On dressing factors of 2-dimensional Toda field theories and multicomponent MKdV equations

Gerdjikov V. S.[1,2]

[1] *National Research Nuclear University MEPHI, 31 Kashirskoe Shosse, 115409 Moscow, Russian Federation*

[2] *Institute of Mathematics and Informatics Bulgarian Academy of Sciences, 1113 Sofia, Bulgaria*

vgerdjikov@math.bas.bg

In 1981 A. V. Mikhailov [1] introduced the reduction group for soliton equations and as a result discovered the class of 2-dimensional Toda field theories (2d-TFT) related to the algebras $sl(n)$. Soon after that it was established that 2d-TFT can be related to each simple Lie algebra [2] and Kac-Moody algebra [3]. With each Kac-Moody algebra one can relate also a hierarchy of nonlinear evolution equations with typical representative a multi-component MKdV equations [3].

The Lax representations of 2d-TFT related to the simple Lie algebra $\mathfrak{g}$ graded by its Coxeter automorphism C is [2]:

$$\begin{aligned} L\psi &\equiv i\frac{\partial\psi}{\partial x} + (U_0(x,t) - \lambda U_1(x,t))\psi = 0, \qquad U_0 = i\frac{\partial\phi}{\partial x}, \qquad U_1 = \sum_{\alpha\in A} e^{\phi_\alpha} E_\alpha \\ M\psi &\equiv i\frac{\partial\psi}{\partial t} + (V_0(x,t) - \lambda^{-1} V_1(x,t))\psi = 0, \qquad V_0 = -i\frac{\partial\phi}{\partial t}, \qquad V_1 = \sum_{\alpha\in A} e^{\phi_\alpha} E_{-\alpha}. \end{aligned} \tag{1}$$

Here the real-valued function $\phi(x,t)\in\mathfrak{h}$ and A is the set of admissible roots of $\mathfrak{g}$, see [2]. The Lax pair (1) possesses $\mathbb{D}_h$ as Mikhailov reduction group where h is the Coxeter number of $\mathfrak{g}$, $C^h=1$. The corresponding 2d-TFT take the form:

$$2\frac{\partial\phi}{\partial x\partial t} = \sum_{\alpha\in A} \alpha\, e^{2\phi_\alpha(x,t)}. \tag{2}$$

The Lax representation of the multicomponent MKdV equations involves the same operator L, while the second operator M typically is a cubic polynomial of λ.

Our aim is to extend the Zakharov-Shabat dressing method [4,5] and to construct dressing factors $u(x,t,\lambda)$ for $\mathbb{D}_h$-graded Lax pairs like (1). This allows us to derive the soliton solutions of the 2d-TFT (2) and for the multicomponent MKdV equations.

REFERENCES

1. *Mikhailov, A. V.* The reduction problem and the inverse scattering problem. Physica D, **3D**, n. 1/2, 73–117, 1981.
2. *Mikhailov, A. V., Olshanetsky, M. A., Perelomov A. M.* Two-Dimensional Generalized Toda Lattice, Commun. Math. Phys. **79**, 473-488 (1981).
3. *Drinfel'd, V., Sokolov. V. V.* Lie Algebras and equations of Korteweg - de Vries type. Sov. J. Math. **30**, 1975–2036 (1985).
4. *Zakharov, V. E., Shabat, A. B.* A scheme for integrating nonlinear equations of mathematical physics by the method of the inverse scattering transform. I. Funct. Annal. and Appl. **8**, no. 3, 43–53 (1974); II. Funct. Anal. Appl. **13**(3) 13-23, (1979).
5. *Zakharov, V. E., Mikhailov, A. V.* On the Integrability of Classical Spinor Models in Two-Dimensional Space-Time. Commun. Math. Phys. **74**, 2140 (1980)