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Low–rank interpolation of tensors for high–precision calculation of high–dimensional integrals

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High–dimensional integrals appear in problems with uncertainty and noise and are common in stochastic calculus, mathematical finance, quantum physics, etc. Textbook quadratures applied to d-dimensional integrals require exponential in d number of function evaluations, which is unfeasible for dimensions exceeding ten (this is notoriously known as curse of dimensionality). The use of Smolyak's sparse grids relaxes, but does not completely remove the problem, which is notoriously known as a curse of dimensionality. Currently such integrals are predominantly treated with Monte Carlo algorithm or its variants, e.g. quasi-MC, MCMC. Due to their relatively slow convergence, MC methods require excessive numerical costs and sometimes do not provide sufficient accuracy of results.

Can we do better by using more structure of the original problem? We propose a new algorithm which interpolates the given function with a low–rank tensor product format using separation of variables. Our method is based on adaptive cross interpolation and maximum–volume principle — well–established algorithms for matrices, which we genelised to high–dimensional tensors and multivariate functions. For functions that admit low-rank tensor–product representation, tensor interpolation converges faster than MC and qMC, and hence can be a promising new algorithm for accurate evaluation of high–dimensional integrals.

We demonstrate the efficiency of the proposed algorithm for a class of Ising integrals, which appear in Ising theory of mathematical physics in relation to magnetic susceptibility of two–dimensional spin lattices. This application encourages evaluation of integrals in dimensions up to 1000 with very high precision. Monte Carlo methods are not up to the challenge. Using tensor low–rank interpolation we compute integrals accurately to 100 decimal digits.

We hope that this example encourages further study and exploitation of low–rank tensor product structure for problems in other subject areas where the curse of dimensionality stands in the way of delivering highly accurate results at reasonable cost.

Keywords: high–dimensional problems, numerical integration, low rank, interpolation

References

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