

TITLES AND ABSTRACTS

Mini-course 1: “ p -adic L -functions for $GL(2n)$ via overconvergent cohomology”

p -adic L -functions attached to automorphic representations and their p -adic families provide powerful tools for attacking important problems such as Birch-Swinnerton-Dyer and Bloch-Kato conjectures. However, they are hard to construct and, except in the case of $GL(2)$, the theory is poorly understood beyond the ordinary case.

We are interested in an approach introduced by G. Stevens based on the study of the overconvergent cohomology of locally symmetric spaces. This method was born in the nineties and at the basis of the most general constructions available for $GL(2)$.

In this mini-course we will describe a construction of p -adic L -functions for certain cuspidal automorphic representations of $GL(2n)$ using overconvergent cohomology. This construction extends previous results of Gehrmann/Dimitrov-Januszewski-Raghuram to the non-ordinary setting and allows variation in p -adic families. More precisely, the mini course is divided in three lectures which we describe below.

- **Lecture 1: ‘Critical L -values’** (Andrei Jorza) We recall general conjectures about the existence of p -adic L -functions attached to motives and automorphic representations. Then the lecture is devoted to the study of the critical values of the complex L -function of cuspidal automorphic representations of $GL(2n)$ admitting a Shalika model. In particular we describe such L -values in terms of classical evaluations constructed using the cohomology of the corresponding locally symmetric space and so-called automorphic cycles.
- **Lecture 2: ‘Overconvergent cohomology’** (Daniel Barrera) We introduce and study the overconvergent cohomology adapted to the Shalika setting. Then we describe how to evaluate this cohomology in order to produce distributions over the expected Galois group. Moreover, we verify that this overconvergent evaluation interpolates the classical evaluations explained in the first lecture. Another consequence of this method is the control of the growth of the distribution obtained. The p -adic L -functions are, as usual, the Mellin transform of these distributions.
- **Lecture 3: ‘ p -adic families’** (Chris Williams) The correct eigenvarieties to be considered in the Shalika setting are constructed using the parabolic subgroup of $GL(n)$ having Levi subgroup $GL(n) \times GL(n)$. After the introduction of these parabolic eigenvarieties the talk is devoted to the study of the local properties of them and the existence of Shalika components. We use such results in order to perform a p -adic variation of the distributions obtained in the second lecture. Using the Mellin transform we produce p -adic families of p -adic L -functions.

Mini-course 2: “Triple product periods and explicit class field theory” In this series we explain how triple product periods lead to insights into explicit class field theory for real quadratic fields, focussing on the case of the diagonal restrictions of p -adic families of Hilbert Eisenstein series, which is the case that we have worked out so far in greatest detail.

- **Lecture 1:** (Henri Darmon). General overview of triple product periods. This lecture will describe the general conjectures on triple product periods formulated over the years in joint work with Alan Lauder and Victor Rotger, and discuss a few of their ramifications, including:
 - The connection with generalised Kato classes and their arithmetic applications.
 - Tame variants and the Harris-Venkatesh conjecture.
 - The special case of the adjoint, and a theorem of Rivero-Rotger.
- **Lecture 2:** (Jan Vonk). Rigid meromorphic cocycles and their RM values. This lecture will introduce the basic structures that arise in a p -adic approach to explicit class field theory based on the values at real quadratic arguments of rigid meromorphic cocycles. These values comprise as special cases the Gross-Stark units arising in Gross’s p -adic analogue of the Stark conjecture on p -adic Artin L-series at $s=0$, Stark-Heegner points on (modular) elliptic curves, and singular moduli for real quadratic fields. They can often be expressed in terms of (twisted variants of) the triple product periods covered in Lecture 1.
- **Lecture 3:** (Alice Pozzi). Diagonal restrictions of Hilbert Eisenstein series. This last lecture explains how the diagonal restrictions of the p -adic family of Hilbert modular Eisenstein series for a real quadratic field can be related to RM values of certain rigid analytic cocycles, leading to an interpretation of Gross-Stark units and Stark-Heegner points as triple product periods. The p -adic deformation theory of the weight one Hilbert Eisenstein series, building on the work of Bellaïche-Dimitrov, Darmon-Lauder-Rotger, and Betina-Dimitrov Pozzi, is a key ingredient in some of the most important arithmetic applications.

Mini-course 3: “Higher Hida theory and p -adic L-functions for symplectic groups” (David Loeffler and Vincent Pilloni)

In the first two lectures, Loeffler will recall Hida’s theory of ordinary p -adic families of modular forms, and how it was used to construct p -adic Rankin–Selberg L-functions for $GL_2 \times GL_2$ (by Hida and Panchishkin), and triple-product L-functions for $GL_2 \times GL_2 \times GL_2$ (by Harris–Tilouine and Darmon–Rotger).

Then he will outline the key statements of Pilloni’s higher Hida theory for the symplectic group GSp_4 , which gives an analogous p -adic interpolation results for higher-degree coherent cohomology of Siegel three-folds, and describe how these techniques can be used to construct p -adic L-functions for GSp_4 , $GSp_4 \times GL_2$, and $GSp_4 \times GL_2 \times GL_2$, as in the recent preprint of Loeffler–Pilloni–Skinner–Zerbes.

In the third lecture, Pilloni will outline the proofs of the main theorems of higher Hida theory for GSp_4 , and describe work in progress to generalise these results to higher-rank symplectic groups.

Baskar Balasubramanyam

Title: Critical values of the Asai L-function for GL_n over CM fields

Abstract: Let F be a totally real field, and K a CM extension. For a cuspidal, automorphic, cohomological representation π over GL_n/K , I will talk about the special values at critical points of the Asai L-function associated to π . I will also talk about the special values of its twists by Hecke characters of F .

Amnon Besser

Title: *The toric regulator*

Abstract: I will present a map - The toric regulator, from motivic cohomology of algebraic varieties over p -adic fields with "totally degenerate reduction", e.g., p -adically uniformized varieties, to "toric intermediate Jacobians" which are quotients of algebraic toruses by a discrete subgroup. The toric regulator recovers part of the ℓ -adic etale regulator map for every prime ℓ , and its logarithm recovers part of the log-syntomic regulator. I expect the toric regulator to be the home of "refined" Beilinson conjectures and I will present some evidence for this claim. This is joint work with Wayne Raskind from Wayne State university.

Antonio Cauchi

Title: On the construction of elements in the Iwasawa cohomology of Galois representations for $GSp_{2n} \times GSp_{2n}$

Abstract: The study of arithmetic invariants associated to Galois representations has often relied on the construction of a special family of elements in their Galois cohomology groups. For instance, it has been a crucial ingredient in the work of Kato in the proof of special cases of the conjecture of Birch and Swinnerton-Dyer and the Iwasawa main conjecture for modular forms.

In this talk, we describe how to construct elements in the Iwasawa cohomology of Galois representations associated to a product of two cohomological cuspidal automorphic representations of the similitude symplectic group GSp_{2n} , and, thus, p -adic L-functions using Perrin-Riou's machinery. This construction generalises the one given by Lei-Loeffler-Zerbes when $n = 1$.

Ellen Eischen

Title: Recent developments for p -adic families automorphic forms and L-functions, in the context of unitary groups

Abstract: The p -adic theory of modular forms plays a key role in modern number theory. Geometric developments have enabled vast expansion of Serre's original notion of p -adic modular forms, including by Hida to the case of automorphic forms on unitary groups. This talk will introduce some challenges that arise in the setting of unitary groups, recent efforts to overcome them, and applications.

Harald Grobner

Title: Rationality for Rankin-Selberg L-functions

Abstract: Investigating critical values of Rankin-Selberg L-functions has a long history, both, on the side of results as well as on the side of conjectures. While most of the known results treat the case of

$\mathrm{GL}(n) \times \mathrm{GL}(n-1)$, in this talk we will shade some light on what can be said in the general case $\mathrm{GL}(n) \times \mathrm{GL}(m)$, when the ground field is CM.

Michael Harris

Title: Square root p -adic L-functions

Abstract: The Ichino-Ikeda conjecture, and its generalization to unitary groups by N. Harris, has given explicit formulas for central critical values of a large class of Rankin-Selberg tensor products. Although the conjecture is not proved in full generality, there has been considerable progress, especially for L-values of the form $L(1/2, \mathrm{BC}(\pi) \times \mathrm{BC}(\pi'))$, where π and π' are cohomological automorphic representations of unitary groups $U(V)$ and $U(V')$, respectively. Here V and V' are hermitian spaces over a CM field, V of dimension n , V' of codimension 1 in V , and BC denotes the twisted base change to $\mathrm{GL}(n) \times \mathrm{GL}(n-1)$.

Francesco Lemma

Title: On higher regulators of Siegel varieties.

Abstract: I will present the construction of classes in the middle degree motivic cohomology of Siegel varieties of almost any dimension and the computation of their image by the regulator in Deligne-Beilinson cohomology by some adelic integrals. In the case of the Siegel sixfold, a computation of Pollack-Shah allows to relate the integral to a non-critical value of degree 8 L-functions of $\mathrm{GSp}(6)$. This is a joint work with Antonio Cauchi and Joaquin Rodrigues Jacinto.

Giovanni Rosso

Title: Spin p -adic L -function for GSp_6 .

Abstract: In this talk we shall present the construction of the Spin p -adic L -function for p -ordinary Siegel modular forms of genus 6, using an integral expression due to Pollack. Joint work (still in progress) with E. Eischen and S. Shah.

Eric Urban

Title: Eisenstein congruences and Euler system for Siegel modular forms.

Abstract: I will discuss on some works in progress for the construction of Euler systems attached to the Standard p -adic L-function attached to ordinary Siegel modular forms using congruences between Klingen-type Eisenstein series and cusp forms.

Sarah Zerbes

Title: An explicit reciprocity law for the $\mathrm{GSp}(4)$ -Euler system

Abstract: I will report on work in progress with David Loeffler and Chris Skinner. I will sketch a proof for of an explicit reciprocity law for the Euler system attached to the spin representation of genus 2 Siegel modular forms, relating the Euler system to the spin p -adic L -function that we constructed in joint work with Vincent Pilloni. As an application, we obtain bounds on Selmer groups, conditional on the nonvanishing of non-critical p -adic L -values.