p-adic Langlands correspondence and Iwasawa theory

Université de Lille

24-26 April 2019

Wednesday 24 April

09:00-09:45 Welcome coffee

09:45-10:45 Fred Diamond (King's College London)
  "The Serre filtration on mod p Hilbert modular forms of level p"

11:00-12:00 James Newton (King's College London)
  "Local-global compatibility and the cohomology of locally symmetric spaces"

12:00-14:00 Lunch break

14:00-15:00 Emmanuel Lecouturier (YMSC, Tsinghua University)
  "An application of a conjecture of Mazur–Tate to supersingular elliptic curves"

15:00-15:30 Coffee break

15:30-16:30 Denis Benois (Université de Bordeaux)
  "On extra zeros of p-adic L-functions"

Thursday 25 April

09:00-09:45 Coffee & pastries

09:45-10:45 Zicheng Qian (Université Paris-Sud)
  "Dilogarithm and higher $\mathcal{L}$-invariants for $\mathrm{GL}_3(\mathbf{Q}_p)$"

11:00-12:00 Gabriel Dospinescu (CNRS, ENS Lyon)
  "p-adic cohomology of some period domains"

12:00-14:00 Lunch break

14:00-15:00 Alice Pozzi (University College London)
  "The eigencurve at Eisenstein weight 1 points"

15:00-15:30 Coffee break

15:30-16:30 Shaunak Deo (TIFR)
  "Hilbert modular eigenvariety at exotic and CM classical points of parallel weight 1"

16:45-17:45 Sheng-Chi Shih (Université de Lille)
  "On the Hilbert cuspidal eigenvariety at weight 1 Eisenstein points"

19:00 Conference dinner
  Restaurant *Le Meunier*, 15 rue de Tournai 59800 Lille

Friday 26 April

09:00-09:30 Coffee & pastries

09:30-10:30 Joaquin Rodrigues Jacinto (Aix-Marseille Université)
"Norm-compatible cohomology classes in Siegel varieties"

10:45-11:45 Adel Betina (University of Sheffield)
"Geometry of Siegel eigenvarieties at Saito–Kurokawa points"

11:45-13:30 Lunch break

13:30-14:30 Stéphane Bijakowski (École Polytechnique)
"On the geometry of Pappas–Rapoport Shimura varieties"

14:30-15:00 Coffee break

15:00-16:00 Benjamin Schraen (Université Paris-Sud)
"On the density of automorphic points in global deformation spaces"

Abstracts

The Serre filtration on mod p Hilbert modular forms of level p (F. Diamond)

A result of Serre relates the space of mod p modular forms of level $\Gamma_1(Np)$ and weight 2 to the spaces of mod p modular forms of level $\Gamma_1(N)$ and weight between 2 and $p + 1$. I'll explain a generalization of this to the context of Hilbert modular forms involving a mod p geometric Jacquet–Langlands correspondence. The resulting filtration on mod p Hilbert modular forms of parallel weight 2 and pro-p Iwahori level mirrors the more evident one in cohomology coming from the mod p representation theory of GL_2. This is joint work with P. Kassaei and S. Sasaki.

Local-global compatibility and the cohomology of locally symmetric spaces (J. Newton)

I will discuss joint work with Allen, Calegari, Caraiani, Gee, Helm, Le Hung, Scholze, Taylor and Thorne on potential automorphy for certain compatible systems of Galois representations over CM fields. I will particularly focus on the local-global compatibility results needed to establish our automorphy lifting theorems in the ordinary case and explain the application of a key local ingredient: the computation (due to Hauseux) of derived ordinary parts of parabolically induced representations.

An application of a conjecture of Mazur–Tate to supersingular elliptic curves (E. Lecouturier)

In 1987, Barry Mazur and John Tate formulated refined conjectures of the "Birch and Swinnerton-Dyer type", and one of these conjectures was essentially proved in the prime conductor case by Ehud de Shalit in 1995. One of the main objects in de Shalit's work is the so-called *refined $\mathcal{L}$-invariant*, which happens to be a Hecke operator. We apply some results of the theory of Mazur's Eisenstein ideal to study in which power of the Eisenstein ideal $\mathcal{L}$ belongs. One corollary of ourstudy is the following elementary identity on supersingular j-invariants.

Let N be a prime number and $p \geq 5$ be a prime dividing $N - 1$. For simplicity, assume $N \equiv 1$ (modulo 12). Fix a surjective group homomorphism $\log : \mathbf{F}_{N^2}^{\times} \to \mathbf{Z}/p\mathbf{Z}$. Let $S = \{E_0, \ldots, E_g\}$ be the set of isomorphism classes of supersingular elliptic curves over $\overline{\mathbf{F}}_N$. We denote by $j(E_i) \in \overline{\mathbf{F}}_N$ the j-invariant of E_i; it is well-known that $j(E_i) \in \mathbf{F}_{N^2}$. Let $\mathcal{T}(S)$ be the set of spanning trees of the complete graph with vertices in S. If $T \in \mathcal{T}(S)$, let $E(T)$ be the set of edges of T. If $0 \leq i \neq j \leq g$, let $[E_i, E_j]$ be the edge between E_i and E_j. We have:

$$\sum_{T \in \mathcal{T}(S)} \prod_{[E_i, E_j] \in E(T)} \log(j(E_i) - j(E_j)) = 0.$$

On extra zeros of p-adic L-functions (D. Benois)

We discuss extra-zeros of motives having good reduction at p. An archetypical example is provided by the Kubota–Leopoldt L-function associated to a character χ such that $\chi(p) = 1$ and the theorem of Ferrero and Greenberg. Other interesting examples arise from some modular forms of odd weight. In this situation, the special value of the p-adic L-function can be expressed in terms of an $\mathcal{L}$-invariant defined using p-adic Hodge theory. In this talk, we are mainly interested in the non-critical case. The basic example we have in mind is provided by the Rankin–Selberg convolution of two modular forms of the same weight (joint work with S. Horte).

Dilogarithm and higher $\mathcal{L}$-invariants for $\mathrm{GL}_3(\mathbf{Q}_p)$ (Z. Qian)

We consider a semi-stable three dimensional p-adic representation ρ of the absolute Galois group of $\mathbf{Q}_p$ and assume that ρ has rank two monodromy and is non-critical. It is known that ρ depends on three $\mathcal{L}$-invariants up to isomorphism. We construct an explicit family of locally analytic representations of $\mathrm{GL}_3(\mathbf{Q}_p)$ depending on three invariants and show that there exists a unique representation (conjecturally depends only on ρ) in this family that embeds

into a suitable given Hecke eigenspace associated with a global Galois representation whose restriction at p is ρ. We will briefly introduce the construction which involves p-adic dilogarithm and then explain the relation between these representations and previous results by Breuil, Ding and Schraen.

p-adic cohomology of some period domains (G. Dospinescu)

We will explain how to adapt Orlik's computation of the compactly supported ℓ-adic cohomology of many p-adic period domains ($\ell \neq p$) to the case $\ell = p$. The key input is a vanishing theorem for extensions between generalized Steinberg representations of p-adic reductive groups, with coefficients mod p. This is joint work with Pierre Colmez, Julien Hauseux and Wieslawa Niziol.

The eigencurve at Eisenstein weight 1 points (A. Pozzi)

In this talk, we discuss the geometry of the Coleman–Mazur eigencurve at weight 1 Eisenstein points. The local nature of the eigencurve is mostly understood at classical points of weight greater than 1, while one observes some unusual behaviours at weight 1. In particular, we study cuspidal Hida families specializing to Eisenstein series at weight 1. Our approach consists in studying the deformation rings of certain (deceptively simple!) Artin representations. We discuss the implications of our analysis on the classicality of a certain overconvergent eigenspace. Finally, we explain how this Galois-theoretic method yields some new insight on Gross's formula relating the leading term of the p-adic L-function to a Stark unit. This is joint work with Adel Betina and Mladen Dimitrov.

Hilbert modular eigenvariety at exotic and CM classical points of parallel weight 1 (S. Deo)

We sketch our recent results about the geometry of the p-adic eigenvariety constructed by Andreatta–Iovita–Pilloni, which interpolates Hilbert modular eigenforms over a totally real field F, at classical, regular points of parallel weight 1 which either are CM or have exotic projective image. To prove these results, we assume the p-adic Schanuel conjecture in most of the cases. The key ingredient in our proof is calculation of the dimension of the tangent spaces of some Galois deformation problems. This talk is based on joint work with A. Betina and F. Fite.

On the Hilbert cuspidal eigenvariety at weight 1 Eisenstein points (S.-C. Shih)

When the p-adic L-function of a finite order totally odd character ϕ of a totally real field F has trivial zeros, any p-stabilization of the corresponding weight

1 Eisenstein series belongs to the Hilbert cuspidal eigencurve. In the case of elliptic modular forms, it was proved by Betina–Dimitrov–Pozzi that such points are étale over the weight space, hence belong to a unique cuspidal Hida family. In this talk, we will first present a generalisation to a real quadratic field in which p splits.

The complexity of the geometry of the Hilbert cuspidal eigencurve at such points growing with the dimension of $\mathrm{H}^1(F, \phi)$ which equals the degree of F, a challenging question is to determine the extension classes occurring in Galois representations attached to cuspidal Hida families. We will provide a partial answer in the case when p is inert in F and satisfied the Leopoldt conjecture. A key step of our work is to construct p-ordinary irreducible Galois representations with values in certain local rings of the eigencurve.

As an application, we give a new proof of the rank one abelian Gross–Stark conjecture relating the leading term of p-adic L-function of ϕ and a non-zero algebraic $\mathcal{L}$-invariant. This conjecture was first proved by Dasgupta–Darmon–Pollack under the assumption that a sum of two analytic $\mathcal{L}$-invariances is non-zero. This is an ongoing work with Adel Betina and Mladen Dimitrov.

Norm-compatible cohomology classes in Siegel varieties (J. Rodrigues Jacinto)

We will explain how to construct towers of interesting classes in the cohomology of Siegel sixfolds. We will study their complex regulator and we will give an application to Iwasawa theory. This is joint work with Antonio Cauchi and Francesco Lemma.

Geometry of Siegel eigenvarieties at Saito–Kurokawa points (A. Betina)

I will report on joint work with T. Berger studying the geometry of Siegel eigenvarieties. Under certain assumptions we show that they are smooth at points corresponding to Saito–Kurokawa lifts when the tame level is paramodular, but singular when it is $\Gamma_0(N)$. Moreover, we give an application to the Bloch–Kato conjecture. Our technique uses pseudorepresentations of p-adic families of cuspidal Siegel eigenforms and analytic continuation of crystalline periods.

On the geometry of Pappas–Rapoport Shimura varieties (S. Bijakowski)

After recalling the geometry of the special fiber of the modular curve, I will talk about possible generalizations to Shimura varieties. I will explain why the situation is more involved when ramification appears, and why one is led to use models defined by Pappas and Rapoport. I will then define an analogous of the ordinary locus in this context. This is joint work with V. Hernandez.

On the density of automorphic points in global deformation spaces
(B. Schraen)

I will discuss the problem of the repartition of automorphic points in global polarized deformation spaces. We can ask the problem in terms of fixed level and varying weight or fixed weight and varying level. I will describe positive answers to these problems and their link with the problem of companion p-adic overconvergent automorphic forms.