

## Exploiting Fast Matrix Arithmetic in Block Low-Rank Factorizations

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We consider the LU factorization of an  $n \times n$  matrix  $A$  represented as a block low-rank (BLR) matrix: most of its off-diagonal blocks are approximated by matrices of small rank  $r$ , which reduces the asymptotic complexity of computing the LU factorization of  $A$  down to  $\mathcal{O}(n^2r)$ . In this work, our aim is to further reduce this complexity by exploiting fast matrix arithmetic, that is, the ability to multiply two  $n \times n$  full-rank matrices together for  $\mathcal{O}(n^\omega)$  flops, where  $\omega < 3$ . This is not straightforward: simply accelerating the intermediate operations performed in the standard BLR factorization algorithm does not suffice to reduce the quadratic complexity in  $n$ , because these operations are performed on matrices whose size is too small. To overcome this obstacle, we devise a new BLR factorization algorithm that, by recasting the operations so as to work on intermediate matrices of larger size, can exploit more efficiently fast matrix arithmetic. This new algorithm achieves an asymptotic complexity of  $\mathcal{O}(n^{(\omega+1)/2}r^{(\omega-1)/2})$ , which represents an asymptotic improvement compared to the standard BLR factorization as soon as  $\omega < 3$ . In particular, for Strassen's algorithm,  $\omega \approx 2.81$  yields a complexity of  $\mathcal{O}(n^{1.904}r^{0.904})$ . Our numerical experiments are in good agreement with this theoretical result.

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