

Mini-course 2: “Triple product periods and explicit class field theory” (Henri Darmon, Alice Pozzi and Jan Vonk)

In this series we explain how triple product periods lead to insights into explicit class field theory for real quadratic fields, focussing on the case of the diagonal restrictions of p -adic families of Hilbert Eisenstein series, which is the case that we have worked out so far in greatest detail.

- **Lecture 1:** (Henri Darmon). General overview of triple product periods. This lecture will describe the general conjectures on triple product periods formulated over the years in joint work with Alan Lauder and Victor Rotger, and discuss a few of their ramifications, including:
 - The connection with generalised Kato classes and their arithmetic applications.
 - Tame variants and the Harris-Venkatesh conjecture.
 - The special case of the adjoint, and a theorem of Rivero-Rotger.
- **Lecture 2:** (Jan Vonk). Rigid meromorphic cocycles and their RM values. This lecture will introduce the basic structures that arise in a p -adic approach to explicit class field theory based on the values at real quadratic arguments of rigid meromorphic cocycles. These values comprise as special cases the Gross-Stark units arising in Gross's p -adic analogue of the Stark conjecture on p -adic Artin L-series at $s=0$, Stark-Heegner points on (modular) elliptic curves, and singular moduli for real quadratic fields. They can often be expressed in terms of (twisted variants of) the triple product periods covered in Lecture 1.
- **Lecture 3:** (Alice Pozzi). Diagonal restrictions of Hilbert Eisenstein series. This last lecture explains how the diagonal restrictions of the p -adic family of Hilbert modular Eisenstein series for a real quadratic field can be related to RM values of certain rigid analytic cocycles, leading to an interpretation of Gross-Stark units and Stark-Heegner points as triple product periods. The p -adic deformation theory of the weight one Hilbert Eisenstein series, building on the work of Bellaïche-Dimitrov, Darmon-Lauder-Rotger, and Betina-Dimitrov Pozzi, is a key ingredient in some of the most important arithmetic applications.