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Exploiting fast linear algebra in the computation of multivariate relations

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We consider the problem of computing multivariate relations in a finite-dimensional setting: for a submodule $\mathcal{M}$ of $\mathbb{K}[x]^n$ such that $Q = \mathbb{K}[x]^n/\mathcal{M}$ has finite dimension D as a $\mathbb{K}$-vector space, and given elements $f_1, \ldots, f_m$ in Q, the problem is to compute relations between the f_i's, that is, polynomials $(p_1, \ldots, p_m)$ in $\mathbb{K}[x]^m$ such that $p_1 f_1 + \ldots + p_m f_m = 0$ in Q. Assume that the multiplication matrices of the r variables with respect to some basis of Q are known. Then, for any monomial order, we give an algorithm for computing the reduced Gröbner basis of the module of such relations using $O(rD^\omega \log(D))$ operations in the field $\mathbb{K}$, where ω is the exponent of matrix multiplication. This is done by interpreting the problem as the computation of a nullspace basis in reduced echelon form for a matrix over $\mathbb{K}$ which has a multi-level Krylov structure. For efficiency, the algorithm both exploits this structure and incorporates fast matrix multiplication and Gaussian elimination.

Keywords: Fast linear algebra, structured matrices, Gröbner basis, algebraic relations.

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