead to nil-potent iteration matrices. We study in this poster if this property can also be obtained for the case of a strip decomposition into many subdomains. We show that only the optimal Schwarz method can lead in this case to a nil-potent iteration matrix, and that there are various choices in the transmission conditions that lead to nil-potent matrices of different degrees.

## A new approach for preconditioning discontinuous Galerkin discretizations

Soheil HAJIAN

Domain decomposition preconditioners and in particular the additive Schwarz method are favorite preconditioners for classical finite element methods (FEM). There is a huge effort in designing similar preconditioners for discontinuous Galerkin (DG) discretizations. It has been shown that additive Schwarz methods use different mechanisms for convergence when applied to a DG discretization compared to the classical FEM. More precisely, additive Schwarz methods, when applied to DG, use a non-overlapping Robin transmission condition for the communication between subdomains. This is exactly the same transmission condition that optimized Schwarz methods (OSM) use to obtain fast convergence. In this poster we present an OSM preconditioner for a particular DG discretization along with theoretical convergence estimates.

## D-solutions for doubly reflected BSDEs

Imen HASSAIRI

We are concerned with the problem of existence and uniqueness of a solution in class D for the backward stochastic differential equations (BSDEs for short) with two continuous reflecting barriers which are completely separated. We consider that the data are  $L_p$ -integrable with  $p = 1$ .

## Space-time domain decomposition methods for mixed formulations of flow and transport problems in porous media

Caroline JAPHET et Michel KERN

Flow and transport problems in porous media are well-known for their high computational cost. In the simulation of an underground nuclear waste disposal site, one has to work with extremely different length and time scales, and highly variable coefficients while satisfying strict accuracy requirements. One strategy for tackling these difficulties is to apply a non-overlapping domain decomposition method which allows local adaptation in both space and time and makes possible the use of parallel algorithms. In this work we present two approaches, one using a time-dependent Steklov-Poincaré operator, another using the optimized Schwarz waveform relaxation for solving a time-dependent advection-diffusion problem in a mixed formulation. We show numerical experiments for various test cases, both academic and more realistic prototypes for nuclear waste disposal simulation, to investigate and compare the behavior of the two methods.

## Méthodes asynchrones de haute précision pour la modélisation de phénomènes multi-échelles

Asma TOUMI

La simulation numérique est de systèmes physiques multi-échelles est souvent synonyme de calculs coûteux et particulièrement longs. En effet, dans les méthodes classiques d'intégration temporelle, le pas de temps local le plus faible est souvent limitant pour le pas de temps global d'intégration. Notre étude porte sur un schéma asynchrone permettant de lever cette limitation. Ce formalisme impose alors de revoir les algorithmes habituellement utilisés pour monter en ordre d'approximation.

## Wave-Ray Multigrid Method for The 1D Helmholtz Equation A precise mathematical formulation and first analysis

Shu-Lin WU

We try to analyze the convergence properties of the wave-ray multigrid method for the Helmholtz equation in the 1D case. We present the details of the method and perform a local Fourier analysis for the convergence behavior. This preliminary study shows no remarkable evidence of advantages by using the wave-ray idea.

## Factorizations, Sweeping, Source Transfer, Potentials and Schwarz : One Algorithm

Hui ZHANG

Many of the modern iterative algorithms for the Helmholtz (or a more general PDE) operator have common ingredients. We show that all these algorithms can be understood in the framework of optimized Schwarz methods. They only differ in the particular choice on how to approximate the optimal transmission condition which contains a Dirichlet to Neumann operator, in the choice of the subdomain configuration, and in details of the implementation.