Mathieu Moonshine: Symmetry Surfing and Quarter BPS states at the Kummer Point

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Integrability, Anomalies and Quantum Field Theory

In honour of Samson Shatashvili @ 60

Paris, 11 February 2020

1987-89

Eguchi-T (N=4 characters)

Eguchi, Ooguri, T & Yang (conformal field theoretic elliptic genus of K3)

2010

Eguchi, Ooguri & Tachikawa (Mathieu Moonshine observation)

2010

T & Wendland

PLAN

1. Elliptic genus of K3, 1/4 BPS states and Mathieu Moonshine
2. Generic and non generic K3 theories
3. $\mathbb{Z}_2$-orbifold CFTs: a quest for a generic space of states
4. Refined elliptic genera

Work done in collaboration with Katrin Wendland (Freiburg)

1. *SU(2) channels the cancellation of K3 BPS states*, to be submitted to arXiv (July 2019)
2. *A twist in the M24 moonshine story*, arXiv:1303.3221 [hep-th]; Confluentes Mathematici 7(1) (2015) 83-113
3. *Symmetry surfing the moduli space of Kummer K3s*, arXiv:1303.2931 [hep-th] (String Maths proceedings (2015))
4. *Overarching finite symmetry group of Kummer surfaces in the Mathieu group M24*, arXiv:1107.3834 [hep-th]; JHEP08 (2013)125

1.1 2d Superconformal Field Theories (SCFT)

2d Euclidean SCFT (unitary)

- $N = (2,2)$ worldsheet supersymmetry and spacetime supersymmetry at central charge $(c, \bar{c} = c)$, $c = 3D, D \in \mathbb{N}$
- space of states: $\mathbb{H} := \mathbb{H}^{\mathrm{NS}} \oplus \mathbb{H}^{\mathrm{R}}$ unitary $\mathbb{C}$-vector space
- $J_0,\ \bar{J}_0,\ T_0 = L_0 - \frac{1}{2}J_0,\ \bar{T}_0 = \bar{L}_0 - \frac{1}{2}\bar{J}_0$ commuting linear operators, self-adjoint, diagonalisable with $T_0 \geq 0, \bar{T}_0 \geq 0$
- $\mathrm{spec}(J_0), \mathrm{spec}(\bar{J}_0) \in \frac{c}{2} + \mathbb{Z}$ on $\mathbb{H}^R$
- $\tilde{R}$ partition function with elliptic and modular properties

$$\boxed{\begin{aligned} Z_{\tilde{R}}(\tau, z) &= \mathrm{STr}_{\mathbb{H}^{\mathrm{R}}}\left(y^{J_0}\, q^{L_0 - \frac{c}{24}}\, \bar{y}^{\bar{J}_0}\, \bar{q}^{\bar{L}_0 - \frac{\bar{c}}{24}}\right) \\ &= \mathrm{Tr}_{\mathbb{H}^{\mathrm{NS}}}\left((-1)^{J_0 - \bar{J}_0}\, y^{J_0 - \frac{c}{6}}\, q^{T_0}\, \bar{y}^{\bar{J}_0 - \frac{\bar{c}}{6}}\, \bar{q}^{\bar{T}_0} \right) \\ & \qquad q = e^{2\pi i \tau}, y = e^{2\pi i z}, \quad \tau, z \in \mathbb{C}, Im(\tau) > 0 \end{aligned}}$$

$$\begin{aligned} Z_{\tilde{R}}(\tau, z+1) &= Z_{\tilde{R}}(\tau, z) \Longrightarrow \mathrm{spec}(J_0 - \bar{J}_0) \in \mathbb{Z} \\ Z_{\tilde{R}}(\tau+1, z) &= Z_{\tilde{R}}(\tau, z) \Longrightarrow \mathrm{spec}(T_0 - \bar{T}_0) \in \mathbb{Z} \end{aligned}$$

$$Z_{\tilde{R}}(\tau, z) = \exp\left(\frac{\pi i z^2\, c}{3\tau} - \frac{\pi i \bar{z}^2\, \bar{c}}{3\bar{\tau}}\right) Z_{\tilde{R}}(-\frac{1}{\tau}, \frac{z}{\tau})$$

1.2 Conformal field theoretic elliptic genus

$$\begin{aligned}\mathcal{E}_{\mathrm{CFT}}(\tau,z) &:= \mathrm{Tr}_{\mathbb{H}^{\mathrm{NS}}}\left(\,(-1)^{J_0-\bar{J}_0}\,y^{J_0-\frac{c}{6}}\,q^{T_0}\,\bar{q}^{\bar{T}_0}\,\right)\\ &= \mathrm{Tr}_{\ker\bar{T}_0}\left(\,(-1)^{J_0-\bar{J}_0}\,y^{J_0-\frac{c}{6}}\,q^{T_0}\,\right)\\ &= \mathrm{Tr}_{\mathbb{H}_0^{\mathrm{NS}}}\left(\,(-1)^{J_0-\bar{J}_0}\,y^{J_0-\frac{c}{6}}\,q^{T_0}\,\right),\end{aligned}$$

Witten 1987

$\mathbb{H}_0^{\mathrm{NS}} \hookrightarrow \ker\bar{T}_0$, generic space of states

The conformal field theoretic elliptic genus is an invariant of $N=(2,2)$ SCFTs

A K3 theory is a 2d SCFT at central charge $c=6$ whose conformal field theoretic elliptic genus given by

$$\mathcal{E}_{\mathrm{CFT}}(\tau,z) = 8\left\{\frac{\vartheta_2(\tau,z)^2}{\vartheta_2(\tau,0)} + \frac{\vartheta_3(\tau,z)^2}{\vartheta_3(\tau,0)} + \frac{\vartheta_4(\tau,z)^2}{\vartheta_4(\tau,0)}\right\}$$

Eguchi, Ooguri, Taormina & Yang 1987

Non linear sigma models on K3 surfaces are examples of K3 theories

1.3 1/4 BPS states and Mathieu Moonshine

$$\mathcal{E}_{\mathrm{CFT}}(\tau,z) = 20\chi_0^{\widetilde{\mathrm{R}}}(\tau,z) - 2\chi_{1/2}^{\widetilde{\mathrm{R}}}(\tau,z) + A(\tau)\,\widetilde{\chi}^{\widetilde{\mathrm{R}}}(\tau,z)$$ *decomposition in N=4 characters*

1/4-BPS states in K3 theory: states which saturate the BPS bound for half of the anti-holomorphic $N=4$ worldsheet supersymmetries and which are massive w.r.t. the holomorphic $N=4$ SCA

$$\mathcal{E}_{\mathrm{CFT}}(\tau,z) = (\,\text{massless} + \underbrace{\text{massive}\,)_{\text{left}} \times (\,\textit{massless}\,)_{\text{right}}}$$

1/4-BPS states encoded in $A(\tau)\widetilde{\chi}^{\widetilde{R}}(\tau,z)$

$A(\tau) = \sum_{n=1}^{\infty} A_n q^n$

n	1	2	3	4	...
A_n	90	462	1540	4554	...

dimensions of representations of M_{24} Eguchi, Ooguri, Tachikawa (2010)

$\mathcal{H}^{\mathrm{BPS}} := \bigoplus_{n=1}^{\infty} \left(H_n \otimes \mathcal{H}_n^{N=4}\right)$: space of 1/4-BPS states accounted for in $\mathcal{E}(\tau,z)$

$\mathcal{H}^{\mathrm{BPS}} \hookrightarrow \mathbb{H}_0^{\mathrm{NS}}$ generic space of states of all K3 theories

H_n: representation of the sporadic group M_{24} with dim $H_n = A_n < \infty\,, \forall n$ Gannon 2016

MATHIEU MOONSHINE

2. Generic and non generic K3 theories

NLSM on K3	pluses	minuses
Generic	*no cancellations in $\mathcal{E}_{\mathrm{CFT}}$	no known theory explicitly
Non generic	• known explicit theories e.g. $\mathbb{Z}_2$-orbifold CFTs • same spectrum of generic states for all $\mathbb{Z}_2$-orbifold CFTs	cancellations in $\mathcal{E}_{\mathrm{CFT}}$

*Wendland (2017); Song (2017)

In non-generic K3 theories:

$$\mathcal{H}^{\mathrm{BPS}} := \bigoplus_{n=1}^{\infty} (H_n \otimes \mathcal{H}_n^{N=4}) \subsetneq \widehat{\mathcal{H}}^{\mathrm{BPS}} = \bigoplus_{n=1}^{\infty} (\widehat{H}_n \otimes \mathcal{H}_n^{N=4})$$

space of all 1/4-BPS states in a K3 theory

dim $H_n = A_n$, dim $\widehat{H}_n = \widehat{A}_n$

n	1	2	3	4	...	
A_n	90	462	1540	4554	...	
$\hat{A}_n$	102	466	1944	4614	...	$\mathbb{Z}_2$-orbifold CFTs

$$\widehat{\mathcal{H}}^{\mathrm{BPS}} := \mathcal{H}^{\mathrm{BPS}} \oplus \mathcal{H}^{+}, \quad \mathcal{H}^{+} := \oplus_{n=1}^{\infty} (H_n^{+} \otimes \mathcal{H}^{N=4})$$

driver of cancellations?

3.1 $\mathbb{Z}_2$-orbifold SCFTs: N=4 free field representation

$\mathcal{C} = \mathcal{T}/\mathbb{Z}_2,\ \mathcal{T}$ a toroidal SCFT at $c = \bar{c} = 6$

Construction induced by the Kummer construction of K3 surfaces:

$T_\Lambda/\mathbb{Z}_2,\ \ T_\Lambda = \mathbb{C}^2/\Lambda,\ \ \Lambda \subset \mathbb{C}^2$

2 Dirac fermions and conjugates: $\chi^a_\pm,\ a \in \{1,2\}$

bosonic superpartners: $\jmath^a_\pm,\ a \in \{1,2\}$

N=4, c=6 FREE FIELD REPRESENTATION

$$J^3 = \frac{1}{2}\{:\chi^1_+\chi^1_-:+:\chi^2_+\chi^2_-:\} \qquad J^\pm = \pm:\chi^1_\pm\chi^2_\pm:$$

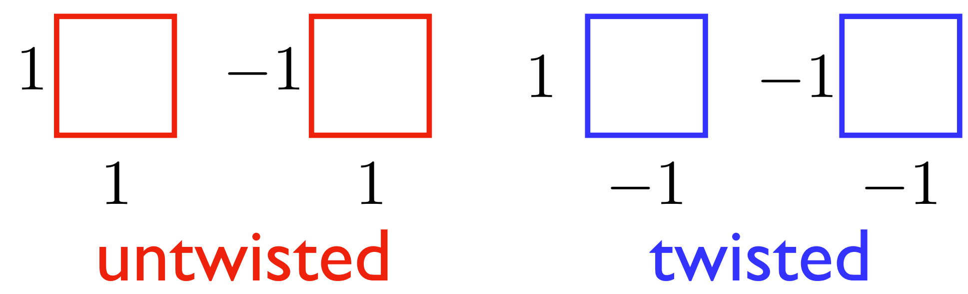
$$G^\pm = \sqrt{2}\,\{:\chi^1_\pm\, \jmath^1_\mp:+:\chi^2_\pm\, \jmath^2_\mp:\} \qquad G'^\pm = \sqrt{2}\,\{:\chi^1_\mp\, \jmath^2_\mp:-:\chi^2_\mp\, \jmath^1_\mp:\}$$

$$T = \sum_{a=1}^{2} :\jmath^a_+\jmath^a_-:+\frac{1}{2}\{\sum_{a=1}^{2} :\partial\chi^a_+\chi^a_-:+:\partial\chi^a_-\chi^a_+:\}$$

3.2 $\mathbb{Z}_2$-orbifold SCFTs: generating functions for BPS states

spectral-flow: $\quad \mathcal{E}^{\mathrm{NS}}(\tau,z) = -q^{1/4}\, y\, \mathcal{E}_{\mathrm{CFT}}(\tau, z+\frac{\tau+1}{2})$

partition function in NS sector



consider generic states: zero momentum and winding

$$\mathcal{Z}^{\mathrm{NS,generic}}(z,\bar z = -\tfrac{\bar\tau+1}{2}) = \mathcal{E}^{\mathrm{NS}}(\tau,z)\,\bar q^{\,-1/4}$$

$$\begin{aligned}
\mathcal{E}^{\mathrm{NS}}(\tau,z) &= 2U_{\ell=1/2}(\tau,z) - U_{\ell=0}(\tau,z) - 16T_{\ell=0}(\tau,z) \\
&= 2\chi_0^{\mathrm{NS}}(\tau,z) - 20\chi_{1/2}^{\mathrm{NS}}(\tau,z) - \underbrace{(-2B(\tau)+C(\tau)+16D(\tau))}_{=A(\tau):=\sum_{n=1}^{\infty} A_n q^n}\widetilde{\chi}^{\mathrm{NS}}(\tau,z)
\end{aligned}$$

$$\mathcal{H}^{\mathrm{BPS}} := \bigoplus_{n=1}^{\infty} (H_n \otimes \mathcal{H}_n^{N=4}) \hookrightarrow \mathbb{H}_0^{\mathrm{NS}}$$

3.3 $\mathbb{Z}_2$-orbifold SCFTs: 1/4 BPS states

Ansatz: $\widehat{\mathcal{H}}^{\mathrm{BPS}} = \mathcal{H}^{\mathrm{BPS}} \oplus \mathcal{H}^{+} = \mathcal{H}^{\perp} \oplus \mathcal{H}^{\mathrm{rest}} \oplus \mathcal{H}^{+}$ (space of all 1/4-BPS)

$$\mathcal{H}^{\perp} = \oplus_{n=1}^{\infty}(H_n^{\perp} \otimes \mathcal{H}_n^{N=4}), \quad \mathcal{H}^{\mathrm{rest}} = \oplus_{n=1}^{\infty}(H_n^{\mathrm{rest}} \otimes \mathcal{H}_n^{N=4})$$

$$H_n \cong H_n^{\perp} \oplus H_n^{\mathrm{rest}}$$

$$\mathcal{H}^{+} = \oplus_{n=1}^{\infty}(H_n^{+} \otimes \mathcal{H}_n^{N=4})$$

dim $H_n^{\perp} = 15D_n, \quad D_n^{\mathrm{diag}} = D_n^{\mathrm{rest}} + D_n^{+}$

dim $H_n^{\mathrm{rest}} = C_n + \underbrace{(D_n - 2B_n)}_{D_n^{\mathrm{rest}} \geq 0}$

dim $H_n^{+} = 2B_n + \underbrace{2B_n}_{D_n^{+}}$

level 1	level 2
dim $H_1^{\perp} = 15D_1 = 15 \times 6 = 90 = 45 + 45$	dim $H_2^{\perp} = 15D_2 = 15 \times 28 = 420$
dim $H_1^{\mathrm{rest}} = C_1 + (D_1^{\mathrm{diag}} - 2B_1) = 0 + (6 - 2 \times 3) = 0$	dim $H_2^{\mathrm{rest}} = C_2 + (D_2^{\mathrm{diag}} - 2B_2) = 16 + (28 - 2 \times 1) = 42$
dim $H_1^{+} = 2B_1 + 2B_1 = 2 \times 3 + 2 \times 3 = 12$	dim $H_2^{+} = 2B_2 + 2B_2 = 2 \times 1 + 2 \times 1 = 4$

3.4 $\mathbb{Z}_2$-orbifold SCFTs: 1/4-BPS states at level $n = 1$

look for a 45-dimensional vector space of states V_{45}^{CFT} that are generic to all $\mathbb{Z}_2$-orbifold CFTs on K3 and carry an action of $G_{\mathrm{octad}} := \mathbb{Z}_2^4 \rtimes A_8 \subset M_{24}$

(symmetry surfing)

Taormina & Wendland (2012,..)

$|\alpha_{\mathrm{diag}}\rangle := \frac{1}{4} \sum_{\beta \in \mathbb{F}_2^4} |\alpha_\beta\rangle$ is invariant under G_{octad}

$\mathcal{F}^{\mathrm{diag}}$: Fock space built on $|\alpha_{\mathrm{diag}}\rangle$

$\mathcal{A}$: orthogonal complement of $\mathcal{F}^{\mathrm{diag}}$ in the space of twisted ground states

$$V_{45}^{CFT} := \mathrm{span}_{\mathbb{C}}\{WA : W \in \{\chi_+^1 j_+^2 + \chi_+^2 j_+^1,\ \chi_+^1 j_+^1,\ \chi_+^2 j_+^2\}, A \in \mathcal{A}\}$$

$$V_{45}^{CFT} = W \otimes \mathcal{A} = \mathbf{3} \otimes \mathbf{15},\ \mathbf{3} \text{ of } SO(3) \text{ and } \mathbf{15} \text{ of } \mathrm{Aff}(\mathbb{F}_2^4)$$

The states in V_{45}^{CFT} and $\overline{V_{45}^{\mathrm{CFT}}}$ are generic and both enjoy an action of G_{octad}

$$\boxed{V_{45}^{\mathrm{CFT}} \oplus \overline{V_{45}^{\mathrm{CFT}}} := H_1^{\perp}}$$

Taormina, Wendland (2015)
Margolin (1993)

If deformation by marginal operator T_{diag}, all BPS states in $\mathcal{F}^{\mathrm{diag}}$ lift off

Keller & Zadeh 2019

3.5 $\mathbb{Z}_2$-orbifold SCFTs: action of octad group at levels $n > 1$

Ansatz: $\widehat{\mathcal{H}}^{\mathrm{BPS}} = \mathcal{H}^{\perp} \oplus \mathcal{H}^{\mathrm{rest}} \oplus \mathcal{H}^{+}$ (space of all 1/4-BPS)

$\mathcal{H}^{\mathrm{BPS}} \hookrightarrow \mathbb{H}_0^{\mathrm{NS}}$

states remain 1/4-BPS against marginal deformation T_{diag} built on $|\alpha_{\mathrm{diag}}\rangle$

1. G_{octad} acts on 45-dim subspace $\mathcal{H}^{\perp}$ at level $n = 1$
2. G_{octad} acts on $\mathcal{H}^{\perp} \oplus \mathcal{H}^{\mathrm{rest}}$ *at all levels* Gaberdiel, Keller, Paul 2017

   $H_n \cong H_n^{\perp} \oplus H_n^{\mathrm{rest}}$, $n \in \mathbb{N}, n > 0$ as representations of G_{octad} and $\overline{J}_0$

3. under deformations to generic $K3$ theories, all 1/4 BPS states accounted for by $U_{\ell=\frac{1}{2}}$ are lifted $(2B_n)$ Wendland 2017

Which 1/4 BPS states partner with the states accounted for by $U_{\ell=\frac{1}{2}}$ to form long representations upon deformation?

3.6 $\mathbb{Z}_2$-orbifold SCFTs: searching for generic space of states

2 Dirac fermions and conjugates: $\chi^a_\pm,\ a \in \{1,2\}$

bosonic superpartners: $\jmath^a_\pm,\ a \in \{1,2\}$

doublets of geometric SU(2)

N=4, c=6 FREE FIELD REPRESENTATION

$$J^3 = \frac{1}{2}\{:\chi^1_+\chi^1_-: + :\chi^2_+\chi^2_-:\} \qquad J^\pm = \pm :\chi^1_\pm\chi^2_\pm:$$

$$G^\pm = \sqrt{2}\,\{:\chi^1_\pm\, \jmath^1_\mp: + :\chi^2_\pm\, \jmath^2_\mp:\} \qquad G'^\pm = \sqrt{2}\,\{:\chi^1_\mp\, \jmath^2_\mp: - :\chi^2_\mp\, \jmath^1_\mp:\}$$

$$T = \sum_{a=1}^{2} :\jmath^a_+\jmath^a_-: + \frac{1}{2}\{\sum_{a=1}^{2} :\partial\chi^a_+\chi^a_-: + :\partial\chi^a_-\chi^a_+:\}$$

The geometric $SU(2)$ action commutes with the $N=4$ SCA action

3.7 $\mathbb{Z}_2$-orbifold SCFTs: excess states $\mathcal{H}^+$ and SU(2) geometric

Ansatz: $\widehat{\mathcal{H}}^{\mathrm{BPS}} = \mathcal{H}^{\perp} \oplus \mathcal{H}^{\mathrm{rest}} \oplus \mathcal{H}^{+}$ (space of all 1/4-BPS)

level n	1	2	3	4	
B_n	3, 3	1, 1	18, $3\,5^3$	15, $1\,3^3\,5$	untwisted sector $(U_{\ell=1/2})$
C_n	0	16, $3^2\,5^2$	8, $1^2\,3^2$	72, $1^2\,3^4\,5^6\,7^4$	untwisted sector $(U_{\ell=0})$
D_n	6, 3^2	28, $1^2\,3^2\,5^4$	98, $1^2\,3^6\,3^2\,5^6\,7^6$	282, $1^6\,1^2\,3^8\,3^6\,5^{16}\,5^2\,7^{10}\,9^8$	one twisted sector $(T_{\ell=0})$
A_n	90	462	1540	4554	dim H_n $(\mathcal{H}^{\mathrm{BPS}})$
$(D_n - 2B_n) + C_n$	0	42	70	324	dim H_n^{rest} $(\mathcal{H}^{\mathrm{rest}})$
$15D_n$	90	420	1470	4230	dim $H_n^{\perp}$ $(\mathcal{H}^{\perp})$
$2B_n + 2B_n$	6+6	2+2	36+36	30+30	dim H_n^{+} $(\mathcal{H}^{+})$

Data on the number of $1/4$-BPS states emerging from different sectors of $\mathbb{Z}_2$-orbifolds CFTs on K3

postulate: *all untwisted states accounted for by $U_{\ell=0}$ $(C_n\ \forall n)$ are generic*

but no means to prove this so far through the $SU(2)_{\mathrm{geom}}$ action

4.1 Refined Elliptic Genera

CFT	GEOMETRIC
2d SCFT with space of states $\mathbb{H}$	X compact Calabi-Yau D-fold
$\mathcal{E}_{\mathrm{CFT}}(\tau,z) = \mathrm{STr}_{\ker \bar{\mathrm{T}}_0}\left(y^{J_0-\frac{c}{6}}q^{T_0}\right)$ $= \mathrm{STr}_{\mathbb{H}_0^{\mathrm{NS}}}\left(y^{J_0-\frac{c}{6}}q^{T_0}\right)$	$\mathcal{E}(X;\tau,z) = y^{-\frac{D}{2}}\sum_{\ell=0}^{\infty}\sum_{m=-D}^{D}\chi(\mathcal{T}_{\ell,m})(-y)^m q^\ell$ COMPLEX ELLIPTIC GENUS Hirzebruch 1988; Witten 1988
$\mathcal{E}_{\mathrm{CFT}}^{\mathrm{Hodge}}(\tau,z,w) = \mathrm{STr}_{\ker \bar{T}_0}\left(y^{J_0-\frac{c}{6}}\, v^{\bar{J}_0-\frac{c}{6}}\, q^{T_0}\right)$ $v=e^{2\pi i w},\, w\in\mathbb{C}$ T	$\mathcal{E}^{\mathrm{Hodge}}(X;\tau,z,w)$ $= (vy)^{-\frac{D}{2}}\sum_{\ell=0}^{\infty}\sum_{m=-D}^{D}\chi^{v}(\mathcal{T}_{\ell,m})(-y)^m q^\ell,$ $\chi^{v}(\mathcal{T}_{\ell,m}) := \sum_{j=0}^{D}(-v)^j\,\dim H^j(X,\mathcal{T}_{\ell,m})$ HODGE ELLIPTIC GENUS T Kachru & Tripathi 2016
$\mathcal{E}_{\mathrm{CFT}}^{\mathrm{Hodge},0}(\tau,z,w) = \mathrm{STr}_{\mathbb{H}_0^{\mathrm{NS}}}\left(y^{J_0-\frac{c}{6}}v^{\bar{J}_0-\frac{c}{6}}\, q^{T_0}\right)$ GENERIC CFT HODGE ELLIPTIC GENUS	$\mathcal{E}^{\mathrm{Hodge},ch}(X;\tau,z,w)$ $= (vy)^{-\frac{D}{2}}\sum_{j=0}^{D}(-v)^j\mathrm{Tr}_{H^j(X,\Omega_X^{\mathrm{ch}})}\left((-y)^{J_0}q^{T_0}\right)$ Ω_X^{ch}: chiral de Rham complex, Malikov et al 1999 CHIRAL HODGE ELLIPTIC GENUS T Wendland 2017; Song 2017

4.2 Refined Elliptic Genera - K3 Theories

Assumption: the generic chiral algebra for all K3 theories is N=4 at c=6

Let $\widehat{X}$ be a K3 surface. Then, Wendland 2017

1. $\mathcal{E}^{\mathrm{Hodge,ch}}(\widehat{X};\tau,z,w)$ is independent of the complex structure
2. $\mathcal{E}^{\mathrm{Hodge},0}_{\mathrm{CFT}}(z,\tau,w) \neq \mathcal{E}^{\mathrm{Hodge}}_{\mathrm{CFT}}(z,\tau,w)$
3. $\mathcal{E}^{\mathrm{Hodge},0}_{\mathrm{CFT}}(z,\tau,w) \neq \mathcal{E}^{\mathrm{Hodge}}(\widehat{X};z,\tau,w)$
4. $\mathcal{E}^{\mathrm{Hodge},0}_{\mathrm{CFT}}(z,\tau,w) = \mathcal{E}^{\mathrm{Hodge,ch}}(\widehat{X};\tau,z,w)$ (see also Kapustin 2005, Song 2016)

   existence of a space of (Ramond ground) states common to all K3 theories that is protected by $\mathcal{E}^{\mathrm{Hodge},0}_{\mathrm{CFT}}(z,\tau,w)$; this space, when spectral flowed, yields the generic space of states $\mathbb{H}_0^{\mathrm{NS}}$ which is modelled by $H^*(\widehat{X},\Omega^{\mathrm{ch}}_{\widehat{X}})$

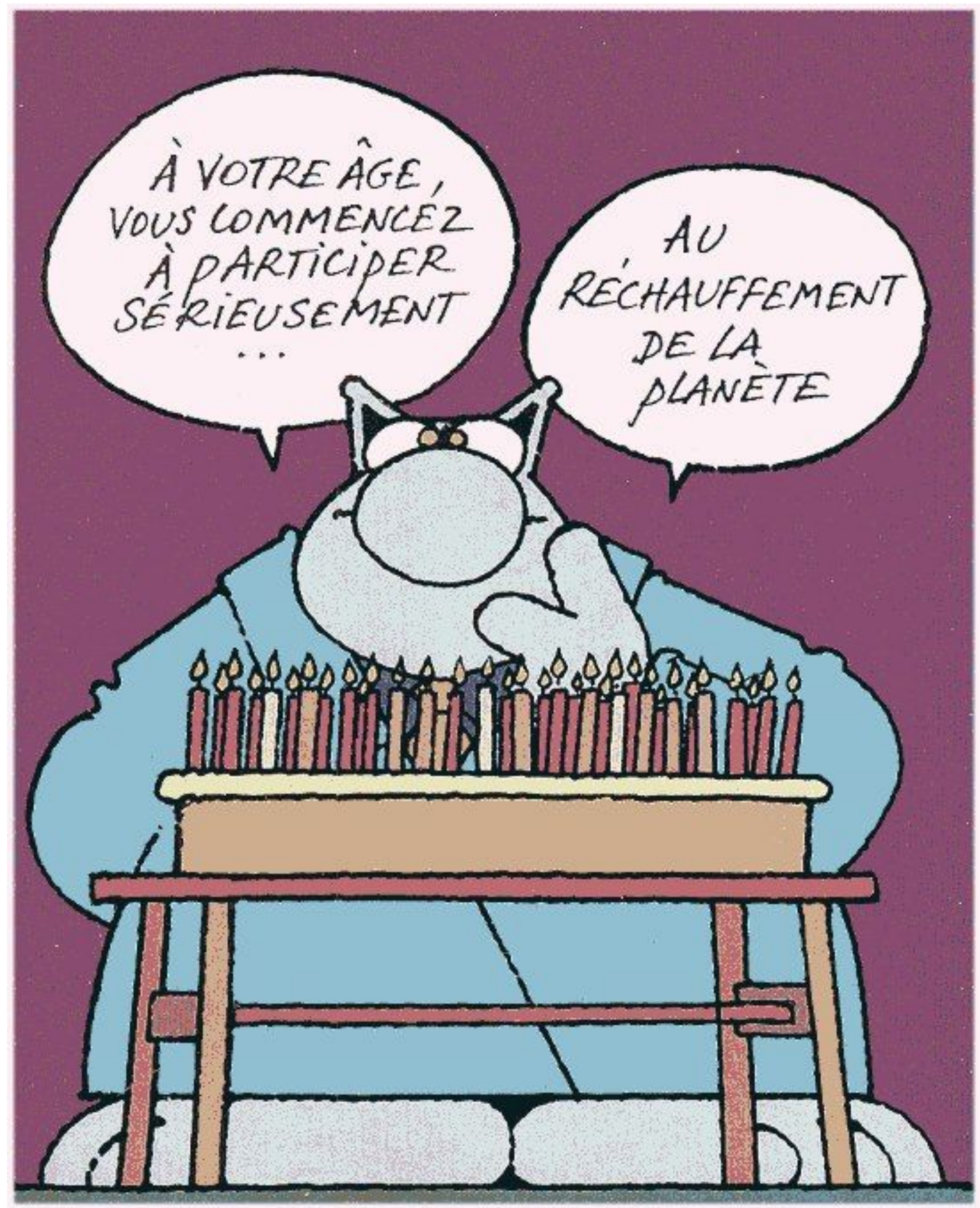
Wrap up

- The conformal field theoretic elliptic genus $\mathcal{E}_{\mathrm{CFT}}(\tau, z)$ is an invariant of $N=(2,2)$ SCFTs that counts the BPS states in any such theory with signs, according to their bosonic or fermionic nature.

$$\mathcal{E}_{\mathrm{CFT}}(\tau, z) = 20\chi_0^{\widetilde{\mathrm{R}}}(\tau, z) - 2\chi_{1/2}^{\widetilde{\mathrm{R}}}(\tau, z) + A(\tau)\widetilde{\chi}^{\tilde{\mathrm{R}}}(\tau, z)$$

- $\mathcal{E}_{\mathrm{CFT}}$ is governed by an action of M_{24}
- M_{24} acts on a generic space of states, common to all K3 theories
- the exact nature of this generic space of states is not known, and we have approached it from the perspective of $\mathbb{Z}_2$-orbifold CFTs. We have provided a strategy to earmark 1/4 BPS states that are not protected under a deformation by the most symmetric marginal operator T_{diag}. This strategy uses a global $SU(2)_{\mathrm{geom}} \subset SO(4)[N=4]$ symmetry as guiding principle.
- the strategy provides a clear proposal for which states should lift off the BPS bound under deformation at level 2, but there remains some ambiguity beyond level 2. At level 1, our strategy has been confirmed by explicit deformation calculations (Keller & Zadeh 2019)
- clearly one needs a cleaner and more powerful approach, which involves the sheaf cohomology of the chiral de Rham complex $H^*(\widehat{X}, \Omega^{\mathrm{ch}}_{\widehat{X}})$, $\widehat{X}$ a K3 surface.

Happy Birthday Samson!



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