

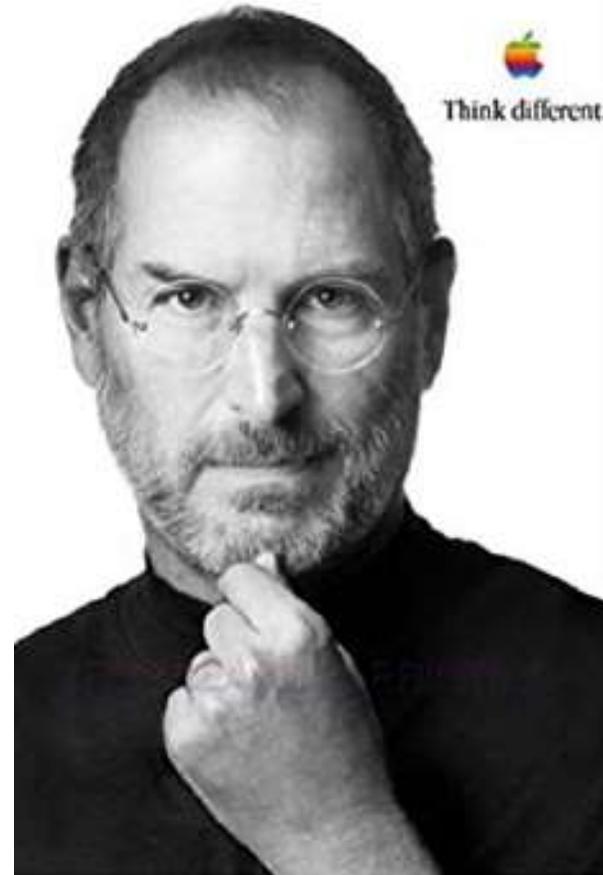


Here's to the crazy ones.

The misfits.
The rebels.
The troublemakers.
The round pegs in the square holes.
The ones who see things differently.
They're not fond of rules.
And they have no respect for the status quo.
You can quote them, disagree with them,
glorify or vilify them.
About the only thing you can't do is ignore them.
Because they change things.
They push the human race forward.
And while some may see them as the crazy ones,
we see genius.

Because the people who are crazy enough
to think they can change the world,
are the ones who do.

 Think different.



$$\frac{1}{4g^2} \text{tr} F^2 + \bar{\psi} \not{D} \psi$$



$$\int d^2x \frac{1}{4g^2} \text{tr} F^2 + \bar{\psi} \not{D} \psi$$



4d $\mathcal{N} = 1$

4d $\mathcal{N} = 2$



2d $\mathcal{N} = (0,2)$

2d $\mathcal{N} = (2,2)$

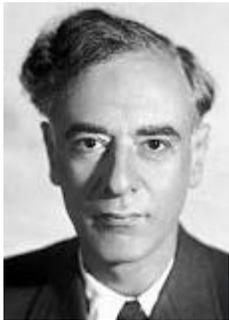
Gauge Invariance and Mass. II*

JULIAN SCHWINGER

Harvard University, Cambridge, Massachusetts

(Received July 2, 1962)

The possibility that a vector gauge field can imply a nonzero mass particle is illustrated by the exact solution of a one-dimensional model.



YOU ARE INVITED TO
WAL-MART
WEEK - LONG
GRAND OPENING
MONDAY JULY 2, THROUGH SAT. JULY 7th.
INCLUDING JULY 4th

22 DEPTS.

- JEWELRY
- DRUGS
- BOOKS
- HOBBIES
- HAIR CARE
- TOYS
- CURTAIN-ACCES.
- PAINT-ACCES.
- SEWING NEEDS
- SPORTING GOODS
- AUTOMOTIVE
- BOYS WEAR

WATCH FOR GRAND OPENING JULY 4

PLENTY OF PARKING

it's here... now!

FREE DOOR PRIZE

NO OBLIGATION - JUST NECESSARY WINNINGS WILL BE NOTIFIED --



- QED₂ : confinement

J.Schwinger (1962)

- QCD₂ : one Regge trajectory

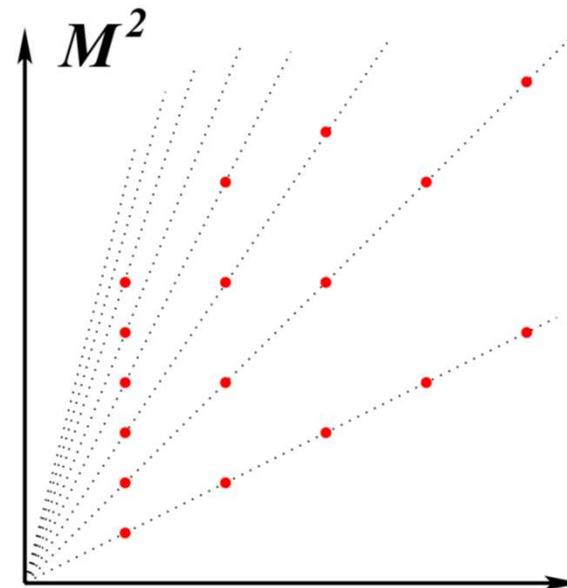
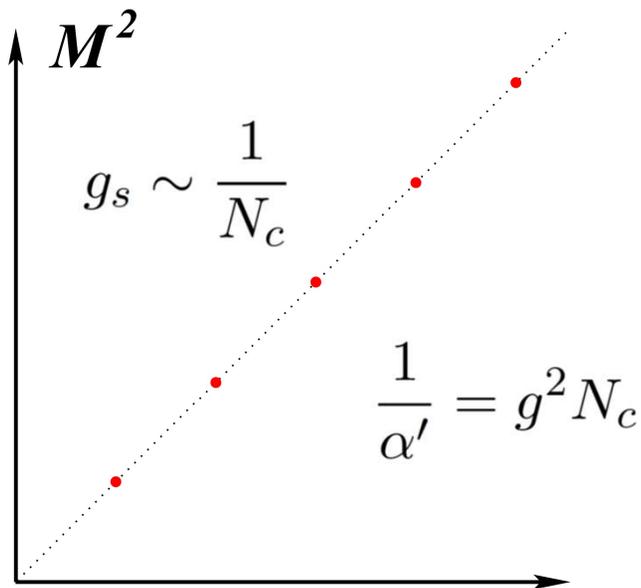
G. 't Hooft (1974)

- adjoint QCD: higher Regge

S.Dalley, I.Klebanov (1992)

D.Kutasov (1993)

D.Gross, I.Klebanov, A.Matytsin, A.Smilga (1995)



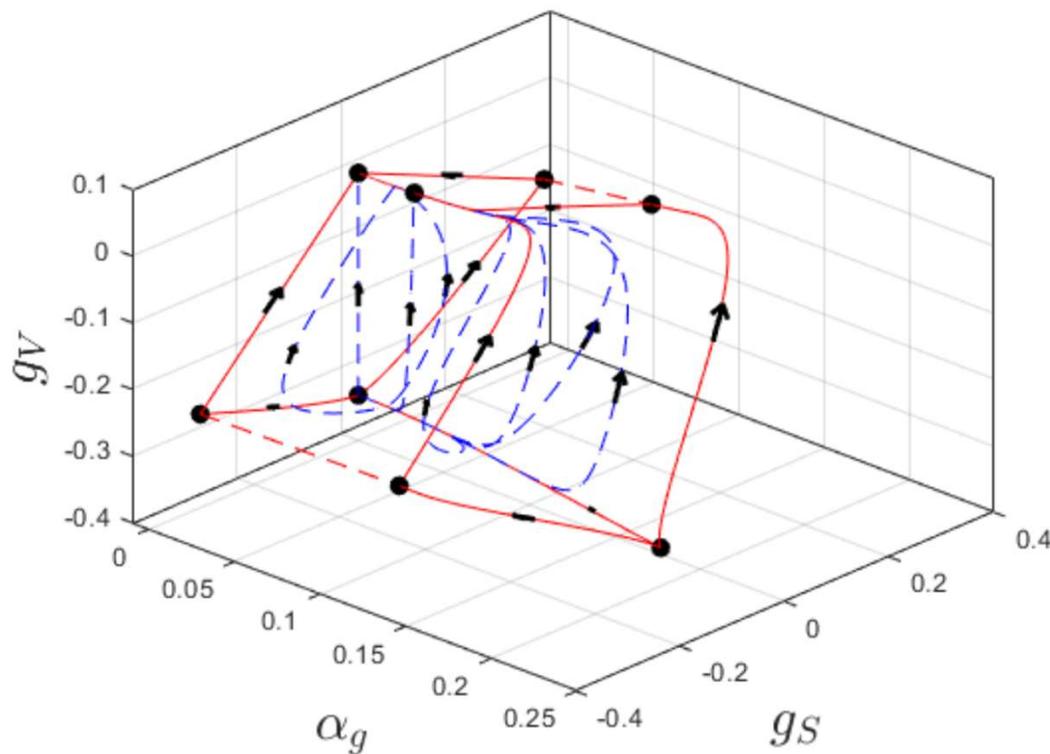
- QED_2 : confinement J.Schwinger (1962)
- QCD_2 : one Regge trajectory G. 't Hooft (1974)
- adjoint QCD: higher Regge S.Dalley, I.Klebanov (1992)
D.Kutasov (1993)
D.Gross, I.Klebanov, A.Matytsin, A.Smilga (1995)
- 2d $\mathcal{N} = (2,2)$ SQCD K.Hori, D.Tong (2006)
O.Aharony, S.Razamat, N.Seiberg, B.Willett (2016)

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- 2d $\mathcal{N} = (0,1)$ SQCD S.G., D.Pei, P.Putrov (2019)

\mathcal{T} = space of Quantum Field Theories
in D dimensions, with a given symmetry,
supersymmetry, ...

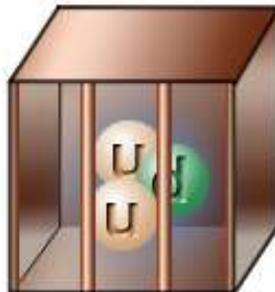
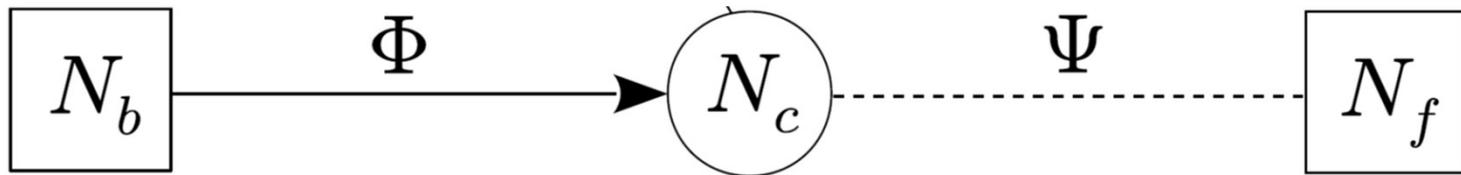


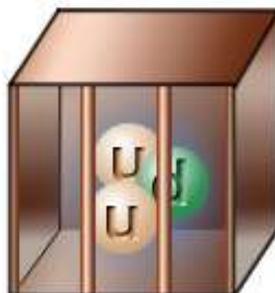
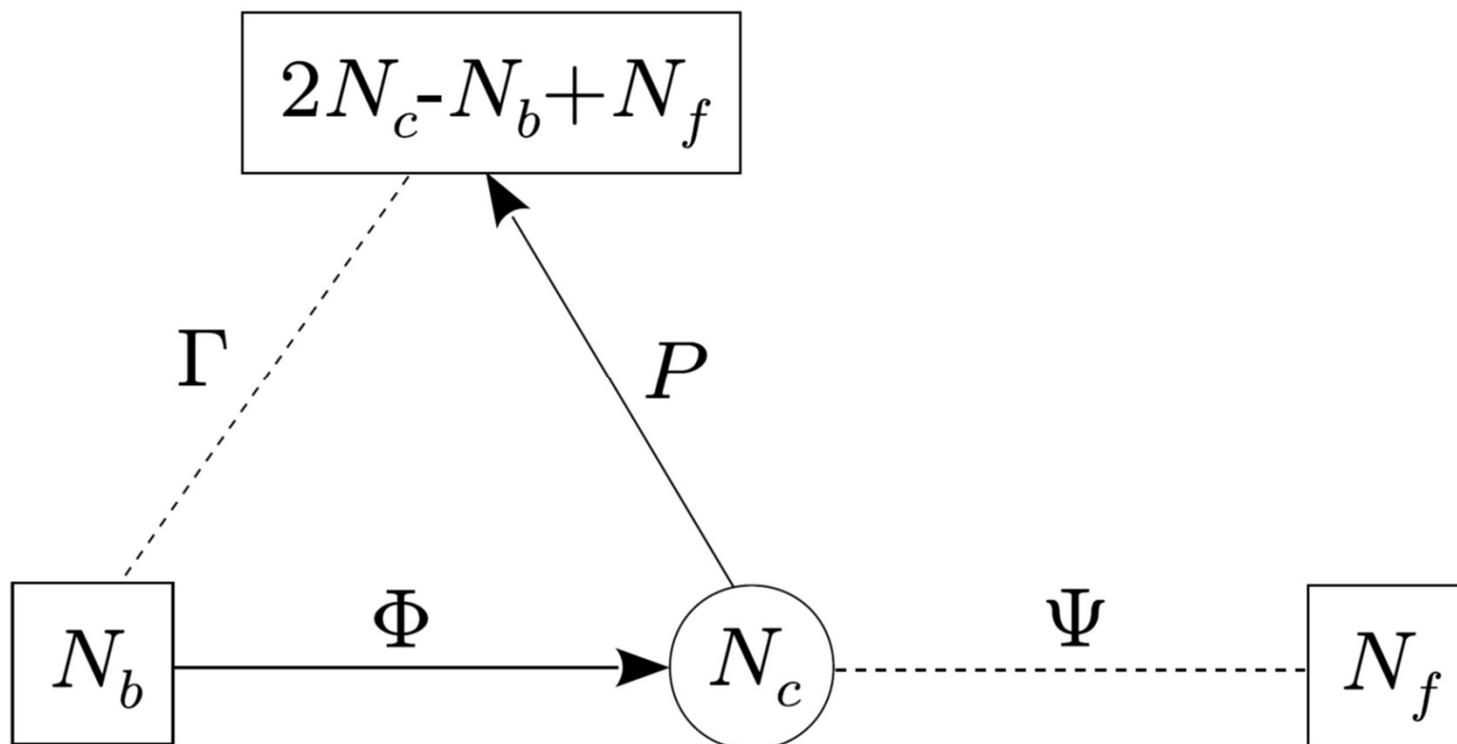
[F.Kuipers, U.Gursoy, Y.Kuznetsov]

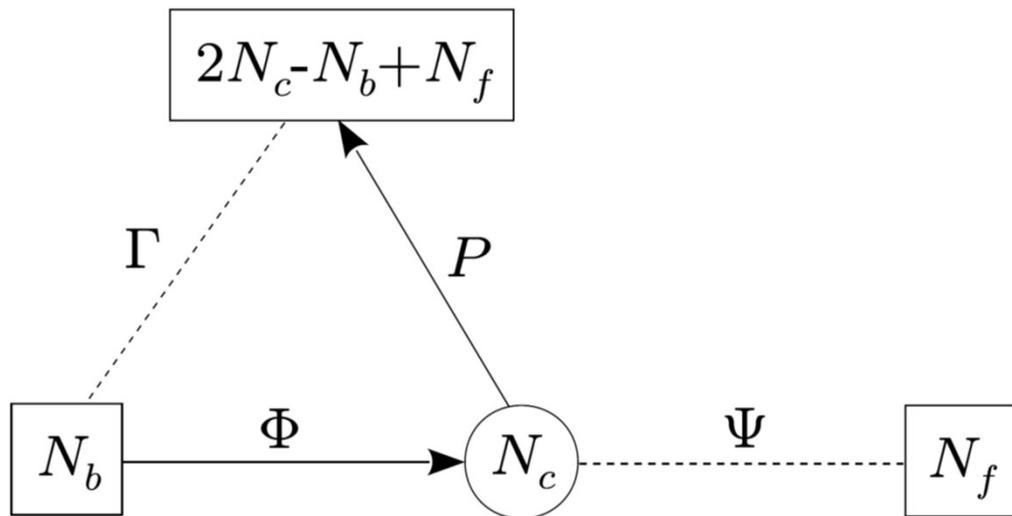
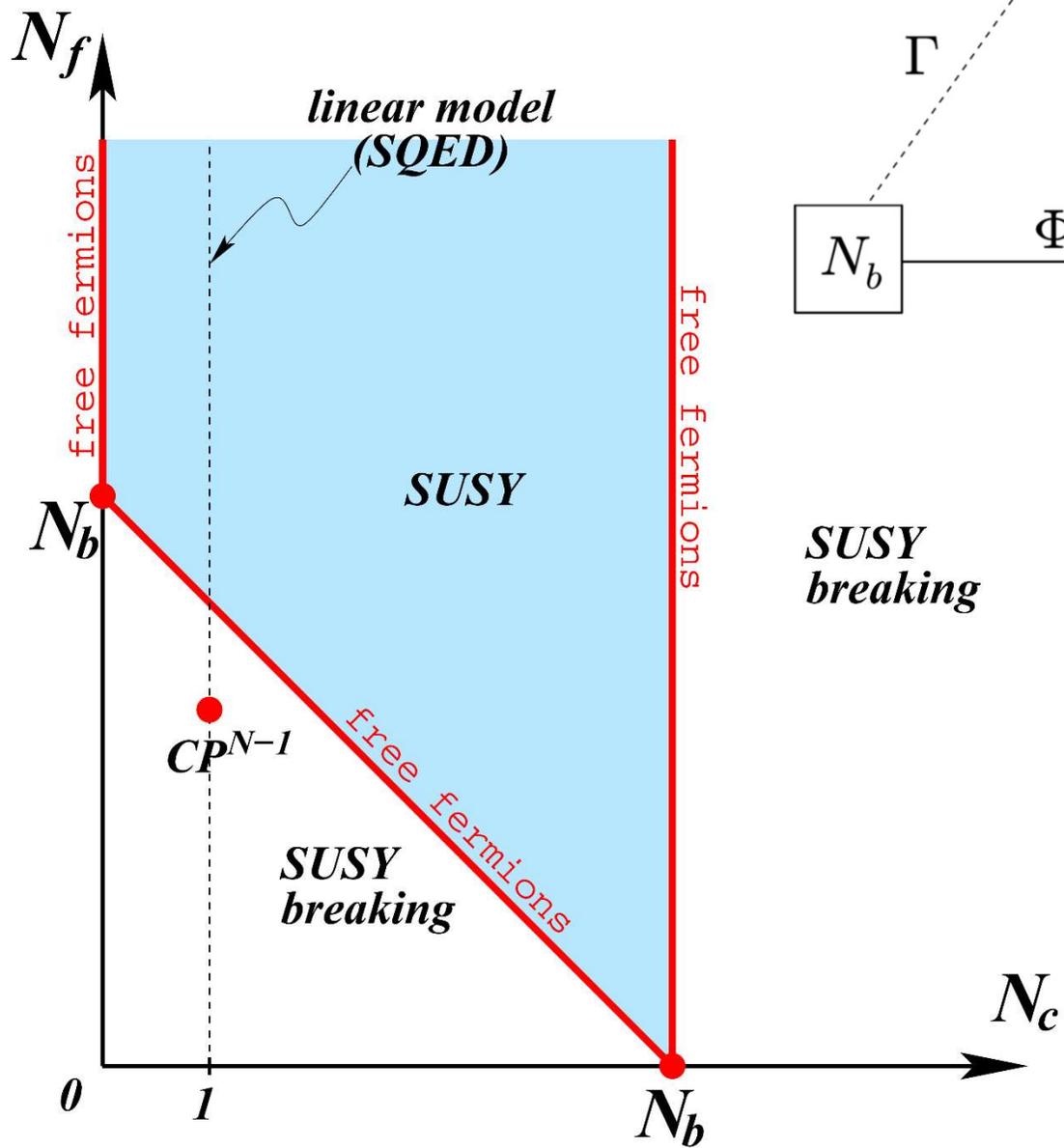
Chapter One

2d (0,2) SQCD

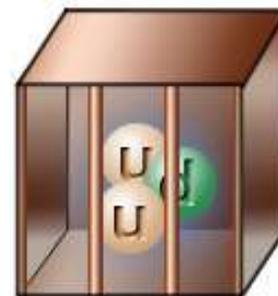
Solving $2d \mathcal{N} = (0,2)$ SQCD



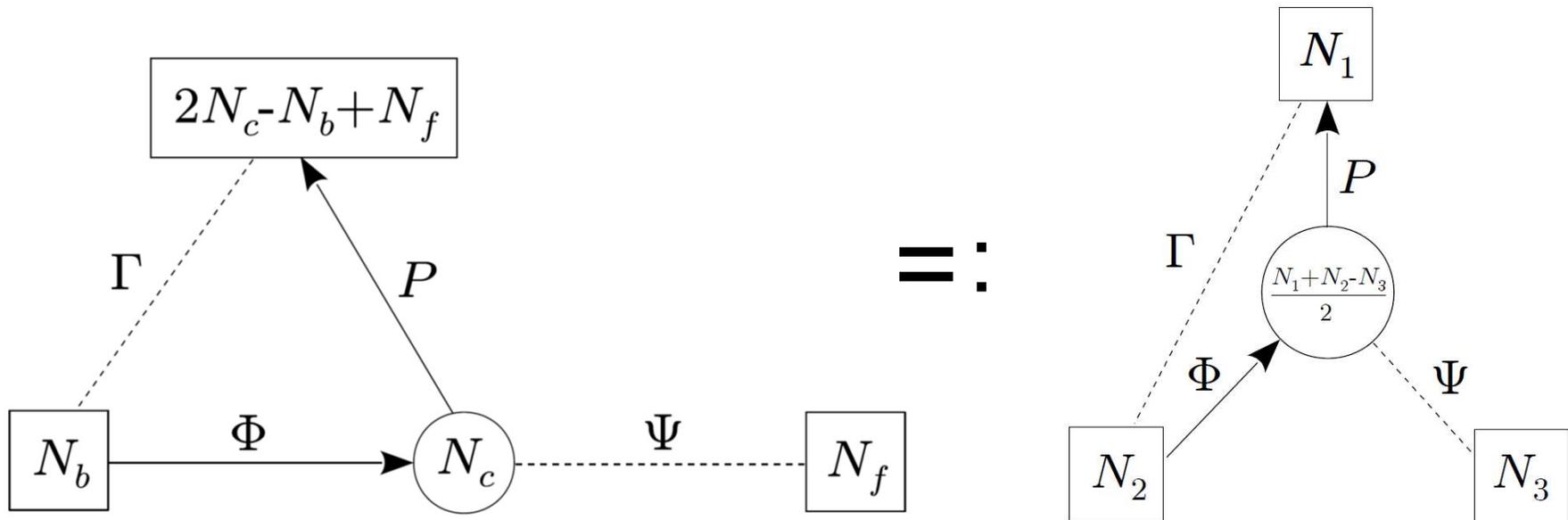




**SUSY
breaking**



New 2d $\mathcal{N} = (0,2)$ dualities



$$\mathcal{T}_{N_1, N_2, N_3} \cong \mathcal{T}_{N_3, N_1, N_2} \cong \mathcal{T}_{N_2, N_3, N_1}$$

$$Gr(k, N) \cong Gr(N - k, N)$$

$$S \rightarrow Gr(k, N) \cong Q^* \rightarrow Gr(N - k, N)$$

$$Q \rightarrow Gr(k, N) \cong S^* \rightarrow Gr(N - k, N)$$

where

$$0 \longrightarrow S \longrightarrow \mathcal{O}^N \longrightarrow Q \longrightarrow 0$$

$$\mathcal{T}_{N_1, N_2, N_3} := \begin{array}{c} \Pi S^{\oplus N_3} \oplus \Pi Q^{\oplus N_2} \\ \downarrow \\ Gr \left(\frac{N_1 + N_2 - N_3}{2}, N_1 \right) \end{array}$$



2d $\mathcal{N} = (0,2)$ SQCD



SU(2) vector

gauge anomaly: $-N_c$

2d $\mathcal{N} = (0,2)$ SQCD



$SU(2)$ vector,
4 fundamental chirals

gauge anomaly: $-2 + 4 \times \frac{1}{2} = 0$

2d $\mathcal{N} = (0,2)$ SQCD



$SU(2)$ vector,
4 fundamental chirals



$\mathcal{N} = (0,2)$ LG model

6 chirals Φ
1 Fermi Ψ

$$J = \Psi \text{Pf}(\Phi)$$

2d $\mathcal{N} = (0,2)$ SQCD



$SU(2)$ vector,
4 fundamental chirals

$\mathcal{N} = (0,2)$ LG model

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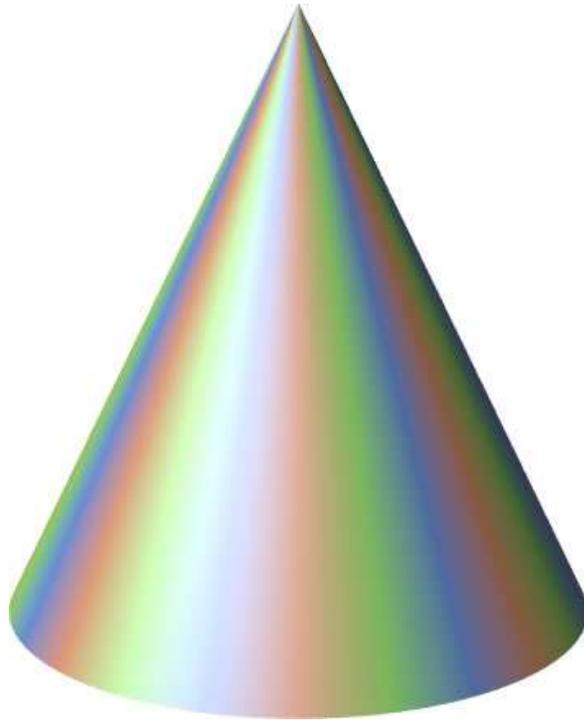
$$J = \Psi \text{Pf}(\Phi)$$



Classical space of vacua = "complex cone" on

$$V^{\otimes 4} // U(2) = \text{Gr}(2, 4) = \{\Phi_{12}\Phi_{12} - \Phi_{13}\Phi_{24} + \Phi_{23}\Phi_{14} = 0\}$$

in $\mathbb{C}\mathbf{P}^5 = \mathbb{C}^6 // U(1)$



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$SU(2)$ vector,
4 fundamental chirals

$\mathcal{N} = (0,2)$ LG model

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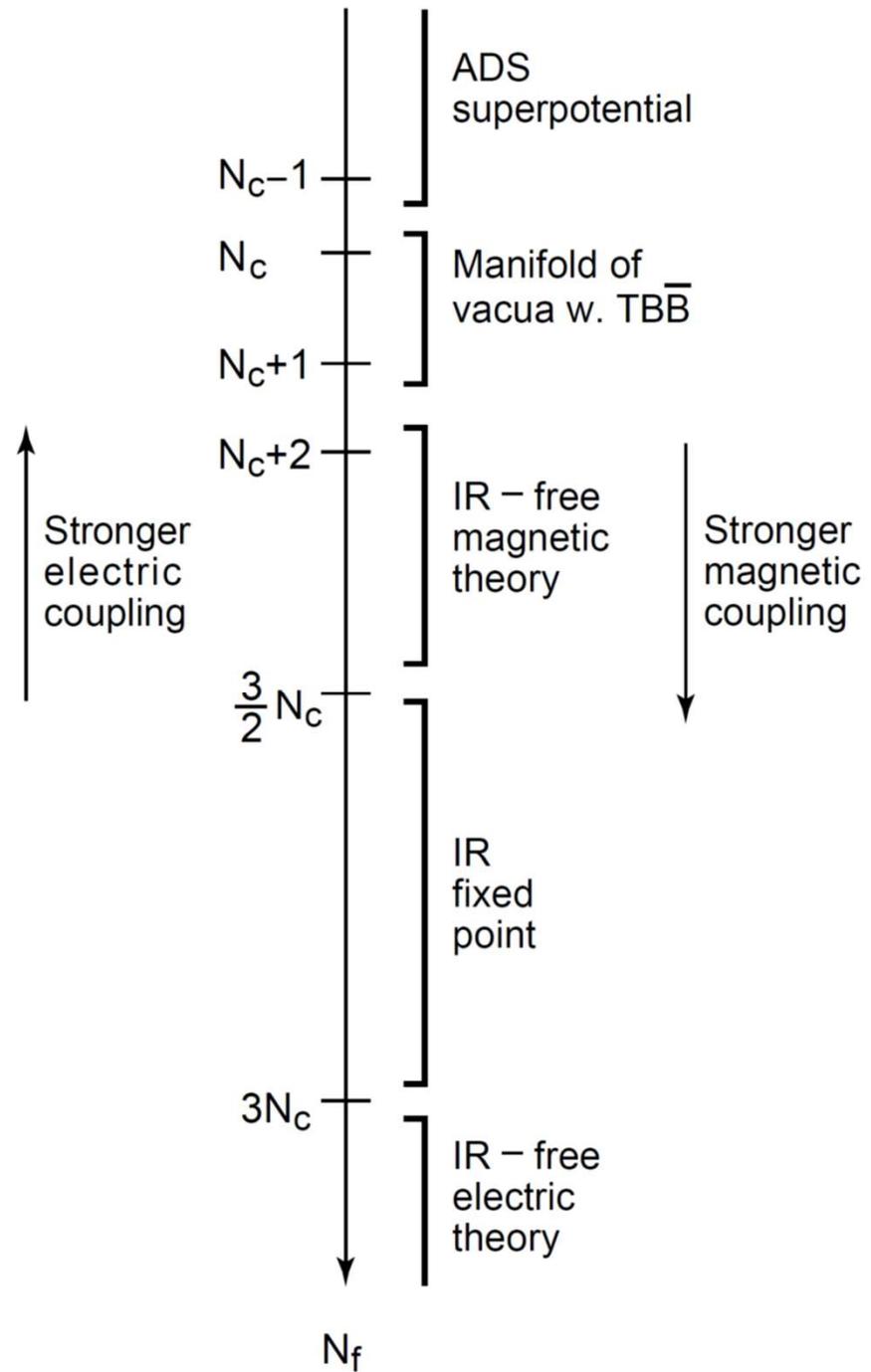
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in $\mathbb{C}\mathbf{P}^5 = \mathbb{C}^6 // U(1)$

Chapter Two

Topological Twists



4d $\mathcal{N} = 1$

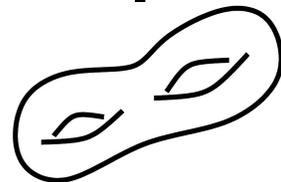
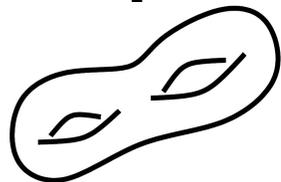
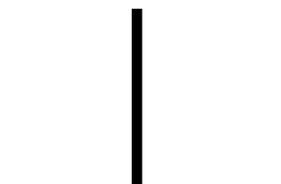
electric



4d $\mathcal{N} = 1$



magnetic



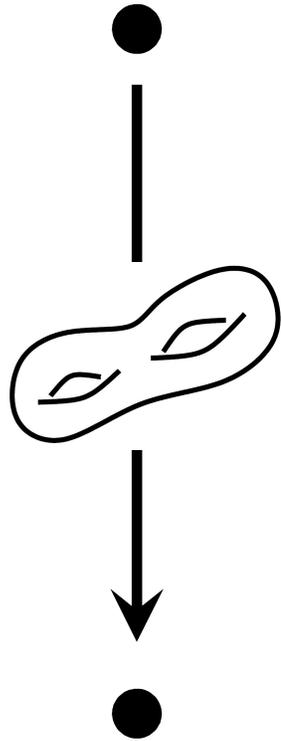
2d $\mathcal{N} = (0,2)$

electric

2d $\mathcal{N} = (0,2)$

magnetic

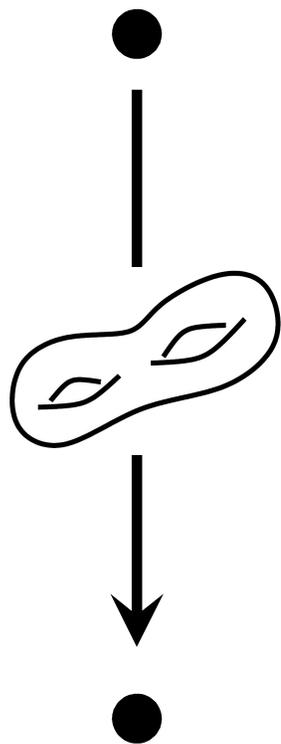
4d $\mathcal{N} = 1$ vector



2d $\mathcal{N} = (0, 2)$

vector,
 \mathfrak{g} adjoint chirals

4d $\mathcal{N} = 1$ chiral



2d $\mathcal{N} = (0, 2)$ $\left\{ \begin{array}{l} H^0(K^{R/2} \otimes L(\mathfrak{m})) \text{ chirals} \\ H^0(K^{1-R/2} \otimes L(-\mathfrak{m})) \text{ Fermi} \end{array} \right.$

4d $\mathcal{N} = 1$ chiral



2d $\mathcal{N} = (0, 2)$ $\left\{ \begin{array}{l} 1-R \text{ chirals (if } R < 1), \\ R-1 \text{ Fermi (if } R > 1) \end{array} \right.$

4d $\mathcal{N} = 1$ SQCD

$SU(2)$

$N_f = 3$



4d $\mathcal{N} = 1$ mesons



$$M_{ij} \sim \epsilon_{ab} q_i^a q_j^b$$



2d $\mathcal{N} = (0,2)$ SQCD

$SU(2)$ with $N_f = 4$

$\mathcal{N} = (0,2)$ LG model

$$J = \Psi \text{Pf}(\Phi)$$

4d $\mathcal{N} = 1$ SQCD

$SU(2)$

$N_f = 3$



4d $\mathcal{N} = 1$ mesons



$$M_{ij} \sim \epsilon_{ab} q_i^a q_j^b$$



$$R = (1, 1, 0, 0, 0, 0)$$

$$R(M_{1i \neq 2}) = R(M_{2j \neq 1}) = 1$$



2d $\mathcal{N} = (0, 2)$ SQCD

$SU(2)$ with $N_f = 4$

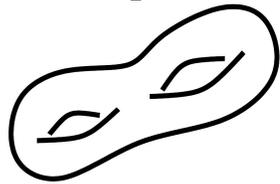
$\mathcal{N} = (0, 2)$ LG model

1 Fermi $\Psi = M_{12}$

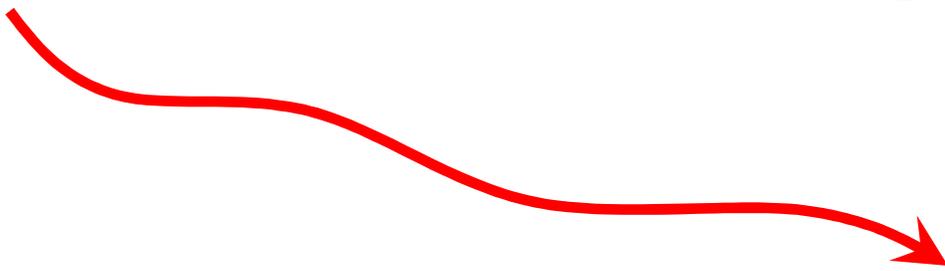
6 chirals $\Phi = M_{ij \neq 1, 2}$

4d $\mathcal{N} = 1$

UV



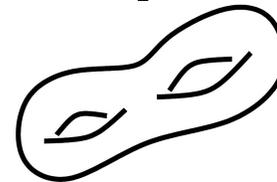
2d $\mathcal{N} = (0,2)$



Non-Lagrangian

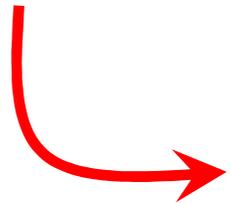
4d $\mathcal{N} = 2$

IR



2d $\mathcal{N} = (2,2)$

4d $\mathcal{N}=2$ theory on $M_4 = S^1 \times M_3$

 3d TQFT on M_3

 MTC (Bethe vacua)

$$Z_{\text{AD}}(S^1 \times M_3) = \sum_{\lambda_v} \prod_{\text{vertices}} S_{0\lambda_v}^{2-\text{deg}(v)} T_{\lambda_v \lambda_v}^{a_v} \prod_{\text{edges}} S_{\lambda_v \lambda'_v}$$

[M.Dedushenko, S.G., H.Nakajima, D.Pei, K.Ye]

Fibonacci MTC

$$S = \frac{2}{\sqrt{5}} \begin{pmatrix} \sin \frac{\pi}{5} & \sin \frac{2\pi}{5} \\ \sin \frac{2\pi}{5} & -\sin \frac{\pi}{5} \end{pmatrix}$$

$$T = \begin{pmatrix} e^{-\frac{\pi i}{15}} & 0 \\ 0 & e^{\frac{11\pi i}{15}} \end{pmatrix}$$



FIBONACCI LEARNS TO COUNT.

$$Z_{\text{AD}}(S^1 \times M_3) = \sum_{\lambda_v} \prod_{\text{vertices}} S_{0\lambda_v}^{2-\text{deg}(v)} T_{\lambda_v \lambda_v}^{a_v} \prod_{\text{edges}} S_{\lambda_v \lambda'_v}$$

[M.Dedushenko, S.G., H.Nakajima, D.Pei, K.Ye]

Chapter Three

2d (0,1) SQCD

- scalar multiplet: $\Phi = \phi + \bar{\theta}_- \psi_+$

$$\frac{1}{2} \int d^2x \left((\partial_\mu \phi)^2 + \bar{\psi}_+ i \not{\partial} \psi_+ \right)$$

- Fermi multiplet: $\Psi = \psi_- + \theta_- F$

$$\frac{1}{2} \int d^2x \left((\bar{\psi}_- i \not{\partial} \psi_- + F^2) \right)$$

[C.Hull, E.Witten]

:

- The (0,1) version of J-interaction:

$$\int d\theta \Psi^a W_a(\Phi) = F^a W_a(\phi) - \frac{\partial W_a}{\partial \phi^i} \bar{\psi}_-^a \psi_+^i$$

- scalar multiplet: $\Phi = \phi + \bar{\theta}_- \psi_+$

$$\frac{1}{2} \int d^2x \left((\partial_\mu \phi)^2 + \bar{\psi}_+ i \not{\partial} \psi_+ \right)$$

- Fermi multiplet: $\Psi = \psi_- + \theta_- F$

$$\frac{1}{2} \int d^2x \left((\bar{\psi}_- i \not{\partial} \psi_- + F^2) \right)$$

[C.Hull, E.Witten]

:

- vector multiplet:

$$L = -\frac{1}{4} \text{Tr} F_{\mu\nu} F^{\mu\nu} + \frac{i}{2} \text{Tr} \bar{\lambda} \not{D} \lambda$$

2d $\mathcal{N} = (0,1)$ SQCD



SU(2) vector

gauge anomaly: $-\frac{1}{2} N_c$

2d $\mathcal{N} = (0, 1)$ SQCD



$SU(2)$ vector,

2 complex fundamental chirals

gauge anomaly:
$$-2 \times \frac{1}{2} + 4 \times \frac{1}{4} = 0$$

2d $\mathcal{N} = (0,2)$ SQCD

$SU(2)$
 $N_f = 4$



$\mathcal{N} = (0,2)$ LG model

$\Psi \text{Pf}(\Phi)$



2d $\mathcal{N} = (0,1)$ SQCD

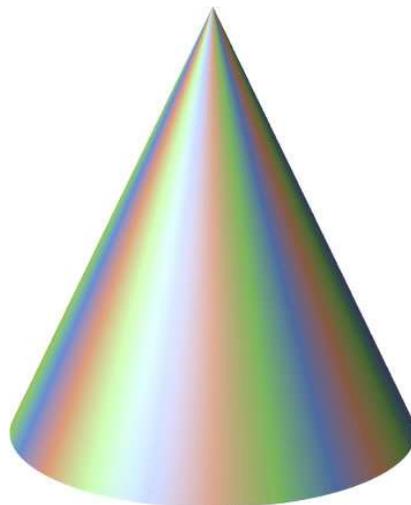
$SU(2)$ with $N_f = 2$

2d $\mathcal{N} = (0,1)$

5 free scalars



Classical space of vacua = cone on $S^7/SU(2) \cong S^4$
 $(\mathbb{C}^2 \otimes \mathbb{C}^2) / SU(2)$

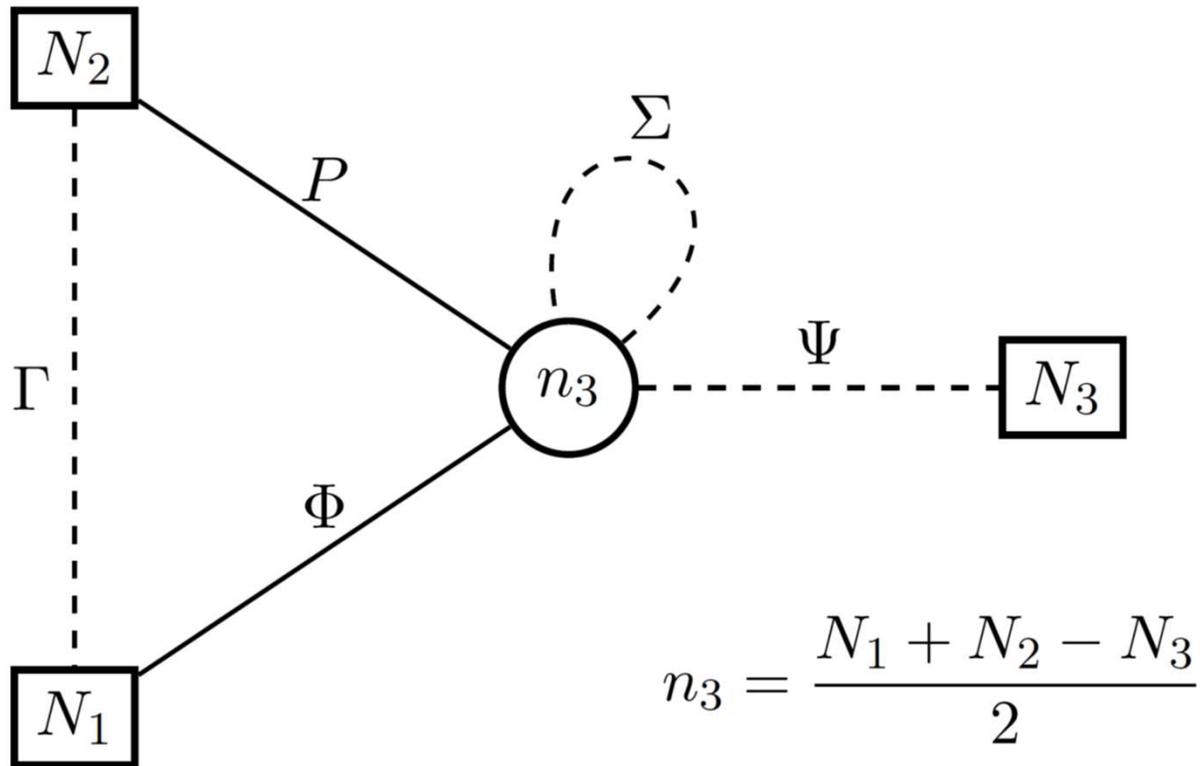


2d $\mathcal{N} = (0, 1)$ SQCD
SU(2) with $N_f = 2$



2d $\mathcal{N} = (0, 1)$
5 free scalars





type	symbol	$SO(n_3)$	$SO(N_1)$	$SO(N_2)$	$SO(N_3)$
scalar	Φ	vector	vector	singlet	singlet
scalar	P	vector	singlet	vector	singlet
Fermi	Ψ	vector	singlet	singlet	vector
Fermi	Γ	singlet	vector	vector	singlet
Fermi	Σ	symmetric	singlet	singlet	singlet

$$\mathcal{T}_{N_1, N_2, N_3} \cong \mathcal{T}_{N_3, N_1, N_2} \cong \mathcal{T}_{N_2, N_3, N_1}$$



generalized $\theta \in \text{Hom} \left(\Omega_2^{\text{Spin}}(BG), U(1) \right)$

non-perturbative
anomaly $\in \text{Hom} \left(\text{Tor } \Omega_3^{\text{Spin}}(BG), U(1) \right)$

Z.-C.Gu, M.Levin
A.Kapustin, R.Thorngren, A.Turzillo, Z.Wang

Example: $G = \mathbb{Z}_2 \rightarrow \text{Hom} \left(\Omega_3^{\text{Spin}}(BG), U(1) \right) \cong \mathbb{Z}_8$

N scalar multiplets and one Fermi

$$\Gamma \left(\sum_{i=1}^N (\phi^i)^2 - r \right)$$





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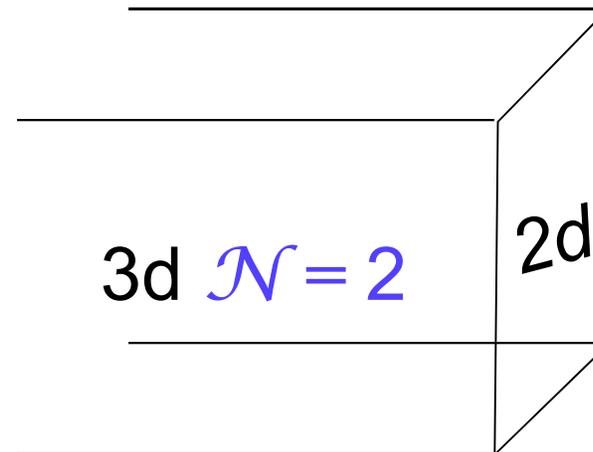
PUBLISHED: May 12, 2014



Walls, lines, and spectral dualities in 3d gauge theories

Abhijit Gadde, Sergei Gukov and Pavel Putrov

*California Institute of Technology,
Pasadena, CA 91125, U.S.A.*

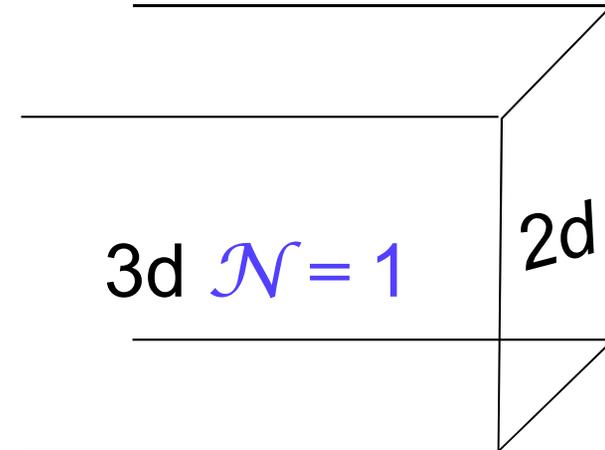




Walls, lines, and spectral dualities in 3d gauge theories

Abhijit Gadde, Sergei Gukov and Pavel Putrov

- 3d gauge with N: $+\frac{1}{2}h$
- 3d gauge with D: $-\frac{1}{2}h$
- 3d level- k Chern-Simons: $+k$
- 3d fermion with dynamical ψ_- on the bdry (D): $+\frac{1}{4}$ or $+\frac{1}{4}T_R$
- 3d fermion with dynamical ψ_+ on the bdry (N): $-\frac{1}{4}$ or $-\frac{1}{4}T_R$
- 2d left MW ψ_- ($= (0, 1)$ Fermi): $+\frac{1}{2}$ or $\frac{1}{2}T_R$
- 2d right MW ψ_+ (as in $(0, 1)$ scalar): $-\frac{1}{2}$ or $-\frac{1}{2}T_R$



Neumann

3d $\mathcal{N} = 1$

$$SU(N)_{k=\frac{N}{2}}$$

Neumann


$$+\frac{N}{2} + \frac{N}{2} = N$$


$$-\frac{N}{2} + \frac{N}{2} = 0$$

2d $\mathcal{N} = (0, 1)$ SQCD

$SU(N)$ with N fundamental chirals

Neumann

3d $\mathcal{N} = 1$

$SU(N_c)$

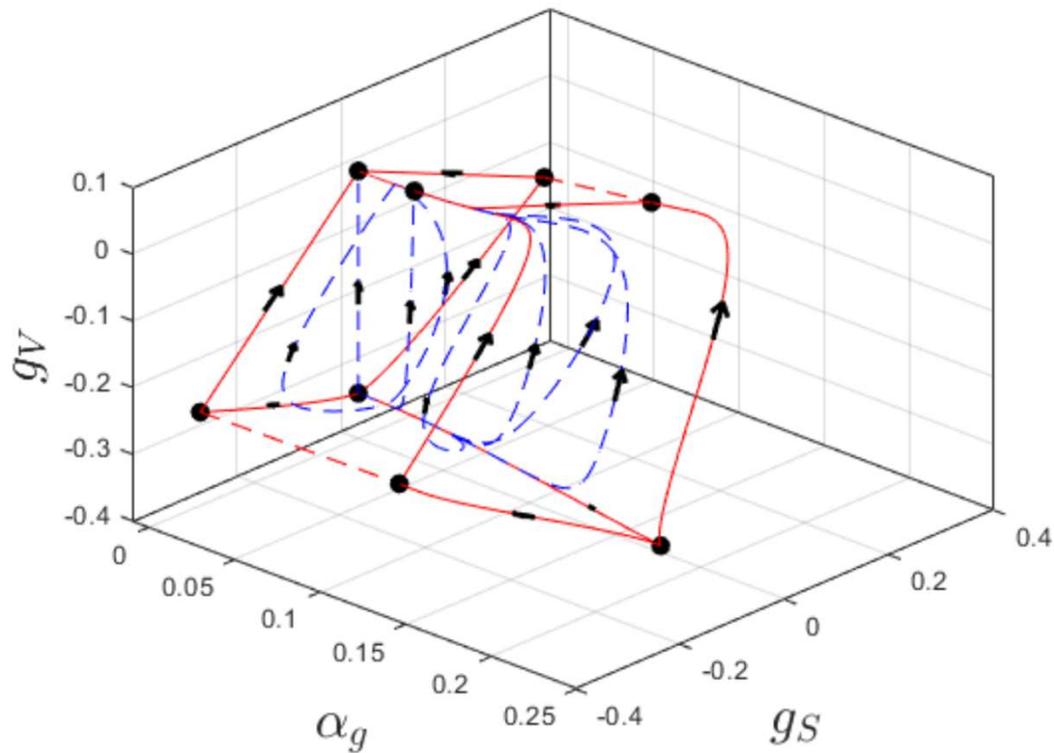
$N_f = N_c$ 3d $\mathcal{N} = 2$ fundamental chirals

Neumann



Chapter Four
New Anomalies

\mathcal{T} = space of 2d (0,1) theories



[F.Kuipers, U.Gursoy, Y.Kuznetsov]

Conjecture:



w/ D.Pei, P.Putrov, C.Vafa

$$\pi_0(\mathcal{T}_n) \cong \pi_*(\mathcal{T}_{*+n})$$



graded by

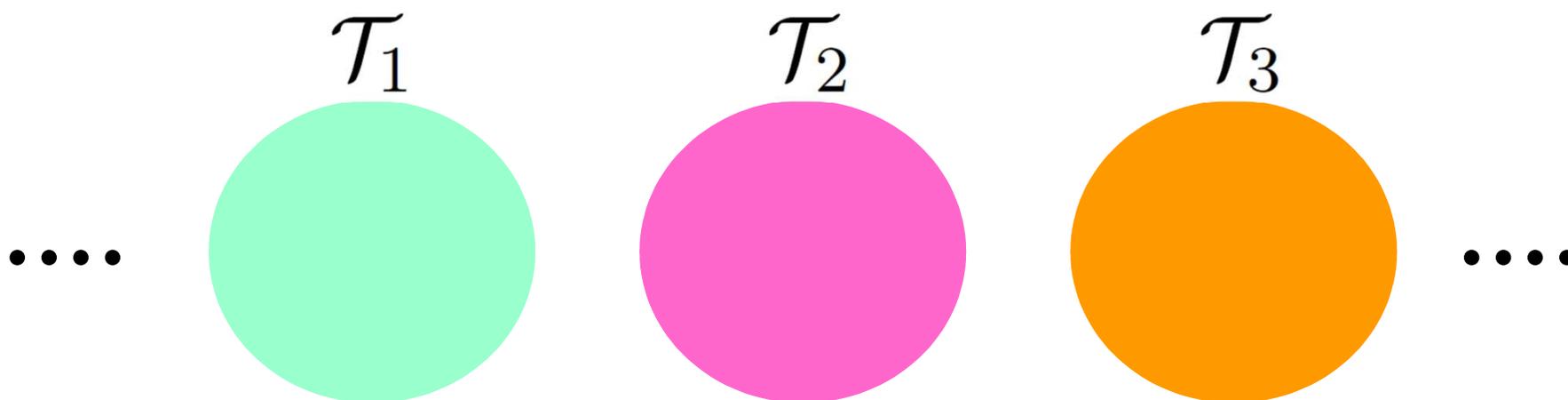
$$n = 2(c_R - c_L)$$

n	...	-4	-3, -2, -1	0	1	2	3	4	...
$\pi_0(\mathcal{T}_n)$...	\mathbb{Z}	0	\mathbb{Z}	\mathbb{Z}_2	\mathbb{Z}_2	\mathbb{Z}_{24}	\mathbb{Z}	...

Conjecture:



w/ D.Pei, P.Putrov, C.Vafa

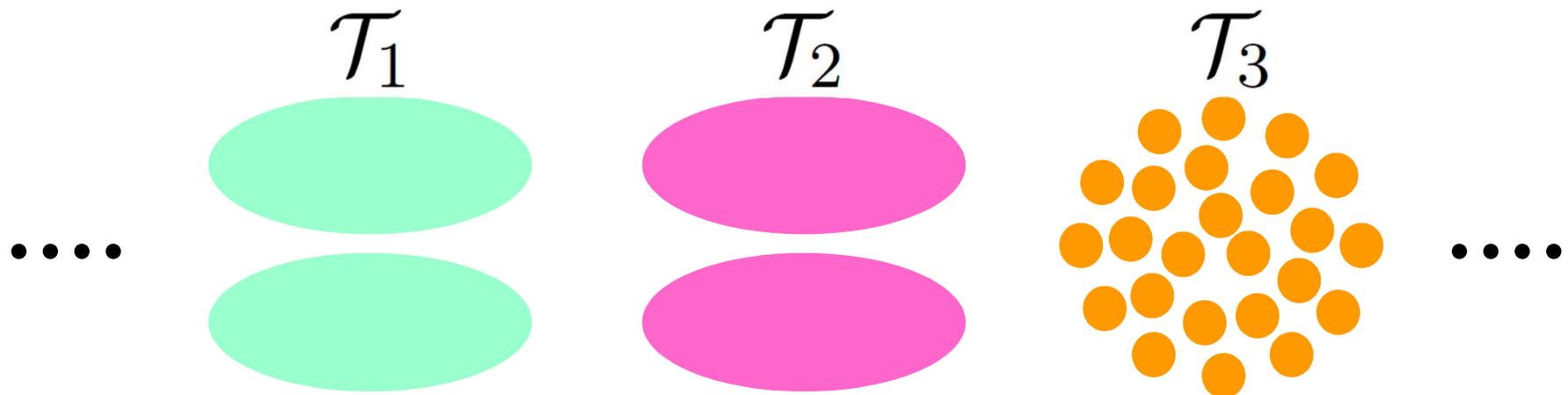


n	\dots	-4	$-3, -2, -1$	0	1	2	3	4	\dots
$\pi_0(\mathcal{T}_n)$	\dots	\mathbb{Z}	0	\mathbb{Z}	\mathbb{Z}_2	\mathbb{Z}_2	\mathbb{Z}_{24}	\mathbb{Z}	\dots

Conjecture:



w/ D.Pei, P.Putrov, C.Vafa



n	\dots	-4	$-3, -2, -1$	0	1	2	3	4	\dots
$\pi_0(\mathcal{T}_n)$	\dots	\mathbb{Z}	0	\mathbb{Z}	\mathbb{Z}_2	\mathbb{Z}_2	\mathbb{Z}_{24}	\mathbb{Z}	\dots

Conjecture:



w/ D.Pei, P.Putrov, C.Vafa

$(\dim \ker Q) \bmod 2$

$(\dim \ker Q|_{F=0}) \bmod 2$



n	...	-4	-3, -2, -1	0	1	2	3	4	...
$\pi_0(\mathcal{T}_n)$...	\mathbb{Z}	0	\mathbb{Z}	\mathbb{Z}_2	\mathbb{Z}_2	\mathbb{Z}_{24}	\mathbb{Z}	...

