

# Uncertainty principle in two-fluid mechanics

*Monday, May 27, 2019 4:30 PM (1 hour)*

Hamilton's principle (or the principle of stationary action) is one of the basic modelling tools in classical mechanics. It states that the reversible motion of a mechanical system is completely determined by the corresponding Lagrangian which is the difference between the kinetic and potential energy of the system.

The extension of Hamilton's principle to the continuum mechanics involving fluid-fluid and solid-fluid interaction can be performed (cf. [1, 2]). The motion of a multi-fluid continuum is described by a coupled system of "Newton's laws" for each component that are completely determined by the Lagrangian. The introduction of dissipative terms compatible with the second law of thermodynamics and natural mathematical restrictions on the potential energy allow us to derive the governing equations having nice mathematical properties.

I will present here a simplest example of two-velocity flows where one of the phases is incompressible (for example, flows of dusty air, or flows of compressible bubbles in an incompressible fluid). A very surprising fact is that one can obtain different governing equations from the same Lagrangian. Different types of the governing equations are due to the choice of independent variables and the corresponding virtual motions. The equations differ from each other in the presence (or not) of gyroscopic forces (also called "lift" forces). The total energy does not depend on these forces, but the velocity distribution depends on them. The gyroscopic forces are not usually taken into account in two-fluid models. Even if these forces have no influence on the hyperbolicity of the governing equations, their presence drastically changes the distribution of the energy of each component.

To the best of my knowledge, such an uncertainty in the governing equations of multi-phase flows was never a subject of discussion in a "multi-fluid" community.

[1] S. Gavriluk, Multiphase flow modelling via Hamilton's principle, In the book : F. dell'Isola, S. L. Gavriluk (Editors), Variational Models And Methods In Solid And Fluid Mechanics, Springer, 2011.

[2] S. Ndanou, N. Favrie, S. Gavriluk, Multi-solid and multi-fluid diffuse interface model: applications to dynamic fracture and fragmentation, J. Comput. Phys. 295(2015) 523–555.

**Presenter:** GAVRILYUK, Sergey (Aix-Marseille Université)