CFT and Black Holes

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Introduction

The holographic principle or AdS/CFT correspondence states that certain QFTs, conformal field theories (or CFTs), have a completely equivalent description in terms of gravity in *AdS* space.

Objective: To deconstruct the holographic principle to learn more about gravity.

Questions:
Which theories have a holographic description?
What restrictions do physical consistency conditions impose?
Can we learn something about black holes?
. . . . . .

Holographic CFTs: Large N, or large c, CFTs with an infinite gap in the spectrum of operators for spin $s \geq 2$.

Current progress:

- The study of the crossing equation reveals the structure of a local gravity theory.
- Unitarity (causality) imply that Einstein's theory of general relativity is the only consistent description.
- Computation of feynman diagrams in gravity via CFT techniques
- $\cdots$

Introduction

Next: black holes physics ?

First step: emergence of black hole geometry from CFT correlation functions.

1. Thermal CFT correlators (canonical ensemble).
2. Correlation functions involving two "heavy" operators, $\mathcal{O}_H|0\rangle$ (microcanonical ensemble).

$$\frac{\Delta_H}{c} = \text{fixed} \qquad \text{when} \qquad c \to \infty.$$

In the dual gravitational description:

$$\mu \equiv \frac{r_H^2}{R_{AdS}^2} = \frac{M_{BH}\ell_p^3}{R_{AdS}^3} = (M_{BH}R_{AdS})\frac{\ell_p^3}{R_{AdS}^3} \sim \frac{\Delta_H}{c}$$

Introduction

The correlator

$$\langle \mathcal{O}_H(\infty)\mathcal{O}_L(1)\mathcal{O}_L(z,\bar{z})\mathcal{O}_H(0)\rangle \sim \langle \mathcal{O}_L(1)\mathcal{O}_L(z,\bar{z})\rangle_T$$

can be studied analytically in the following regimes:

- Regge/eikonal limit

$$z \to z\, e^{2\pi i}, \quad (z,\bar{z}) \to (1,1) \quad \text{with} \quad \frac{1-z}{1-\bar{z}} = \mathit{fixed},$$

[MK, Ng, Parnachev][Karlsson, MK, Parnachev, Tadic][Fitzpatrick,Huang, Li][Karlsson]

- Lightcone limit

$$\bar{z} \to 1, \quad z \leq 1$$

[MK, Ng, Parnachev][Karlsson, MK, Parnachev, Tadic][Fitzpatrick, Huang][Li]

Outline

- CFT basics
- Holographic CFTs: assumptions
- HHLL correlator
- Results
- Summary and open questions

CFT: general aspects

CFTs are ordinary quantum field theories which are invariant under the conformal group. This includes the d-dimensional Poincare group and: The dilatation D and the special conformal transformations generator K_μ.

The dilatation operator D scales the coordinates of spacetime:

$$D : x \to \lambda x, \quad \lambda \geq 0.$$

Operators of the theory, are eigenstates of D with eigenvalue Δ.

Together with the special conformal transformations K_μ and the d-dimensional Poincare group, they form a group isomorphic to $SO(d, 2)$.

CFT: general aspects

The operators of a CFT are classified by their spin s and conformal dimension Δ. The basic building blocks are *primary* operators $\mathcal{O}^s_\Delta$:

$$K_\mu \mathcal{O}^s_\Delta = 0\,.$$

All other operators are *descendants*: they can be obtained from the repeated action of the translation generator P_μ on the primary ones.

Conformal symmetry determines the form of the two- and three-point correlation functions up to a few independent parameters.

Example: (scalar operators)

$$\left\langle \mathcal{O}^i(x_1)\mathcal{O}^k(x_2)\right\rangle = \frac{\delta^{ik}}{x_{12}^{2\Delta}}, \qquad x_{ik} = x_i - x_k$$

$$\langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\mathcal{O}_3(x_3)\rangle = \frac{\lambda_{123}}{x_{12}^{\Delta_{12}} x_{23}^{\Delta_{23}} x_{13}^{\Delta_{13}}}, \qquad \Delta_{ik} = \Delta_i - \Delta_k$$

CFT: general aspects

A special example is the stress-energy tensor $T_{\mu\nu}(x)$.

2-point function:

$$\langle T_{\mu\nu}(x) T_{\rho\sigma}(0) \rangle = c \frac{\mathcal{I}_{\mu\nu,\rho\sigma}(x)}{x^{2d}}$$

3-point function:

$$\langle T^{\mu\nu}(x_3) T^{\rho\sigma}(x_2) T^{\tau\kappa}(x_1) \rangle = \frac{a f_1^{\mu\nu\rho\sigma\tau\kappa}(x) + c f_2^{\mu\nu\rho\sigma\tau\kappa}(x) + b f_3^{\mu\nu\rho\sigma\tau\kappa}(x)}{|x_{12}|^d |x_{13}|^d |x_{23}|^d}$$

CFT: general aspects

A conformal field theory is characterized by:

- Its spectrum. A set of primary operators $\mathcal{O}_s^t$ with spin s and twist $t \equiv \Delta - s$.

- The coefficients of the Operator Product Expansion (OPE):

  Example:

$$\mathcal{O}_1(x)\mathcal{O}_2(0) = \sum_{s,\,t} \frac{\lambda_{12\mathcal{O}}}{|x|^{\Delta_1+\Delta_2-\Delta_3+s}} \mathcal{O}^t_{\mu_1\cdots\mu_s} x^{\mu_1}\cdots x^{\mu_s}$$

Clearly, the undetermined parameters in the three-point functions and the OPE coefficients represent the same set of data.

CFT: general aspects

The four point function is fixed by conformal invariance to be of the form:

$$\langle \mathcal{O}_1(x_4)\mathcal{O}_2(x_3)\mathcal{O}_2(x_2)\mathcal{O}_1(x_1)\rangle = \frac{\mathcal{A}(u,v)}{x_{14}^{2\Delta_1} x_{23}^{2\Delta_2}}$$

with (u, v) *the conformal cross ratios*:

$$z\bar{z} = v \equiv \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2}, \qquad (1-z)(1-\bar{z}) = u \equiv \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2}$$

and $\mathcal{A}(u, v)$ an undetermined function.

CFT: general aspects

Using the "T-channel" OPE, $\mathcal{O}_1\mathcal{O}_1 \to \mathcal{O}_t^s$, the four-point function is expanded as:

$$\mathcal{A}(u,v) = \sum_{\mathcal{O}_t^s} \lambda_{11\mathcal{O}}\lambda_{22\mathcal{O}}\, g_{\mathcal{O}}(u,v)$$

with $g_{\mathcal{O}}(u,v)$ known as *the conformal block*.

Due to conformal symmetry, the conformal block satisfies a 2nd order differential equation, *the Casimir differential equation*. Solutions are explicitly known in any even d and as integral representations or power series in odd d.

CFT: general aspects

Similarly, using the "S-channel" OPE $\mathcal{O}_1\mathcal{O}_2 \to \mathcal{O}^\ell_\Delta$,

$$\mathcal{A}(u,v) = \sum_{\mathcal{O}^\ell_\Delta} \lambda_{12\mathcal{O}}\lambda_{21\mathcal{O}} g^{\Delta_{12}}_{\mathcal{O}}(u,v)$$

with $\Delta_{12} = \Delta_1 - \Delta_2$.

This leads to the crossing equation:

$$\sum_{\mathcal{O}^j_\Delta} \lambda_{11\mathcal{O}}\lambda_{22\mathcal{O}} g_{\mathcal{O}}(u,v) = \sum_{\mathcal{O}^\ell_\Delta} \lambda_{12\mathcal{O}}\lambda_{21\mathcal{O}} g^{\Delta_{12}}_{\mathcal{O}}(u,v)$$

Combined with other consistency conditions, e.g., unitarity, these properties allow us to solve or constrain theories significantly.

Holographic CFTs

To examine CFTs which may have a gravity dual description, we consider the following general assumptions:

The CFT has a stress-tensor operator $T_{\mu\nu}$ and two large paramaters:

1. Large number of degrees of freedom N.

   At $N = \infty$ the CFT correlations functions factorize:

   $$\langle \mathcal{O}_1 \mathcal{O}_1 \mathcal{O}_2 \mathcal{O}_2 \rangle = \langle \mathcal{O}_1 \mathcal{O}_1 \rangle \langle \mathcal{O}_2 \mathcal{O}_2 \rangle + \frac{1}{N^2}(\cdots)$$

2. A characteristic scale Δ_{gap}.

   When $\Delta_{gap} = \infty$ the CFT contains only a finite number of primary single-trace operators with spin $j \leq 2$.

Holographic CFTs

- "single-trace" primaries: $\mathcal{O}_1, \mathcal{O}_2, \cdots, J^\mu, \cdots, T^{\mu\nu}$.
- "double-trace" primaries:

$$M_2 : \mathcal{O}_1 \partial_{\mu_1 \ldots} \partial_{\mu_\ell} (\partial^2)^n \mathcal{O}_2, \quad \mathcal{O}_1 \partial_{\mu_1 \ldots} \partial_{\mu_\ell} (\partial^2)^n J^\mu, \cdots$$

- "multi-trace" primaries:

$$M_{n>2} : \mathcal{O}_1 \partial_{\mu_1 \ldots} \partial_{\mu_a} (\partial^2)^n \mathcal{O}_2 \partial_{\mu_1 \ldots} \partial_{\mu_b} (\partial^2)^m \mathcal{O}_1 \partial_{\mu_1 \ldots} \partial_{\mu_c} (\partial^2)^k J^\mu, \cdots$$

$$\langle \mathcal{O}_1 \mathcal{O}_1 \rangle \sim 1 + \cdots, \qquad \langle M_2 M_2 \rangle \sim 1 + \cdots$$
$$\left\langle \mathcal{O}_2 \mathcal{O}_2 M_2^{\mathcal{O}_2 \mathcal{O}_2} \right\rangle \sim 1 + \cdots, \qquad \langle \mathcal{O}_1 \mathcal{O}_1 M_2 \rangle \sim \frac{1}{N^2} + \cdots,$$
$$\langle \mathcal{O}_1 \mathcal{O}_1 \, T \rangle \sim \frac{1}{N} + \cdots$$

HHLL in the lightcone limit

Objective: Study the correlator by solving the crossing equation order by order in the parameter $\mu \equiv \frac{\Delta_H}{c}$ and in the lightcone limit $1 - \bar{z} \ll 1$.

$$\mathcal{G}(z, \bar{z}) = \lim_{x_4 \to \infty} x_4^{2\Delta_H} \langle \mathcal{O}_H(x_4) \mathcal{O}_L(1) \mathcal{O}_L(z, \bar{z}) \mathcal{O}_H(0) \rangle = \frac{\mathcal{A}(z, \bar{z})}{[(1-z)(1-\bar{z})]^{\Delta_L}}$$

Note: In effect our focus is on the stress-tensor sector of the correlator.

HHLL in the lightcone limit

Method: Establish the leading contributions by studying the correlator in both the T- and S- channels.

$$\mathcal{O}_L \times \mathcal{O}_L \to 1 + \mu(T_{\mu\nu} + \cdots) + \cdots \to \mathcal{O}_H \times \mathcal{O}_H, \quad \text{T-channel}$$

$$\mathcal{O}_H \times \mathcal{O}_L \to [\mathcal{O}_H \mathcal{O}_L]_{\ell,n} \to \mathcal{O}_H \times \mathcal{O}_L, \qquad \text{S-channel}$$

HHLL in the lightcone limit: T-channel

$$\mathcal{G}(z,\bar{z}) = \frac{1}{[(1-z)(1-\bar{z})]^{\Delta_L}} \sum_{t,s} P^{HHLL}_{t,s}\, g_{t,s}(1-z,1-\bar{z})$$

s=spin, $t = (\Delta - s)$=twist

In the lightcone limit, the T-channel blocks behave as follows:

$$g_{t,s}(1-z,1-\bar{z}) \simeq (1-\bar{z})^{\frac{t}{2}}\, f_{\frac{t}{2}+s}(z)$$

where

$$f_{\frac{t}{2}+s}(z) \equiv (1-z)^{\frac{t}{2}+s}\, {}_2F_1\left[\frac{t}{2}+s, \frac{t}{2}+s, t+2s, 1-z\right]$$

Operators with lowest twist dominate the sum.

HHLL in the lightcone limit: T-channel

- Lowest twist $t = 0$ corresponds to the Identity operator, responsible for the disconnected contribution to the correlator $\langle \mathcal{O}_H \mathcal{O}_H \rangle \langle \mathcal{O}_L \mathcal{O}_L \rangle$.
- In the absence of additional symmetries, $T_{\mu\nu}$ provides the next significant contribution.

$$\begin{aligned} t \geq 2, &\quad s \geq 1 \\ t \geq 1, &\quad s = 0 \end{aligned}$$

This contribution is completely determined from a Ward Identity

$$P_T^{HHLL} = \# \frac{\Delta_H}{c} \frac{\Delta_L}{4} = \# \mu \frac{\Delta_L}{4}$$

HHLL in the lightcone limit: T-channel

The correlator admits an expansion in powers of μ.

$$P_{t,s}^{HHLL} = \sum_k P_{t,s}^{(k)} \mu^k$$

In the T-channel the contribution of composite stress-tensor exchanges is enhanced due to Δ_H as opposed to that of other operators suppressed in the $\frac{1}{c}$ expansion. New operators contribute at each order.

$$\begin{array}{lll} \mathcal{O}(\mu) & T_{\mu\nu} & t=2 \\ \mathcal{O}(\mu^2) & :T_{\mu_1\mu_2}\partial_{\mu_5}\partial_{\mu_6}\cdots\partial_{\mu_s}T_{\mu_3\mu_4}: & t=4 \\ & \cdots\cdots\cdots\cdots\cdots & \\ \mathcal{O}(\mu^k) & :T_{\mu_1\mu_2}T_{\mu_3\mu_4}\cdots\partial_{\mu_{2k+1}}\partial_{\mu_{2k+2}}\cdots\partial_{\mu_s}T_{\mu_{2k-1}\mu_{2k}}: & t=2k \end{array}$$

HHLL in the lightcone limit: T-channel

To obtain the leading contribution to the correlator at each order in μ requires summing over the contributions of an infinite number of operators.

$\mathcal{O}(\mu^2)$:

A handful of OPE coefficients $P_{4,s}$ were computed holographically

$$P_{4,s} = \frac{\Delta_L}{\Delta_L - 2} a_s^2 (\Delta_L^2 + b_s \Delta_L + c_s)\,.$$

- What are the functions a_s, b_s, c_s?
- Can we evaluate the sums,

$$\sum_{s=4}^{\infty} P_{4,s}\, f_{2+s}(z) = ?$$

HHLL in the lightcone limit:T-channel

We find the explicit form of the $P_{4,s}$ by combining their form with:

- Geodesic computation at large Δ_L.

$$\lim_{\Delta_L\to\infty}\langle\mathcal{O}_H|\mathcal{O}_L\mathcal{O}_L|\mathcal{O}_H\rangle \simeq e^{-\Delta_L\sigma(0)}\times$$
$$\times\left(1-\Delta_L\mu\sigma_{(1)}+\mu^2\left(\frac{1}{2}\sigma_{(1)}^2\Delta_L^2+\mathcal{O}(\Delta_L)\right)+\mathcal{O}(\mu^3)\right)$$

$$T:\mu\,\Delta_L\,f_3(z)\quad\Rightarrow\quad\mu^2\Delta_L^2\sum_{s=4}^{\infty}a_s^2 f_{2+s}(z)=f_3(z)^2$$

- Identity for product of hypergeometrics.
- Information from the S-channel computation.

HHLL in the lightcone limit: S-channel

$$\mathcal{G}(z,\bar{z}) = (z\bar{z})^{-\frac{\Delta_H+\Delta_L}{2}} \sum_{\tau,\ell} P^{HL,HL}_{\tau,\ell} g^{\Delta_{HL}}_{\tau,\ell}(z,\bar{z})$$

The contribution to the correlator comes from corrections in μ to the mean field theory OPE data of operators

$$: \mathcal{O}_H \partial^{2n} \partial_{\mu_1} \cdots \partial_{\mu_\ell} \mathcal{O}_L :$$

$$\tau = \Delta_H + \Delta_L + 2n + \gamma_{n,\ell}(\mu),$$

$$\gamma_{n,\ell} = \sum \mu^k \gamma^{(k)}_{n,\ell}, \qquad P^{HL,HL}_{n,\ell} = \sum \mu^k P^{HL,HL}_{n,\ell}$$

HHLL in the lightcone limit: S-channel

We analyse the S-channel in the lightcone limit and for $z << 1$.
The lightcone limit correponds to $\ell >> n$:

$$g_{\tau,\ell}^{\Delta_{HL}} \simeq (z\bar{z})^{\frac{\Delta_H+\Delta_L+\gamma_{n,\ell}}{2}}$$

$$P_\ell^{(k)} = P_\ell^{(0)} \frac{P^{(k)}}{\ell^{\frac{k(d-2)}{2}}}, \qquad P_\ell^{(0)} \sim \frac{\ell^{\Delta_L-1}}{\Gamma(\Delta_L)} \qquad \gamma_\ell^{(k)} = \frac{\gamma^{(k)}}{\ell^{\frac{k(d-2)}{2}}}$$

At $\mathcal{O}(\mu^0)$ we verify the crossing equation:

$$\mathcal{G}(z,\bar{z})\Big|_{\mu^0} \simeq \int_0^\ell d\ell P_\ell \bar{x}^\ell = -(\ln\bar{z})^{\Delta_L} \underset{\bar{z}\to 1\, z\to 0}{\simeq} \frac{1}{(1-\bar{z})^{\Delta_L}}$$

HHLL in the lightcone limit: S-channel

At $\mathcal{O}(\mu)$:

$$\mathcal{G}(z,\bar{z})\Big|_{\mu} \underset{\bar{z}\to 1\, z\to 0}{\simeq} \frac{1}{(1-\bar{z})^{\Delta_L-1}}\left(\frac{P^{(1)}}{\Delta_L-1}+\frac{\gamma^{(1)}\ln z}{2(\Delta_L-1)}\right),$$

we determine the unknown data from the contribution of the stress-tensor in the T-channel expansion:

$$P^{(1)}=\frac{3}{2}\gamma^{(1)}, \qquad \gamma^{(1)}=-\frac{\Delta_L(\Delta_L-1)}{2}$$

This completely determines the $\mathcal{O}(\mu^2\ln^2 z)$ data and precisely matches the result from the T-channel expansion for $z<<1$

$$\mathcal{G}(z\bar{z})\Big|_{\mu^2} \simeq \frac{\Delta_L}{(1-\bar{z})^{\Delta_L-2}(\Delta_L-2)}\left[\frac{\Delta_L(\Delta_L-1)}{32}\ln^2 z+\frac{3\Delta_L^2-7\Delta_L-1}{16}\ln z\right]$$

HHLL in the lightcone limit: check

Check against the large impact parameter region in the Regge limit:

$$z = 1 - \sigma e^{\rho}, \quad \bar{z} = 1 - \sigma e^{-\rho}, \quad z \to z e^{-2\pi i}, \quad \sigma << 1$$

$$\mathcal{G}(z,\bar{z})\Big|_{\mu^2} \simeq \frac{1}{\sigma^{2\Delta_L}} \{ \# \frac{\Delta_L(\Delta_L+1)(\Delta_L+2)}{\Delta_L - 2} e^{\frac{-2\rho}{\sigma^2}} + i \# \frac{\Delta_L(\Delta_L+1)}{\Delta_L - 2} \frac{e^{-5\rho}}{\sigma} + \cdots \}$$

HHLL in the lightcone limit

Performing the infinite sums

$$\mathcal{G}(z,\bar{z})\Big|_{\mu^2,\,\ell.c.} \propto \frac{1}{[(1-z)(1-\bar{z})]^{\Delta_L}}(1-\bar{z})^2 \times$$
$$\times \frac{\Delta_L}{\Delta_L-2}\left((\Delta_L-4)(\Delta_L-3)f_3^2+\frac{15}{7}(\Delta_L-8)f_2 f_4+\frac{40}{7}(\Delta_L+1)f_1 f_5\right).$$

where

$$f_a(z)=(1-z)^a\,{}_2F_1[a,a,2a,1-z]$$

An interesting observation... : $3+3=2+4=1+5$

Further Results - Comments

Example $\mathcal{O}(\mu^3)$:

$$\mathcal{G}(z,\bar{z})\Big|_{\mu^3} = \frac{(1-\bar{z})^3}{((1-z)(1-\bar{z}))^{\Delta_L}}\left\{a_{333}f_3^3 + a_{112}f_1^2 f_7 + a_{126}f_1 f_2 f_6 + \right.$$
$$\left. + a_{135}f_1 f_3 f_5 + a_{225}f_2^2 f_5 + a_{234}f_2 f_3 f_4 + a_{114}f_1 f_4^2\right\}$$

$$a_{333} = \frac{\Delta_L^5 + \cdots}{(\Delta_L - 2)(\Delta_L - 3)}, \quad a_{234}, a_{135} = \frac{\Delta_L^4 + \cdots}{(\Delta_L - 2)(\Delta_L - 3)},$$
$$a_{117}, a_{126}, a_{225} = \frac{\Delta_L^3 + \cdots}{(\Delta_L - 2)(\Delta_L - 3)}$$

- Products of f_a functions are not all independent of one another.

The two-dimensional case.

The structure is very similar to the 2d Virasoro vacuum block

$$\langle \mathcal{O}_H | \mathcal{O}_L \mathcal{O}_L | \mathcal{O}_H \rangle \sim e^{\Delta_L g(z)} e^{\Delta_L g(\bar{z})}$$

$$g(z) = -\frac{1}{2} \ln z - \ln \left(2 \sinh \left(\frac{\sqrt{1-\mu}}{2} \ln z \right) \right) + \ln \sqrt{1-\mu}$$

An earlier observation [MK, Ng, Parnachev]:

$$g(z) \sim - \ln (1-z) + \frac{\mu}{24} f_2(z) + \frac{\mu^2}{24^2} \left(-f_2^2 + \frac{6}{5} f_1 f_3 \right) + \\ + \frac{\mu^3}{24^3} \left(\frac{4}{3} f_2^3 - \frac{14}{5} f_1 f_2 f_3 + \frac{54}{35} f_1^2 f_4 \right) + \cdots$$

Further results - Comments

Claim 1: The stress-tensor sector of the HHLL correlator is

$$\mathcal{G}(z,\bar{z}) = \sum \mathcal{G}^{(k)}(z,\bar{z})\mu^k$$

with

$$\mathcal{G}^{(k)}(z,\bar{z}) \underset{\bar{z}\to 1}{\approx} \frac{(1-\bar{z})^{k(\frac{d}{2}-1)}}{[(1-z)(1-\bar{z})]^{\Delta_L}} \sum_{\{i_p\}} a_{i_1\ldots i_k} f_{i_1}(z)\ldots f_{i_k}(z),$$

and where

$$\sum_{p=1}^{k} i_p = k\left(\frac{d+2}{2}\right), \quad i_p \in \mathbb{N}$$

Claim 2: The correlator exponentiates similarly to what happens in two dimensions.

Further results - Comments

- We have shown that this solves the crossing equation in principle. All $\log^k z$-terms can be determined from the S-channel expansion in terms of OPE data of $\mathcal{O}(\mu^k)$.
- Have computed OPE coefficients with the Lorentzian inversion formula (up to $\mathcal{O}(\mu^3)$).

[Li][Karlsson, MK, Parnachev, Tadic]

- Explicitly determined the relevant coefficients $a_{i_1 i_2 \cdots i_k}$ to $\mathcal{O}(\mu^6)$.
- We have also determined the relevant OPE coefficients (e.g. triple stress-tensors).
- Established exponentiation:

$$\mathcal{G}(z,\bar{z}) = [(1-z)(1-\bar{z})]^{-\Delta_L}\, e^{\Delta_L \mathcal{F}(z,\bar{z})}$$

where

$$\mathcal{F}(z,\bar{z}) = \sum_{k=1}^{\infty} \mu^k (1-\bar{z})^k \mathcal{F}_k(z), \quad \text{with} \quad \mathcal{F}_k(z) \simeq_{\Delta_L \to \infty} \mathcal{O}(1)$$

where $\mathcal{F}_k(z)$ is again given by products of f_a functions.

Open Questions

- What underlies this structure?
- Can we resum the series as in 2d?
- What if Δ_L is an integer?
- Beyond the lightcone limit?

  Include the contribution of operators with subleading twists

  $$\mathcal{O}(\mu^2) \quad : T_{\mu_1\mu_2}\partial_{\mu_5}\partial_{\mu_6}\cdots\partial_{\mu_s}T_{\mu_3\mu_4} : \quad + \quad \text{contractions.}$$

  $$\text{e.g.} \quad : T_{\mu_1\mu_2}\partial_{\mu_5}\partial_{\mu_6}\cdots\partial_{\mu_s}\partial^{2n}T^{\mu_2}{}_{\mu_4} : \quad t = 6 + 2n$$

  Similar structure persists up to sub-sub-subleading order in the lightcone limit. [Karlsson, MK, Parnachev, Tadic]
- Quasi-normal modes.
- Beyond large C_T.
- Address the physics close to the horizon.

Thank you !