Modelling liquid foams with an elastoviscoplastic fluid

Prediction of shearband and smooth or abrupt localization

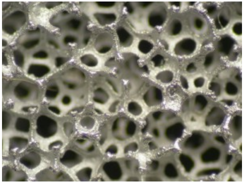
Pierre Saramito[1]

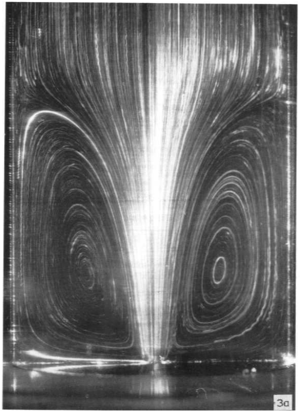
Ibrahim Cheddadi[2] and François Graner[3]

1: LJK, CNRS and univ. Grenoble, France
2: TIMC, univ. Grenoble, France
3: MSC, CNRS and univ. Paris Diderot, France

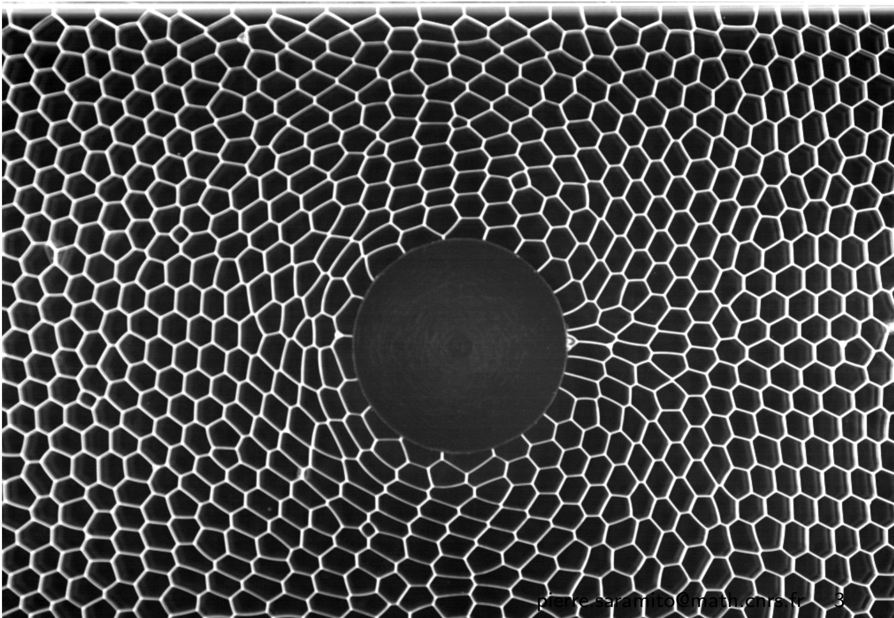
pierre.saramito@math.cnrs.fr

Motivations





Liquid foam: a model material



Outline

1. Mathematical model
2. Test: Couette flow
3. Test: flow around a cylinder

Viscoplastic fluid: Herschel & Bulkley (1926)

$$\begin{cases} \tau = K|\dot{\varepsilon}|^{n-1}\dot{\varepsilon} + \tau_Y \dfrac{\dot{\varepsilon}}{|\dot{\varepsilon}|} & \text{when } \dot{\varepsilon} \neq 0 \\ |\tau| \leqslant \tau_Y & \text{when } \dot{\varepsilon} = 0 \end{cases}$$

$\Leftrightarrow$

$$\max\left(0,\ \frac{|\tau| - \tau_Y}{K|\tau|^n}\right)^{\frac{1}{n}} \tau = \dot{\varepsilon}$$

and

$$\sigma = -pI + \tau = \text{Cauchy stress}$$

Notes

- material = rigid solid, then Newtonian fluid
- $|\tau| \leqslant \tau_Y \Rightarrow \tau$ undetermined
- $n \in \mathbb{R}^+$: power law index
- $n = 1$: Bingham (1920)

Viscoelastic fluid: Oldroyd (1950)

$$\frac{1}{G} \overset{\nabla}{\tau} + \frac{1}{\eta_m} \tau = \dot{\varepsilon}$$

and

$$\sigma = -pI + 2\eta_s D(\mathbf{v}) + \tau = \text{Cauchy stress}$$

Notes

- *objective* tensor derivative

$$\overset{\nabla}{\tau} = \frac{\partial \tau}{\partial t} + \mathbf{v}.\nabla \tau - \tau.\nabla \mathbf{v}^T - \nabla \mathbf{v}.\tau.$$

- requires an initial condition:

$$\tau(t=0) = \tau_0$$

- $\eta_s = 0$: Maxwell (1967)

Elastoviscoplastic fluid

$$\frac{1}{G}\overset{\nabla}{\tau} + \max\left(0,\ \frac{|\tau| - \tau_Y}{K|\tau|^n}\right)^{\frac{1}{n}} \tau = \dot{\varepsilon}$$

and

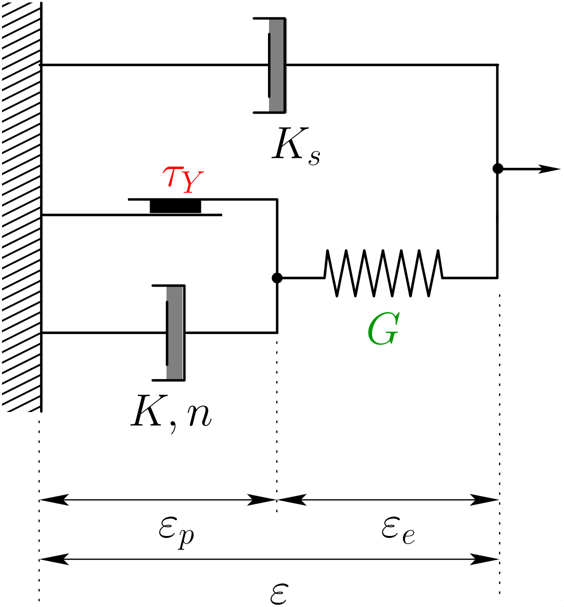
$$\sigma = -pI + 2K_s D(\mathbf{v}) + \tau = \text{Cauchy stress}$$

Notes

- $1/G = 0$: Herschel-Bulkley
- $\tau_Y = 0$ and $n = 1$: Oldroyd
- $1/G = \tau_Y = 0$ and $n = 1$: Navier-Stokes

[Saramito, JNNFM, 2007 & 2009]

Elastoviscoplastic fluid



Thermodynamics

$$\varepsilon = \varepsilon_e + \varepsilon_p$$

Free energy and dissipation potential:

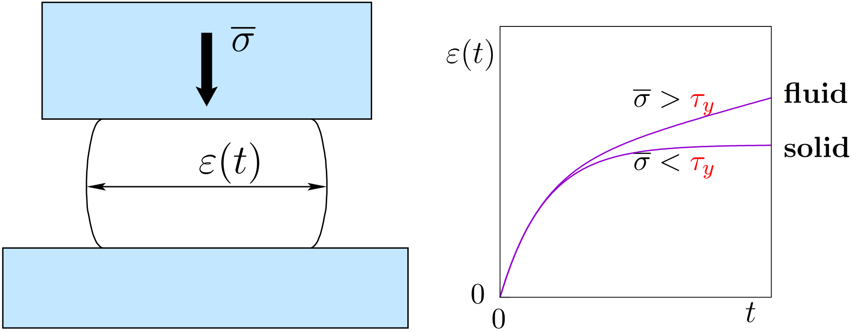
$$\begin{aligned} \mathcal{E}\left(\varepsilon,\, \varepsilon_e\right) &= G\left|\varepsilon_e\right|^2 \\ \mathcal{D}\left(\dot\varepsilon,\, \dot\varepsilon_e\right) &= K_s\left|\dot\varepsilon\right|^2 + K\left|\dot\varepsilon - \dot\varepsilon_e\right|^n + \tau_Y\left|\dot\varepsilon - \dot\varepsilon_e\right| \end{aligned}$$

Properties

- Onsager symmetry principle satisfied
- $\mathcal{D}$ convex $\Longrightarrow$ the second law is satisfied

[Halphen & NGuyen, 1975]

Creeping test



material $=$ viscoelastic **solid**, then viscoelastic **fluid**

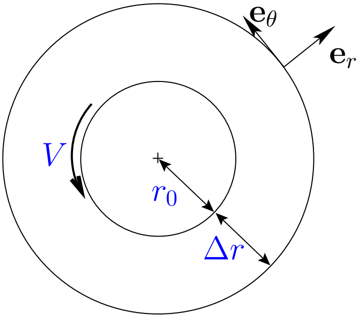
Problem statement

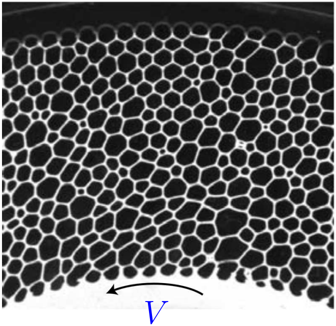
(P): find τ, $\mathbf{v}$ and p such that

$$\begin{aligned}
\frac{1}{G}\stackrel{\nabla}{\tau} + \max\left(0, \frac{|\tau| - \tau_Y}{K|\tau|^n}\right)^{\frac{1}{n}} \tau - 2D(\mathbf{v}) &= 0 \\
\rho\left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}.\nabla\mathbf{v}\right) - \mathbf{div}\{-p.I + 2K_s D(\mathbf{v}) + \tau\} &= \mathbf{f} \\
-\mathrm{div}\,\mathbf{v} &= 0 \\
+B.C. + I.C.
\end{aligned}$$

Implementation: Rheolef FEM C++ library (free software)

Couette Geometry





$$\mathbf{v} = \begin{pmatrix} 0 \\ v_\theta(r) \end{pmatrix} \qquad \tau = \begin{pmatrix} \tau_{rr}(r) & \tau_{r\theta}(r) \\ \tau_{\theta r}(r) & \tau_{\theta\theta}(r) \end{pmatrix}$$

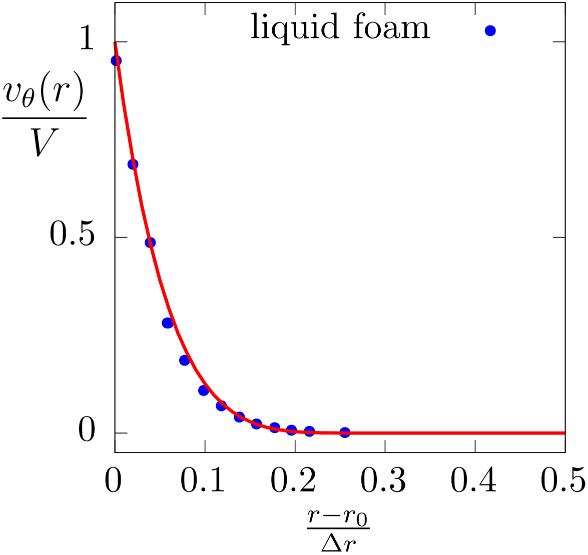
Two dimensionless numbers

$$We = \frac{\eta\dot{\gamma}}{G} \qquad \text{viscoelasticity}$$

$$Bi = \frac{\tau_Y}{\eta\dot{\gamma}} \qquad \text{viscoplasticity}$$

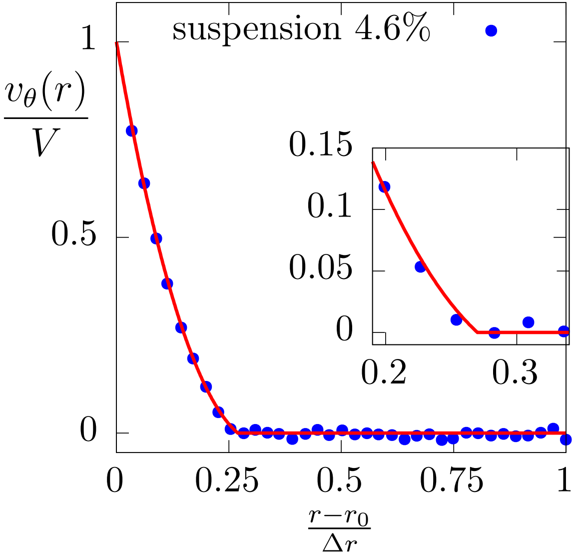
with $\eta = K\dot{\gamma}^{n-1}$: dimension of a viscosity
$\dot{\gamma} = V/\Delta r$: representative shear rate

Observation: (1) smooth profile



experiment: *Debrégeas et al., PRL, 2001*

Observation: (2) abrupt profile



experiment: *Coussot et al., PRL, 2002*

10 years of questioning

year	profil	authors	material
2001	smooth	*Debrégeas* et al.	liquid foam
2002	abrupt	*Coussot* et al.	suspensions, emulsions
2003	abrupt	*Salmon* et al.	wormlike micelles
2004	abrupt	*Lauridsen* et al.	liquid foam
2006	abrupt	*Gilbreth* et al.	liquid foam
2008	abrupt	*Dennin* et al.	liquid foam

$\Longrightarrow$ are profils abrupts ?

10 years of questioning

year	profil	authors	material
2001	smooth	*Debrégeas* et al.	liquid foam
2002	abrupt	*Coussot* et al.	suspensions, emulsions
2003	abrupt	*Salmon* et al.	wormlike micelles
2004	abrupt	*Lauridsen* et al.	liquid foam
2006	abrupt	*Gilbreth* et al.	liquid foam
2008	abrupt	*Dennin* et al.	liquid foam
2008	smooth	*Kätgert* et al.	liquid foam
2010	smooth	*Coussot* et al.	suspensions, emulsions
2010	smooth	*Ovarlez* et al.	liquid foam
2010	smooth	*Kätgert* et al.	liquid foam

$\Longrightarrow$ are profiles smooth ?

$\Longrightarrow$ computations + theoretical interpretation

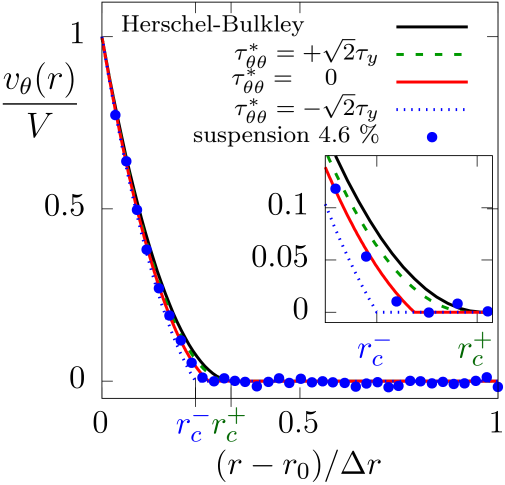
Initial stress influence ?

$$\tau(t=0) = \begin{pmatrix} \tau_{rr} & \tau_{r\theta} \\ \tau_{\theta r} & \tau_{\theta\theta} \end{pmatrix} \Leftarrow \begin{pmatrix} 0 & 0 \\ 0 & \tau^*_{\theta\theta} \end{pmatrix}$$

Three extremal cases:

$\tau^*_{\theta\theta}$	
0	no pre-stress
$+\sqrt{2}\tau_Y$	pre-stress
$-\sqrt{2}\tau_Y$	pre-stress

Computation: (1) abrupt profile

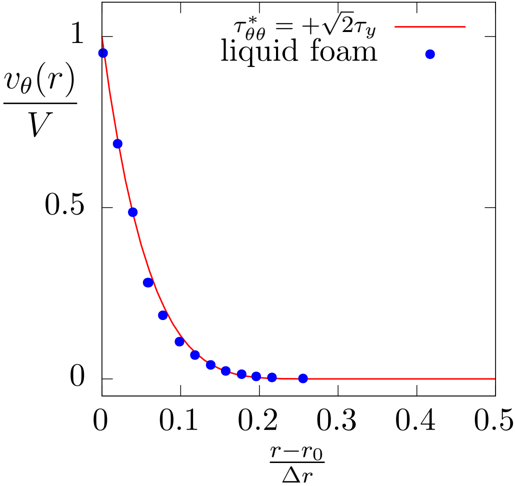


computation: *We* = 0.026, *Bi* = 27, $n = 1$

Cheddadi, Saramito, Graner, JoR, 2012

experiment: *Coussot et al., PRL, 2002*

Computation: (2) smooth profile



computation: *We* $= 0.035$, *Bi* $= 10$, $n = 1/3$
Cheddadi, Saramito, Graner, JoR, 2012

experiment: *Debrégeas et al., PRL, 2001*

Discussion

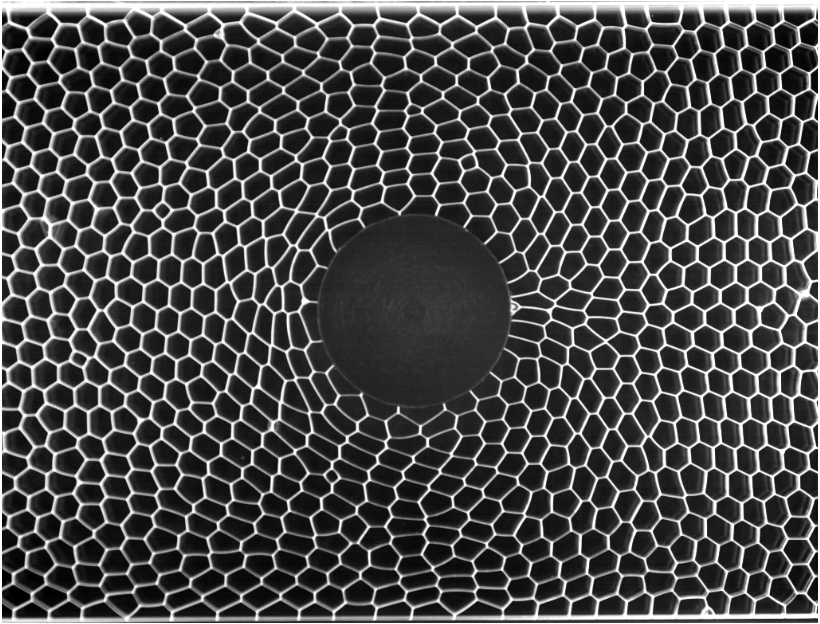
- " *To eliminate transient effects, we ran the experiment a full round before taking data.*"
  [Debrégeas, Tabuteau & di Meglio, 2001]

- [Coussot et al., 2002]: not presheared

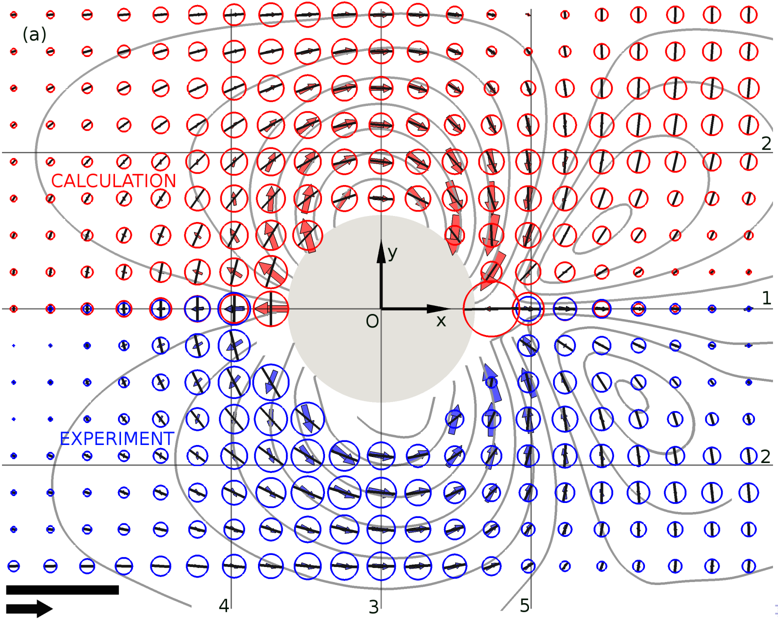
- " *Before any measurement, the Laponite sample is presheared for* 1 *min at* $+1500\ s^{-1}$ *and for* 1 *min at* $-1500\ s^{-1}$ *to erase most of the sample history. We checked that this procedure leads to reproducible results over a few hours.*"
  [Gibaud, Barentin & Manneville, 2008]

- " *The preshear protocol prior to the experiment may strongly influence the results.*"
  [Divoux, Fardin, Manneville & Lerouge, 2016]

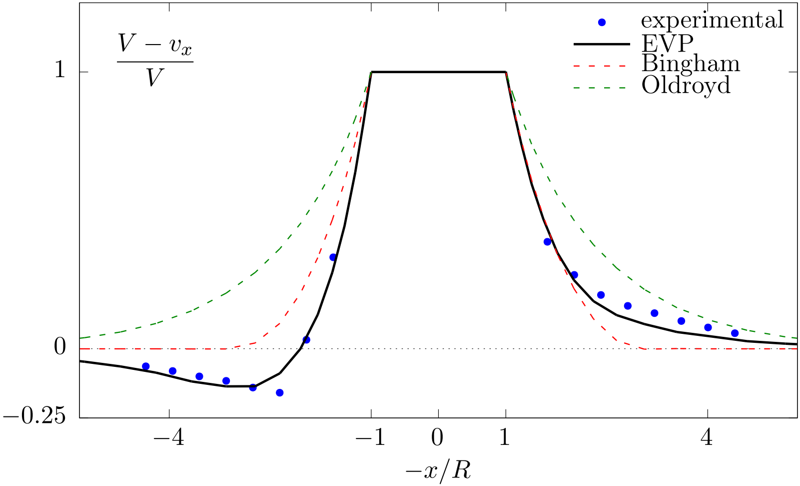
Flow around an obstacle



Computation / experiment



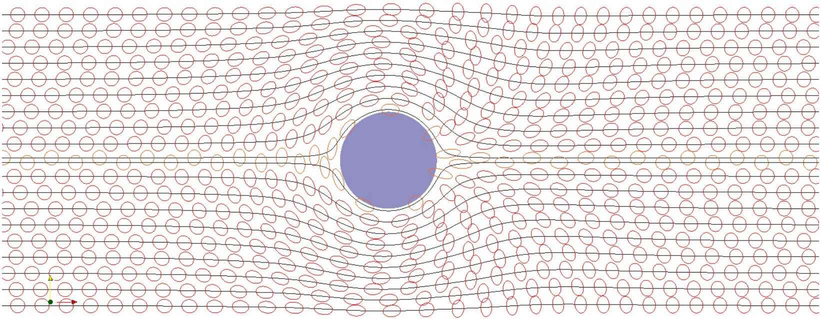
Velocity along the axis: negative wake



Cheddadi, Saramito, JNNFM, 2013

Cheddadi, Saramito, Dollet, Raufaste, Graner, EPJE, 2011

Flow around an obstacle



Conclusion

- modeling: 3D, objective derivative

  few parameters, all measurable:

  $$G, \tau_Y, K, n.$$

- Couette: predict both smooth or abrupt profiles
  $\Rightarrow$ pre-stress dependence
  quantitative predictions

- obstacle: quantitative predictions on complex geometry
  $\Rightarrow$ negative wake
  prediction - measure $\leqslant 5\%$

  success story: Carbopol arround an obstacle
  [Fraggedakis, Dimakopoulos, Tsamopoulos, Soft Mat. 2016]

Perspectives

- lab: others complex geometries (contractions, etc)
- geophysics: sea ice cover (add plasticity), earthquake, crustal large deforations, meteorite impact ...
- biology: tissues and embryogenesis, blood flow...

More reading

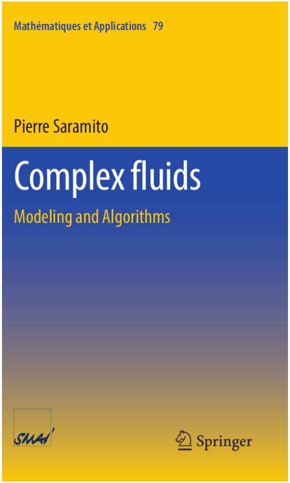
paper: Cheddadi, Saramito, JNNFM, 2013

Cheddadi, Saramito, Graner, JoR, 2012

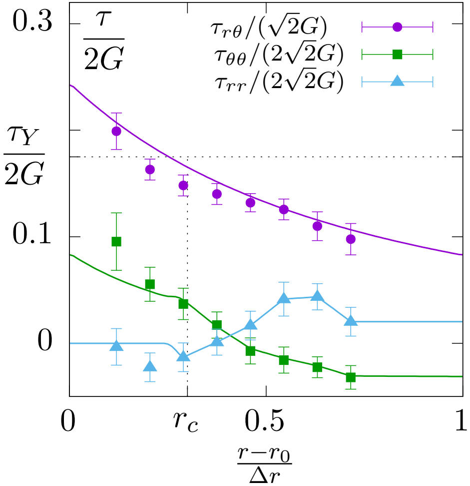
Saramito, JNNFM, 2009, 2007

book: Saramito, *Complex fluids*
Springer, 2016

code: Saramito, 2018
Rheolef FEM C++ library
Free software: GPL licence
http://www-ljk.imag.fr/membres/Pierre.Saramito/rheolef

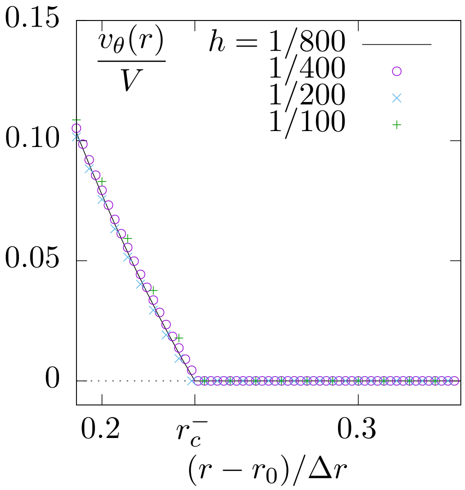


Full stress tensor comparisons



computation: *We* $= 0.035$, *Bi* $= 10$, $n = 1/3$
experiment: *Debrégeas* et al. *(2001), PRL 87:178305.*

Convergence of abrupt solutions



Convergence of abrupt solutions