Continuous and Discrete Lagrangian Systems with Set-valued Controllers

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A joint work with S. Adly and B. Brogliato.

Abstract

The aim of this talk is to study the well-posedness, stability and robustness of a class of nonlinear Lagrangian dynamical systems with a multivalued controller of the form:

$$M(q(t))\ddot{q}(t)+C(q(t),\dot{q}(t))\dot{q}(t)+\nabla\mathcal{V}(q(t))+F(t,q(t),\dot{q}(t)) \in -\partial\Phi(\dot{q}(t)) \quad (1)$$

for a.e. $t \geq t_0$ where $t_0 \in \mathbb{R}$ is fixed, $\Phi : \mathrm{dom}(\Phi) = \mathbb{R}^n \to \mathbb{R}$ is a convex function, $\mathcal{V} \in C^1(\mathbb{R}^n;\mathbb{R})$, $F : \mathbb{R} \times \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$ is continuous in t, uniformly locally Lipschitz in x_1, x_2, the matrices $M(q), C(q,\dot{q}) \in \mathbb{R}^{n\times n}$, and $\partial\Phi$ stands for the convex subdifferential of Φ. The model in (1) appears widely in many scientific fields such as physics, economics, biology, electrical engineering, and especially in unilateral mechanics. The vector q represents the generalized coordinates, n is the number degrees of freedom, $M(q)$ is the inertia matrix, $C(q,\dot{q})$ is the centripetal - Coriolis matrix. The function $F(t,q(t),\dot{q}(t))$ represents a perturbation force which is usually bounded by a constant. Historically, numerous articles deal with the case when M and C are constant matrices, but the study of (1) is very limited.

In addition, the piecewise linear approximate function constructed by the following implicit time-discretized scheme:

$$\begin{cases} M(Q^k)\frac{\dot{Q}^{k+1}-\dot{Q}^k}{h_k} + C(Q^k,\dot{Q}^k)\dot{Q}^{k+1} + \nabla\mathcal{V}(Q^k) + F(t_k,Q^k,\dot{Q}^k) \in -\partial\Phi(\dot{Q}^{k+1}), \\ Q^{k+1} = Q^k + h_k\dot{Q}^k. \end{cases} \quad (1)$$

is proved to converge to a solution of (1) with order $1/2$.

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References

[1] S. ADLY, B. BROGLIATO, B. K. LE, *"Well-posedness, Robustness and Stability Analysis of a Set- Valued Controller for Lagrangian Systems"*, SIAM J. Control Optim., 51(2), 1592–1614, 2013.

[2] S. ADLY, B. BROGLIATO, B. K. LE, *" Implicit Euler Time-Discretization of a Class of Lagrangian Systems with Set-Valued Controller"*, submitted.