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A Local Construction of Stable Motivic Homotopy Theory (3/3)

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V. Voevodsky [6] invented the category of framed correspondences with the hope to give a new construction of stable motivic homotopy theory SH(k) which will be more friendly for computational purposes. Joint with G. Garkusha we used framed correspondences to develop the theory of framed motives in [4]. This theory led us in [5] to a genuinely local construction of SH(k). In particular, we get rid of motivic equivalences completely.

In my lectures I will recall the definition of framed correspondences and describe the genuinely local model for SH(k) (assuming that the base field k is infinite and perfect). I will also discuss several applications. Let Fr(Y,X) be the pointed set of stable framed correspondences between smooth algebraic varieties Y and X. For the first two applications I choose k = ℂ for simplicity. For further two applications k is any infinite and perfect field.

(1) The simplicial space Fr(□alg,S^1) has the homotopy type of the topological space Ω∞Σ∞(S^1_top). So the topological space Ω^∞S1Σ^∞*S1(S^1_top) is recovered as the simplicial set Fr(□alg,S^1), which is described in terms of algebraic varieties only. This is one of the computational miracles of framed correspondences.*

(2) The assignment X □ π(Fr(□alg,X□S^1)) is a homology theory on complex algebraic varieties. Moreover, this homology theory regarded with ℤ/n-coefficients coincides with the stable homotopies X □ π ^S(X+^S^1_top;ℤ/n) with ℤ/n-coefficients.

The latter result is an extension of the celebrated Suslin–Voevodsky theorem on motivic homology of weight zero to the stable motivic homotopy context.

(3) Another application of the theory is as follows. It turns out that π^s_0,0(X+) = H0(ℤF(□,X)), where (ℤF(□,X)) is the chain complex of stable linear framed correspondences introduced in [4]. For X = G_m^^n this homology group was computed by A. Neshitov as the nth Milnor–Witt group K_n^MW (k) of the base field k recovering the celebrated theorem of Morel.

(4) As a consequence of the theory of framed motives, the canonical morphism of motivic spaces can : C_Fr(X) → Ω^∞ℙ^1 Σ^∞_ℙ^1 (X+) is Nisnich locally a group completion for any smooth simplicial scheme X. In particular, if CFr(X) is Nisnevich locally connected, then the morphism can is a Nisnevich local weak equivalence. Thus in this case C_Fr(X) is an infinite motivic loop space and π_n(C_FR(X)(K)) = π^A1_n,0 (Σ^∞_ℙ^1 (X+))(K).

In my lectures I will adhere to the following references:

[1] A. Ananyevskiy, G. Garkusha, I. Panin, Cancellation theorem for framed motives of algebraic varieties, arXiv:1601.06642

[2] G. Garkusha, A. Neshitov, I. Panin, Framed motives of relative motivic spheres, arXiv:1604.02732v3.

[3] G. Garkusha, I. Panin, Homotopy invariant presheaves with framed transfers, Cambridge J. Math. 8(1) (2020), 1-94.

[4] G. Garkusha, I. Panin, Framed motives of algebraic varieties (after V. Voevodsky), J. Amer. Math. Soc., to appear.

[5] G. Garkusha, I. Panin, The triangulated categories of framed bispectra and framed motives, arXiv:1809.08006.

[6] V. Voevodsky, Notes on framed correspondences, unpublished, 2001, www.math.ias.edu/vladimir/publications

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