A discrete time Dynamic Programming approach on a tree structure for finite horizon optimal control problems

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Abstract

The classical Dynamic Programming (DP) approach to optimal control problems is based on the characterization of the value function as the unique viscosity solution of a Hamilton-Jacobi-Bellman (HJB) equation. The DP scheme for the numerical approximation of viscosity solutions of those equations is typically based on a time discretization which is projected on a fixed space triangulation of the numerical domain. The time discretization can be done by a one-step scheme for the dynamics and the projection on the grid typically uses a polynomial interpolation. This approach, which allows to get information on optimal controls in feedback form, works in any dimension although its practical application has been limited to rather low dimensional problems. Several methods have been proposed to mitigate the curse of dimensionality of DP schemes, e.g. static and dynamic domain decomposition, fast-marching and fast-sweeping methods and discrete representation formulas.

We will discuss a new approach for finite horizon optimal control problems where we compute the value function on a tree structure constructed directly by the time discrete dynamics and we do not use a space discretization to solve the HJB equation. This allows to drop the cost of space interpolation, moreover the tree will guarantee a perfect matching with the discrete dynamics. We prove convergence and a-priori error estimates. In the simple case, we discretize the dynamics with an Euler scheme and we will prove first order convergence to the value function in the framework of viscosity solutions. We will also discuss how this approach can be extended to high-order schemes, show some examples of second order approximation schemes and applications to the control of evolutive PDEs.

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