RESURGENCE IN MATHEMATICS AND PHYSICS

A conference organised by
Maxim Kontsevich and Yan Soibelman

11-14 June 2019

Abstracts

- **Jørgen E. ANDERSEN**
  - **Title**: Geometric Recursion with a View Towards Resurgence
  - **Abstract**: We shall review the geometric recursion and its relation to topological recursion. In particular, we shall consider the target theory of continuous functions on Teichmüller spaces and we shall exhibit a number of classes of mapping class group invariant functions, which satisfies the geometric recursion. Many of these classes of functions are integrable over moduli spaces and we prove that there averages over moduli spaces satisfies topological recursion. The talk will end with a discussion of possible resurgence perspectives. The construction of geometric recursion and the results relating it to topological recursion is joint work with Borot and Orantin.

- **Philip BOALCH**
  - **Title**: Topology of the Stokes Phenomenon

- **Olivia DUMITRESCU**
  - **Title**: Lagrangian Fibration of the de Rham Moduli Space and Gaiotto Correspondence
  - **Abstract**: There have been new developments in understanding Lagrangian fibrations of the de Rham moduli space in connection to Lagrangian stratifications of the Dolbeault moduli space through biholomorphic isomorphisms of the Lagrangian fibers. I will report recent results by different groups of authors.

- **Gerald V. DUNNE**
  - **Title**: Resurgence and Phase Transitions

- **Jean ÉCALLE**
  - **Title**: Resurgence's two Main Types and Their Signature Complications: Tessellation, Isography, Autarchy
  - **Abstract**: Quite specific challenges attend the move from equational resurgence (i.e. resurgence in a singular variable – the main type in frequency and importance) to coequational resurgence (i.e. resurgence in a singular parameter – a close second, roughly dual to the first): complexity soars; two Bridge equations are required instead of one; the complex valued Stokes constants make way for discrete tessellation coefficients; the acting alien algebra remains isomorphic to an algebra of ordinary differential operators, but these are now subject to isographic invariance (meaning that they annihilate some specific differential two-form); and lastly, the new resurgence coefficients

possess the paradoxical property of autarchy, combining sectorial resurgence with global entireness.
We shall attempt a comprehensive, up-to-date survey of the field, with emphasis on the rather unexpected and quite novel structures spawned by these two regimes of resurgence.

- **Bertrand EYNARD**
  - **Title**: Considerations about Resurgence Properties of Topological Recursion
  - **Abstract**: To a spectral curve S (e.g. a plane curve with some extra structure), topological recursion associates a sequence of invariants: some numbers $F_g(S)$ and some n-forms $W_{g,n}(S)$. First we show that $F_g(S)$ grow at most factorially at large g, $F_g = O((\beta g)! r^{-g})$ with $r>0$ and $\beta\leq 5$. This implies that there is a Borel transform of $\sum_g \hbar^{2g-2} F_g$ that is analytic in a disk of radius r. The question is whether this is a resurgent series or not?
    We give arguments for this, and conjecture what are the singularities of the Borel transform, and we show how it works on a number of examples.

- **Toshiaki FUJIMORI**
  - **Title**: Bion Saddle Points and Resurgence in CP^N Model
  - **Abstract**: Perturbation series in quantum field theory are generically divergent asymptotic series. Resurgence theory relates such perturbation series and non-perturbative effects which cannot be captured by the perturbative expansion. It has been shown that the so-called bion saddle points play an important role in resurgence theory in a certain class of quantum systems. In this talk, I will overview the recent studies on the bion saddle points in the CP^{N −1} models based on the complexified path integral and the bion saddle points.

- **Stavros GAROUFALIDIS**
  - **Title**: Arithmetic Resurgence of Quantum Invariants
  - **Abstract**: I will explain some conjectures concerning arithmetic resurgence of quantum knot and 3-manifold invariants formulated in an earlier work of mine in 2008, as well as numerical tests of those conjectures and their relations to quantum modular forms, state integrals and their q-series. Joint work (in parts) with R. KASHAEV and D. ZAGIER.

- **Sergei GUKOV**
  - **Title**: Resurgence, Topology and Modularity

- **Mikhail KAPRANOV**

  to be confirmed

- **Maxim KONTSEVICH**
  - **Title**: Resurgence through Path Integrals
  - **Abstract**: I will review the approach to the resurgence phenomenon via integration over rapid decay cycles (Lefschetz thimbles) in path integrals. Examples include WKB asymptotics, heat kernels, WZW models and Chern-Simons theory.

- **Marcos MARIÑO**
  - **Title**: Resurgence for superconductors
  - **Abstract**: One of the most important non-perturbative effects in Nature is the energy gap of superconductors, which is exponentially small in the coupling constant. A natural question is whether this effect can be incorporated in the theory of resurgence.

In this talk I will argue that this is the case. More precisely, I conjecture that the perturbative series for the ground state energy of a superconductor is factorially divergent, and its leading Borel singularity corresponds to the superconducting gap. In the case of the attractive Gaudin-Yang model (a superconductor in one dimension), I develop techniques that make it possible to calculate the exact perturbative series of the ground state energy up to very high order, providing a non-trivial test of this conjecture. For superconductors in three dimensions, evidence for this conjecture can be given by using diagrammatic methods. We also argue that the leading Borel singularity is of the renormalon type, associated to factorially divergent subdiagrams.

- **Takuro MOCHIZUKI**
  - **Title**: Non-abelian Hodge Theory for Monopoles with Periodicity
  - **Abstract**: Recently, we obtained equivalences between monopoles with periodicity and difference modules of various types, i.e., periodic monopoles and difference modules, doubly periodic monopoles and q-difference modules, and triply periodic monopoles and difference modules on elliptic curves.
    In this talk, we shall give a review on the equivalences. If possible, we would like to discuss some deeper aspects of the correspondences from non-abelian Hodge theoretic viewpoints.

- **Jean-Pierre RAMIS**
  - **Title**: The Mano Decompositions and the Space of Monodromy Data of the q-Painlevé V I equation
  - **Abstract**: The talk is based upon a joint work with Y. OHYAMA and J. SAULOY. Classically the space of Monodromy data (or character variety) of PV I (the sixth Painlevé differential equation) is the space of linear representations of the fundamental group of a 4-punctured sphere up to equivalence of representations. If one fixes the local representation data it "is" a cubic surface. We will describe a q-analog: the space of q-Monodromy data of the q-Painlevé V I equation. For the q-analogs of the Painlevé equations (which are non-linear q-difference equa-tions), according to H. SAKAI work, "*everything*" is well known on the "*left side*" of the (q-analog of the) Riemann-Hilbert map (the varieties of "initial conditions"), but the "*right side*" (the q-analogs of the spaces of Monodromy data or character varieties) remained quite mysterious.
    We will present a complete description of the space of Monodromy data of q–PV I (some local data being fixed). It is a "*modification*" of an elliptic surface and we will explicit some "*natural*" parametrizations. This surface is analytically, but not algebraically isomorphic to the Sakai surface of "*initial conditions*". Our description uses a new tool, the Mano decompositions, which are a q-analog of the classical pants decompositions of surfaces. We conjecture that our constructions can be extended to the others q–Painlevé equations. This involves q-Stokes phenomena.

- **David SAUZIN**
  - **Title**: On the Resurgent WKB Analysis
  - **Abstract**: I will report on a work in progress with F. FAUVET *(Université de Strasbourg)* and R. SCHIAPPA *(University of Lisbon)* about the WKB formal expansions solutions to the 1D stationary Schrödinger equation with polynomial coefficients. Our emphasis is on the coequational resurgent structure, in the sense of J. ÉCALLE, which is a way of using the singularity structure in the Borel plane to access the connection formulae for the WKB solutions and the Voros data.

- **Ricardo SCHIAPPA**
  - **Title**: Resurgence, Matrices and Strings
  - **Abstract**: I will review older work, and report on work in progress, concerning applications of resurgence and trans series within the realms of matrix models and minimal/topological strings.

- **Carlos SIMPSON**
  - **Title**: On the Laplace Transform of the Monodromy as a Function of the Perturbation Parameter in WKB-Voros Resurgence
  - **Abstract**: We consider the Voros resurgence or WKB problem of Monodromy for a family of connections of the form \nabla + t.\Phi, and look at the transport along a path as a function of t. Taking the Laplace transform, we discuss the analytic continuation properties leading to asymptotic estimates for the Monodromy as t --> \infty. If time permits, we'll discuss possible relations with spectral networks and harmonic mappings to buildings.

- **Yan SOIBELMAN**
  - **Title**: Wall-crossing formulas and resurgence
  - **Abstract**: In the joint paper with M. KONTSEVICH (*arXiv:0811.2435*) among other things we introduced the notion of stability data on graded Lie algebras, upgraded later to the notion of wall-crossing structure in arXiv:1303.3253. Both notions turned out to be suitable for spelling out wall-crossing formulas in various circumstances, in particular in Donaldson-Thomas theory of 3-dimensional Calabi-Yau categories as well as supersymmetric gauge theories in dimensions two and four.
    Few years ago, we wrote a paper (yet unpublished) devoted to applications of that to algebraic, analytic and resurgent properties of series arising in wall-crossing formulas. Aim of the talk is to discuss some of these ideas.

- **Mithat ÜNSAL**
  - **Title**: Semi-classics, mixed anomalies and resurgence in 2d QFT
  - **Abstract**: I will discuss new methods in QFTs which allows us to study the non-perturbative dynamics of (non-)supersymmetric theories. The main tools are mixed anomalies, semi-classical methods, resurgence and quantum distillation. As an application, I will describe the charge-q Schwinger model and two-dimensional sigma models, such as CP^{N-1} model compactified on cylinder. Using certain boundary conditions which guarantee persistence of mixed anomaly upon compactification reveals a large set of new saddle points, which play instrumental role in non-perturbative dynamics.

- **André VOROS**
  - **Title**: Resurgent Theta-functions: a conjectured gateway into dimension D>1 quantum mechanics
  - **Abstract**: Resurgent analysis of the stationary Schrödinger equation (exact-WKB method) has remained exclusively confined to 1D systems due to its underlying linear-ODE techniques. Here, building on a solvable 2D case (a Selberg trace formula, as analyzed with P. CARTIER), and on a Balian-Bloch abstract quantum framework in any dimension using complex orbits, we isolate a very special generalized-heat-trace function as best candidate to start some resurgent description of quantum mechanics in general dimension. The latter statement is still quite embryonic and speculative - our main hope is to encourage future research.