Degeneration, Geometry and Duality

Nipol Chaemjumrus and CH:

[arXiv:1907.04040] **Degenerations of K3, Orientifolds and Exotic Branes**

[arXiv:1908.04623] **Special Holonomy Manifolds, Domain Walls, Intersecting Branes and T-folds**

[arXiv:1909.12348] **The Doubled Geometry of Nilmanifold Reductions**

K3 and Duality

- IIA strings on K3 dual to heterotic or type I on T^4
- Duality between type I on T^4 and IIA on $T^4/\mathbb{Z}_2$ is chain of T and S dualities. Gives K3 at one point in moduli space
- Given this identification, can then move to other points in moduli space, where there are no isometries and so no T-dualities
- Chain of dualities taking type I to IIA on K3 takes D-branes to KK monopoles. Orientifold-planes to some geometric dual?
- Gives “new” picture of K3, and new understanding of O planes and strong coupling. Explicit picture of dualities.

- Type I on circle T-dual to Type I’ on interval

- 16 D8-branes moving on interval, O8-planes at ends

- At strong coupling: mysterious O* planes

- Dual to *degenerate* limit of K3, long neck region, KK monopoles moving on neck

- Ends of neck capped by geometries “dual” to O-planes or O*-planes. Strong coupling mysteries now accessible at weak coupling

- Intriguing relation with del Pezzo Surfaces

Further directions

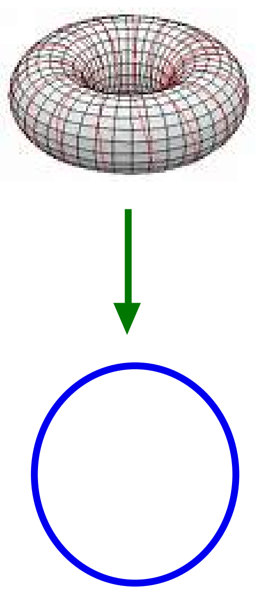
- Generalisation to special holonomy, relation to intersecting branes
- Dualising type I’ gives consistent string configurations for various branes that can’t exist in isolation
- Dualities relate branes to *exotic branes*, low co-dimension
  Elitzur,Giveon,Kutasov,Rabinovici; Blau,O’Loughlin; CH; Obers &Pioline
- Takes K3 to non-geometric backgrounds: T-folds etc

Torus Bundle with Duality Twist

Simple case: bundle of torus CFTs fibred over a circle *Dabholkar & CH*

Stringy version of Scherk-Schwarz

Monodromy in Duality group $O(d,d;\mathbb{Z})$



e.g. T^2 fibre: $SL(2,\mathbb{Z}) \times SL(2,\mathbb{Z})$ acting on $\tau, \rho = B + iA$

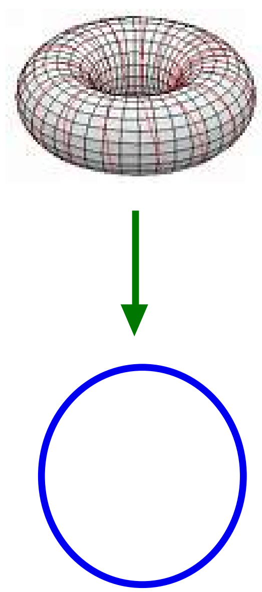
“Bundle” with fibre torus CFT

T^2 Fibred over S^1

<u>3-torus with flux</u>, H=m x Vol

$$ds^2 = dx^2 + dy^2 + dz^2 \qquad B = mx\,dy \wedge dz$$

Monodromy: $\rho \to \rho + m$



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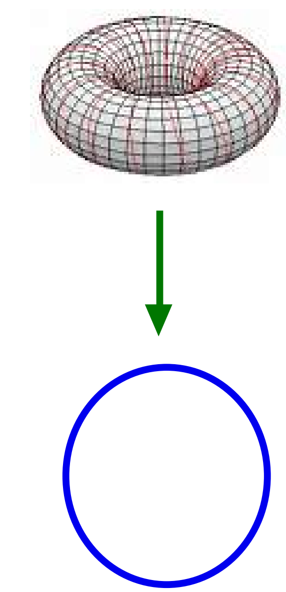


<u>3-torus with flux,</u> H=m x Vol

$$ds^2 = dx^2 + dy^2 + dz^2 \qquad B = mx dy \wedge dz$$

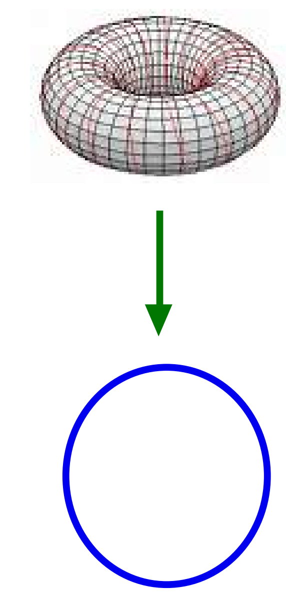
Monodromy: $\rho \to \rho + m$

<u>Nilfold</u> S^1 Bundle over T^2

$$ds^2_{\mathcal{N}} = dx^2 + (dy + mx dz)^2 + dz^2$$

Monodromy: $\tau \to \tau + m$

T^2 Fibred over S^1



3-torus with flux, H=m x Vol

$$ds^2 = dx^2 + dy^2 + dz^2 \qquad B = mx\,dy \wedge dz$$

Monodromy: $\rho \to \rho + m$

Nilfold S¹ Bundle over T²

$$ds^2_{\mathcal{N}} = dx^2 + (dy + mx\,dz)^2 + dz^2$$

Monodromy: $\tau \to \tau + m$

T-fold T² fibration over S¹, T-duality monodromy

$$ds^2_{\text{T-Fold}} = dx^2 + \frac{1}{1+(mx)^2}(dy^2 + dz^2) \qquad B = \frac{mx}{1+(mx)^2} dy \wedge dz$$

Monodromy: $\rho \to \dfrac{\rho}{1 + m\rho}$

String solutions

- None of these are solutions of string theory
- Can find bundle solutions in which these are fibres CH
- Duality then acts fibre wise
- Simplest case: fibre over a line
- Nilfold fibred over a line: hyperkahler CH
  Gibbons +Rychenkova
  Lavrinenko, Lu, Pope

Gibbons-Hawking Metric

Hyperkahler metric with S^1 symmetry

$$g = V(d\tau^2 + dx^2 + dz^2) + V^{-1}(dy + \omega)^2$$

$V(\tau, x, z)$ a harmonic function on $\mathbb{R}^3$

$$\vec{\nabla} \times \vec{\omega} = \vec{\nabla} V$$

Delta-function sources at points (m an integer)

$$V = a + \sum_i \frac{m}{|\vec{r} - \vec{r}_i|}$$

S^1 Bundle on $\mathbb{R}^3$ - {points}

Regular at sources if m=1: multi-Taub-NUT

Orbifold singularities for m>1

Smeared GH Metrics

$V(\tau, x, z)$ a harmonic function on $\mathbb{R}^3$

“Smeared” solutions: V independent of one or more coordinates

Can then take those coordinates to be periodic
Metric typically singular

Smear on x,y: $V(\tau) = m\tau + c$

or
$$V(\tau) = \begin{cases} c + m'\tau, & \tau \leq 0 \\ c + m\tau, & \tau > 0. \end{cases}$$

Singular at kink at $\tau = 0$

Domain wall: 2-plane dividing space into 2 parts

N=m-m’: energy density (tension) of domain wall (2-brane)

Smeared GH & Nilfolds

$$ds^2 = V(\tau)(d\tau^2 + dx^2 + dz^2) + \frac{1}{V(\tau)}(dy + mxdz)^2 \qquad V(\tau) = m\tau + c$$

Take x,y,z periodic

Fixed τ: **nilfold**

$$ds^2_{\mathcal{N}} = dx^2 + (dy + mxdz)^2 + dz^2$$

S^1 Bundle over T^2

$F = mdx \wedge dz$ Degree $m \in \mathbb{Z}$

Quotient of the group manifold of the Heisenberg group by a cocompact discrete subgroup

4-d space: nilfold fibred over a line Wall: jump in degree m

Smeared GH & Nilfolds

$$ds^2 = V(\tau)(d\tau^2 + dx^2 + dz^2) + \frac{1}{V(\tau)}(dy + mxdz)^2 \qquad\qquad V(\tau) = m\tau + c$$

Take x,y,z periodic

Fixed τ: **nilfold**

$$ds^2_{\mathcal{N}} = dx^2 + (dy + mxdz)^2 + dz^2$$

S^1 Bundle over T^2

$$F = mdx \wedge dz \qquad \text{Degree} \qquad m \in \mathbb{Z}$$

Quotient of the group manifold of the Heisenberg group by a cocompact discrete subgroup

4-d space: nilfold fibred over a line   Wall: jump in degree m

String solution: product with $\mathbb{R}^{1,5}$. Smeared KK monopole

T-duality

Nilfold fibred over line → T^3 fibred over line

$$ds^2 = V(\tau)(d\tau^2 + dx^2 + dy^2 + dz^2)$$

$$H_{ijk} = -\epsilon_{ijkl}\delta^{lm}\partial_m V \qquad H_{xyz} = -V'(\tau) \qquad e^{2\Phi} = V$$

$$V(\tau) = m\tau + c$$

or

$$V(\tau) = \begin{cases} c + m'\tau, & \tau \leq 0 \\ c + m\tau, & \tau > 0. \end{cases}$$

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$$V(\tau) = \begin{cases} c + m'\tau, & \tau \leq 0 \\ c + m\tau, & \tau > 0. \end{cases}$$

10-d solution

$$ds^2_{10} = V\, ds^2(\mathbb{R} \times T^3) + ds^2(\mathbb{R}^{1,5})$$

NS5-brane smeared over x,y,z. These are identified to give transverse space

$$\mathbb{R} \times T^3$$

T-fold fibred over line

$$ds^2 = V(\tau)(d\tau)^2 + V(\tau)(dx)^2 + \frac{V(\tau)}{V(\tau)^2 + (M(\tau)x)^2}(dy^2 + dz^2)$$

$$B = \frac{M(\tau)x}{V(\tau)^2 + (M(\tau)x)^2} dy \wedge dz$$

$$\Phi = \frac{1}{2}\log\left(\frac{V(\tau)}{V(\tau)^2 + (M(\tau)x)^2}\right)$$

$$V(\tau) = \begin{cases} c + m'\tau, & \tau \leq 0 \\ c + m\tau, & \tau > 0. \end{cases}$$

T-fold fibred over line

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String solution: product with $\mathbb{R}^{1,5}$. Smeared exotic brane.

Smeared $(5,2^2)$ brane or 5^2_2 brane.

$$M \sim \frac{R_1 R_2 R_3 R_4 R_5 R_6^2 R_7^2}{g^2 l_s^{10}}$$

Multi-domain wall solutions

V Piecewise linear: **multi-wall solution** with domain walls at $\tau = \tau_1, \tau_2, \ldots \tau_n$

$$V(\tau) = \begin{cases} c_1 + m_1\tau, & \tau \le \tau_1 \\ c_2 + m_2\tau, & \tau_1 < \tau \le \tau_2 \\ \vdots & \\ c_n + m_n\tau, & \tau_{n-1} < \tau \le \tau_n \\ c_{n+1} + m_{n+1}\tau, & \tau > \tau_n\,. \end{cases}$$

The charge of the domain wall at τ_r is the integer $\quad N_r = m_{r+1} - m_r$

e.g. GH: $$ds^2 = V(\tau)(d\tau^2 + dx^2 + dz^2) + \frac{1}{V(\tau)}(dy + M(\tau)x dz)^2 \qquad M(\tau) \equiv V'(\tau)$$

Can take x,y,z periodic

Single-sided domain wall

$$V = c + m|\tau|$$

Quotient by reflection $\tau \to -\tau$ gives
"single-sided" wall at $\tau = 0$

Not consistent string backgrounds

- Hyperkahler space + duals give CFTs away from walls
- Domain walls singular
- Linear dilaton and V blow up unless end with single-sided walls
- Need negative brane charges to give net charge zero

Dualities: Singular Solns

Smeared KK Monopole	Nilfold fibred over line
NS5-brane Smeared on T^3	T^3 with flux fibred over line
D8-brane Wrapped on T^3	D8-brane: domain wall in 9+1 dimensions

- D8-brane: string background needs orientifold planes
- Singularities at walls: reflect presence of physical objects (D8-branes)
- Type I' string: 16 D8-branes and 2 O8-planes
- Dualise to get consistent backgrounds for nilfold, T-fold and T^3 with H-flux with duals of O8-planes. How are singularities resolved?

Type I' String Theory

Interval x $\mathbb{R}^{1,8}$

16 D8-branes of charge 1: N_i branes at points τ_i on interval

Orientifold 8-planes of charge -8 at end-points $\tau = 0, \pi$

$$ds^2 = V^{-1/2} ds^2(\mathbb{R}^{1,8}) + V^{1/2} d\tau^2 \qquad V(\tau) = \begin{cases} c_1 + m_1\tau, & 0 \le \tau \le \tau_1 \\ c_2 + m_2\tau, & \tau_1 < \tau \le \tau_2 \\ \vdots & \\ c_n + m_n\tau, & \tau_{n-1} < \tau \le \tau_n \\ c_{n+1} + m_{n+1}\tau, & \tau_n < \tau \le \pi \end{cases}$$

$$N_i = m_{i+1} - m_i \qquad \sum_{i=1}^{n} N_i = 16$$

Or, if at $\tau = 0$ there are N_- branes giving charge $b_- = -8 + N_-$
and if at $\tau = \pi$ there are N_+ branes giving charge $b_+ = -8 + N_+$

$$b_- = -m_1, b_+ = m_{n+1} \qquad 0 \le b_\pm \le 8$$

$$\sum_{i=1}^{n} N_i = b_- + b_+ \le 16$$

Dualise Type I'

- Dualise supergravity solution wrapped on T^3 to get smeared GH and NS5 with same potential V on interval

- 16 sources: KK monopole or NS5-brane smeared over T^3

- Smeared KK, NS5 singular. How are singularities resolved?

- At ends of interval: duals of O8 planes.

- Orientifold analogues of KK monopoles and (5,2) branes?

$$D8 \xrightarrow{T} D5 \xrightarrow{S} NS5 \xrightarrow{T} KK \xrightarrow{T} (5,2)$$

$$O8 \xrightarrow{T} O5 \xrightarrow{S} ON \xrightarrow{T} ?? \xrightarrow{T} ??$$

String theory

To address these issues, look at full string theory and dualities

Type I' string on $S^1/\mathbb{Z}_2 \times \mathbb{R}^{1,8}$ $\longleftrightarrow$ Type I string on $S^1 \times \mathbb{R}^{1,8}$

Type I' string on $S^1/\mathbb{Z}_2 \times T^3 \times \mathbb{R}^{1,5}$ $\longleftrightarrow$ Type I string on $T^4 \times \mathbb{R}^{1,5}$

$$D9 \xrightarrow{T} D8 \xrightarrow{T} D5 \xrightarrow{S} NS5 \xrightarrow{T} KK$$

$$\mathrm{I} \equiv \frac{\mathrm{IIB}}{\Omega} \xrightarrow{\mathrm{T}_9} \mathrm{I}' \equiv \frac{\mathrm{IIA}}{\Omega \mathrm{R}_9} \xrightarrow{\mathrm{T}_{678}} \frac{\mathrm{IIB}}{\Omega \mathrm{R}_{6789}} \xrightarrow{\mathrm{S}} \frac{\mathrm{IIB}}{(-1)^{\mathrm{F_L}}\mathrm{R}_{6789}} \xrightarrow{\mathrm{T}_6} \frac{\mathrm{IIA}}{\mathrm{R}_{6789}}$$

String theory

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Type I' string on $S^1/\mathbb{Z}_2 \times \mathbb{R}^{1,8}$ $\longleftrightarrow$ Type I string on $S^1 \times \mathbb{R}^{1,8}$

Type I' string on $S^1/\mathbb{Z}_2 \times T^3 \times \mathbb{R}^{1,5}$ $\longleftrightarrow$ Type I string on $T^4 \times \mathbb{R}^{1,5}$

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Last step gives IIA on $T^4/\mathbb{Z}_2$, orbifold limit of K3

Duality between heterotic/type I on T^4 and IIA on K3 from T&S dualities

Orientifolds

$$\mathrm{I} \equiv \frac{\mathrm{IIB}}{\Omega} \xrightarrow{\mathrm{T}_9} \mathrm{I}' \equiv \frac{\mathrm{IIA}}{\Omega \mathrm{R}_9} \xrightarrow{\mathrm{T}_{678}} \frac{\mathrm{IIB}}{\Omega \mathrm{R}_{6789}} \xrightarrow{\mathrm{S}} \frac{\mathrm{IIB}}{(-1)^{\mathrm{F_L}} \mathrm{R}_{6789}} \xrightarrow{\mathrm{T}_6} \frac{\mathrm{IIA}}{\mathrm{R}_{6789}}$$

$$D9 \xrightarrow{T} D8 \xrightarrow{T} D5 \xrightarrow{S} NS5 \xrightarrow{T} KK$$

Branes → gravitational solitons

O9 → 2 O8's →16 O5's →16 ON's → ?

Smooth geometric dual to orientifolds?

Dualising Supergravity Soln with D8's to one with KK monopoles:

Space which is nilfold fibred over line, with smeared KK monopoles

Ends of line: geometric dual of orientifold planes

Dualising Type I' string

Same dualities take I' on T^3 to IIA on K3

"Predicts" a region of K3 moduli space where the K3 looks like a nilfold fibred over a line interval with 16 KK monopole insertions, and where the regions of K3 at the ends of the interval look like the duals of O8 planes?

Degenerate Limit of K3

Hein, Sun, Viaclovsky and Zhang [HSVZ]

- Family of K3 Metrics g(t), limit t=0 is line interval
- Long Neck Region at small t
- Segment of neck is nilfold fibred over a line.
- Nilfold is S^1 bundle over T^2, with degree (Chern number) m. Different values of m in different segments.
- Jump in m: insertion of gravitational instanton (Kaluza Klein monopole)
- Ends of neck capped by Tian-Yau spaces: complete non-compact hyperkahler manifolds asymptotic to nilfold fibred over a line

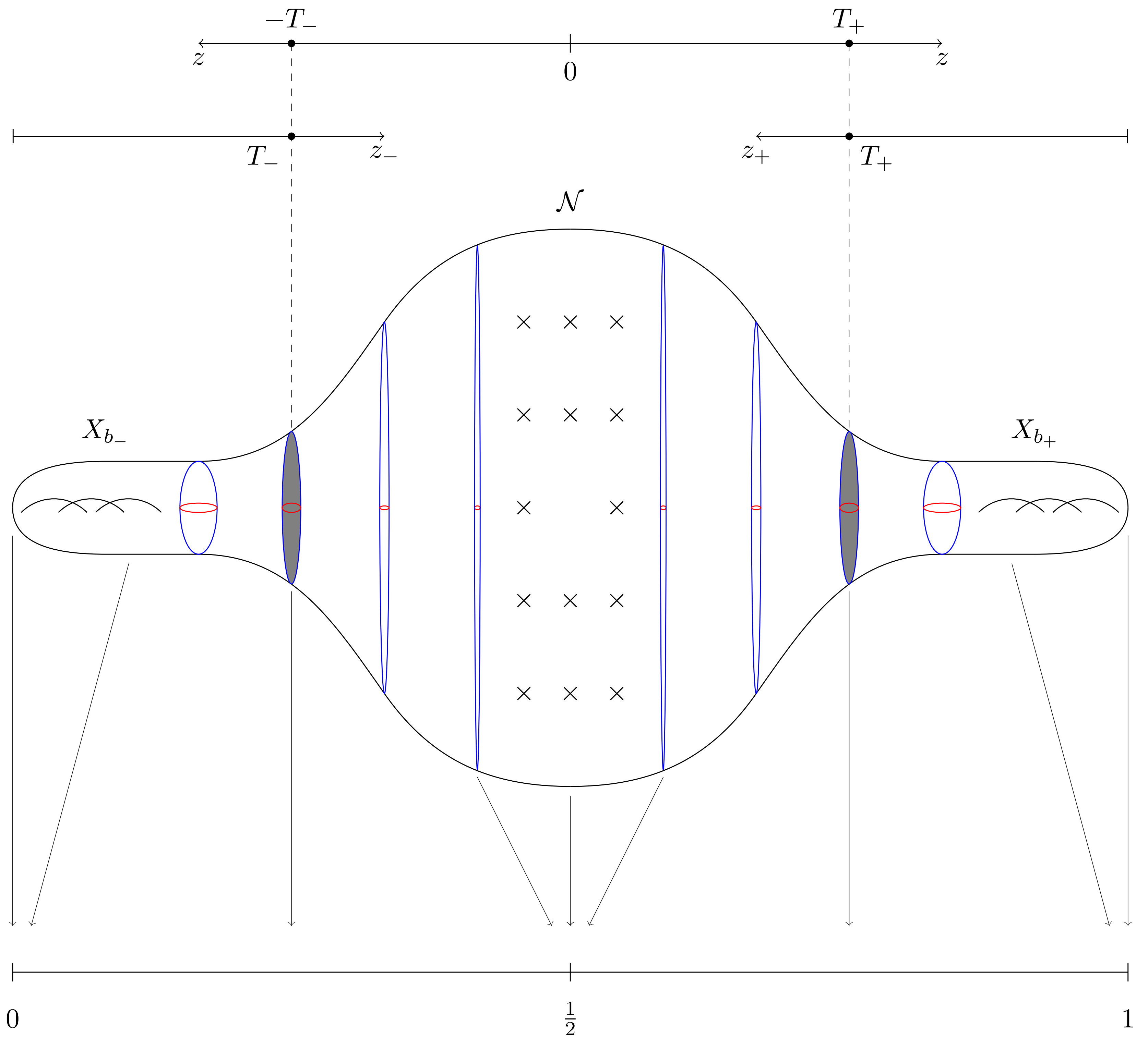

FIGURE 1. The vertical arrows represent collapsing to a one-dimensional interval. The red circles represent the S^1 fibers and the blue curves represent the base $\mathbb{T}^2$s of the nilmanifolds. The $\times$s are the monopole points in the neck region $\mathcal{N}$. The gray regions are in the "damage zones".

1st approximation to HSVZ K3

Interval $\tau \in [0,\pi]$
Multi-domain wall solution with domain walls at $\tau = \tau_1, \tau_2, \ldots \tau_n$
Single-sided domain walls at $\tau = 0,\pi$

$$ds^2 = V(\tau)(d\tau^2 + dx^2 + dz^2) + \frac{1}{V(\tau)}(dy + M(\tau)xdz)^2$$

$$V(\tau) = \begin{cases} c_1 + m_1\tau, & 0 \leq \tau \leq \tau_1 \\ c_2 + m_2\tau, & \tau_1 < \tau \leq \tau_2 \\ \vdots & \\ c_n + m_n\tau, & \tau_{n-1} < \tau \leq \tau_n \\ c_{n+1} + m_{n+1}\tau, & \tau_n < \tau \leq \pi \end{cases} \qquad M(\tau) \equiv V'(\tau)$$

<u>HSVZ resolve singularities:</u>

- Resolve domain walls with Ooguri-Vafa construction
- Resolve single-sided domain walls with Tian-Yau spaces

Ooguri-Vafa Metric

Want Gibbons-Hawking metric, $\mathbb{R}^3$ replaced with $\mathbb{R} \times T^2$
1st approximation: smear over T^2

Ooguri-Vafa:

- On $\mathbb{R}^3$, take periodic array of sources in (x,z) plane
- Regularised sum of potentials gives harmonic function
- Can now periodically identify x,z directions, to get single source on $\mathbb{R} \times T^2$.
- Near source, non-singular, looks like Taub-NUT
- Can then take superpositions to get multiple sources on $\mathbb{R} \times T^2$.
- Solutions regular on finite interval in $\mathbb{R}$

Resolve GH metric with

$$V(\tau) = \begin{cases} c + m'\tau, & \tau \leq 0 \\ c + m\tau, & \tau > 0. \end{cases}$$

Charge N=m-m'

by OV metric with V harmonic on $\mathbb{R} \times T^2$

Monopole charge N

Near sources, N-centre multi Taub-NUT, or one source of charge N, orbifold singularity: bubbling limit to Taub-NUT

For N sources, regular hyperkahler metric for some interval

$$-T < \tau < T'$$

Far enough away from $\tau = 0$, tends to GH with

$$V(\tau) = \begin{cases} c + m'\tau, & \tau \leq 0 \\ c + m\tau, & \tau > 0. \end{cases}$$

Tian-Yau Spaces

- Complete non-singular non-compact hyperkahler space
- Asymptotic to a nilfold bundle over a line.
- Of the form M \ D, where M is a del Pezzo surface, D ⊂ M is a smooth anti-canonical divisor
- Del Pezzo surfaces are complex algebraic surfaces classified by their degree b, where b = 1, 2, . . . , 9. They are Kahler 4-manifolds, $c_1 > 0$.
- The del Pezzo surface of degree nine is CP^2
- A degree b del Pezzo surface can be constructed from blowing up 9 – b points in CP^2
- A 2nd del Pezzo surface of degree 8 is $CP^1 \times CP^1$
- The TY space M_b of degree b is constructed from del Pezzo of degree b
- M_b is asymptotic to GH metric on $N_b \times \mathbb{R}$ where N_b is nilfold of degree b
- Degree zero: Take M to be rational elliptic surface, $N_0 = T^3$, M_0 is ALH, asymptotic to cylinder given by $T^3 \times \mathbb{R}$

1st approximation to HSVZ K3

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Multi-domain wall solution with domain walls at $\tau = \tau_1, \tau_2, \ldots \tau_n$

Single-sided domain walls at $\tau = 0,\pi$

$$ds^2 = V(\tau)(d\tau^2 + dx^2 + dz^2) + \frac{1}{V(\tau)}(dy + M(\tau)xdz)^2$$

$$V(\tau) = \begin{cases} c_1 + m_1\tau, & 0 \leq \tau \leq \tau_1 \\ c_2 + m_2\tau, & \tau_1 < \tau \leq \tau_2 \\ \vdots & \\ c_n + m_n\tau, & \tau_{n-1} < \tau \leq \tau_n \\ c_{n+1} + m_{n+1}\tau, & \tau_n < \tau \leq \pi \end{cases} \qquad M(\tau) \equiv V'(\tau)$$

<u>HSVZ resolve singularities</u>:

Glue together Ooguri-Vafa spaces, Tian-Yau spaces to get complete K3 metric

Tian-Yau spaces of degree b_-, b_+

$$b_- = -m_1, b_+ = m_{n+1} \qquad 0 \leq b_\pm \leq 9$$

$$N_i = m_{i+1} - m_i$$

$$\sum_{i=1}^{n} N_i = b_- + b_+ \leq 18$$

1st approximation to HSVZ K3

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$$b_- = -m_1, b_+ = m_{n+1} \qquad 0 \le b_\pm \le 9$$

$$N_i = m_{i+1} - m_i$$

$$\sum_{i=1}^{n} N_i = b_- + b_+ \le 18$$

Almost agrees with type I' picture

But 18 instead of 16?

Type I’: 16 D8 branes & 2 O8-planes

This is correct for *perturbative* type I’ theory

At *strong coupling*, O8 plane can emit one D8 brane to leave

O8* plane of charge -9 Morrison and Seiberg

Then O8* planes at either end and 18 D8-branes on interval

If at $\tau = 0$ there are N_- branes giving charge $b_- = -9 + N_-$

and at $\tau = \pi$ there are N_+ branes giving charge $b_+ = -9 + N_+$

$$b_- = -m_1, b_+ = m_{n+1}$$

$$\sum_{i=1}^{n} N_i = b_- + b_+ \leq 18$$

Same equations as for degenerate K3

Both cases have 18 sources

Allows e.g. SU(18) gauge symmetry from coincident sources

Matching Moduli Spaces

Type I' moduli space $O(1,17;\mathbb{Z})\backslash O(1,17)/O(17) \times \mathbb{R}^+$

16 D8-brane positions, dilaton, length of S^1

$$\mathrm{I}' \equiv \frac{\mathrm{IIA}}{\Omega\mathrm{R}_9} \xrightarrow{\mathrm{T}} \frac{\mathrm{IIB}}{\Omega\mathrm{R}_{6789}} \xrightarrow{\mathrm{S}} \frac{\mathrm{IIB}}{(-1)^{\mathrm{F_L}}\mathrm{R}_{6789}} \xrightarrow{\mathrm{T}} \frac{\mathrm{IIA}}{\mathrm{R}_{6789}}$$

Embed in moduli space of duals: region where dual has long throat

Orientifold of IIB on $T^4/\mathbb{Z}_2$

Regard $T^4/\mathbb{Z}_2$ as $T^3 \times I$, where at ends of I identify T^3 to $T^3/\mathbb{Z}_2$

Long neck $T^3 \times I$, but ends "pinch off". Moduli from positions of branes

K3: long neck $Nilfold \times I$. Moduli from positions of KK's

- Duality between Heterotic or type I on T^4 and IIA on K3 understood as T + S dualities at orbifold point when $K3 \sim T^4/\mathbb{Z}_2$
- But <u>not</u> at general points in K3 moduli space — no isometries
- However, moduli space of type I on T^4 and IIA on K3 are the same
- Duality at one point in moduli space leads to duality at all points
- Can translate moving in type I mod space into moving in IIA mod space

Non-Geometric

- K3: no isometries, so no conventional T-duals (if not orbifold)
- Move to region of mod space with long neck, HSVZ metric
- In long neck region, approximately nilfold x interval (I)
- T-dual: T-fold x I, essentiality doubled space x I
- Doubled geometry: nilfold and all its duals arise as different polarisations of doubled space, 6-dim nilmanifold N_6. *CH and Reid-Edwards*
- Doubled formulation: N_6 fibred over I. *Chaemjumrus and CH*
- Sources: exotic branes, moving in non-geometric background

Special Holonomy Generalisations

- Replace 3-d nilfold with higher dim nilmanifold
- Quotient of nilpotent Lie group by discrete subgroup
- T^n bundle over T^m
- Special holonomy metrics on nilmanifold fibred over a line
  Gibbons, Lu, Pope and Stelle [GLPS]
- T-Dualise: intersecting NS5-branes Chaemjumrus and CH

Dimension	Nilmanifold: torus bundle over torus	Holonomy
4	S^1 over T^2	SU(2)
6	S^1 over T^4	SU(3)
6	T^2 over T^3	SU(3)
7	T^2 over T^4	G_2
7	T^3 over T^3	G_2
8	S^1 over T^6	SU(4)
8	T^3 over T^4	Spin(7)

T-duality

Taub-NUT → NS 5-brane [CH+Townsend]

ALF Multi-instanton → Multi 5-brane

GLPS Special Holonomy → Intersecting 5-brane solution with one function

↓ [Chaemjumrus and CH]

Special Holonomy with several functions ← Intersecting 5-brane solution with several functions

↓

Semi-local solution

Localised Solutions and Degenerate Limits

- Are there fully localised non-singular solutions?
- Use duality to intersecting branes, brane webs to motivate ansatz
- Are these part of neck region of some compact special holonomy space, just as the hyperkahler metrics were model metrics for part of degenerate limit of K3?
- Relation to other configurations via string dualities?

Del Pezzo Magic?

Del Pezzo surfaces have intriguing relations to U-duality, branes etc

Much of structure of toroidal compactifications of M-theory mirrored in mathematics of Del Pezzo surfaces

A Mysterious duality
A. Iqbal, A. Neitzke, C. Vafa
Noncritical strings, Del Pezzo singularities and Seiberg-Witten curves
W. Lerche, P. Mayr, N. Warner
Exotic Branes from del Pezzo Surfaces
J. Kaidi
Borcherds symmetries in M theory
P. Henry-Labordere, B. Julia, L. Paulot

Tian-Yau spaces and their relation to K3 give another link to Del Pezzo surfaces

Conclusions

- Nilfold and its duals: local string solutions by fibring over I
- Dualise Type I’: full string theory solutions
- Realise K3 as nilfold fibred over interval with KK monopole insertions and Tian-Yau end caps
- Good approximate geometry for K3 that allows explicit duality transformations
- TY: geometric dual of orientifolds, reveals non-perturbative structure of orientifolds: O8* etc.
- Singularities of smeared branes resolved
- Generalisation to special holonomy manifolds, non-geometries