

The Ising Model, the Saga of the Critical Exponents

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The Ising model, being one of the simplest statistical systems, the properties of its phase transition have been studied very early. Of particular interest are its critical exponents.

The first guess was obtained from the mean-field approximation. However, starting with Onsager, the exact values in two dimensions could be calculated and were found non-mean-field like.

In three dimensions, using various mathematical techniques, approximate values were extracted from the high-temperature expansion. The first calculated values were somewhat biased by not taking into account possible confluent singularities at the critical temperature. The breakthrough came from the renormalization group (RG) method (Wilson). It showed the existence of confluent singularities and led to improved exponent estimates. Moreover, soon, as a solution to the RG equations, and quantum field techniques, exponents could be calculated as $\epsilon=4-D$ expansions but also as perturbative expansions in the interaction strength. It took some time to generate a long enough series. Moreover, in both cases, large order estimates showed that the series was always divergent, in the mathematical sense. Original summation methods based on Borel transformation and conformal mapping could be found that led to the first precise exponent estimates.

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