

Quantum Theory and Causality

An Exercise in “Natural Philosophy”

*“I leave to several futures (not to all) my garden of forking paths”
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Contents and credits

Contents:

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Credits:

I am indebted to my last PhD student *Baptiste Schubnel* for enjoyable collaboration, to *M. Ballesteros*, *Ph. Blanchard* and *M. Fraas*, for cooperation, and to these colleagues as well as many further ones, including some of the “*Bohmians*”, several colleagues at ETH Zurich, and *D. Buchholz* for useful discussions on *QM*.

1. What this lecture is about

New Foundations of Quantum Mechanics are proposed: the

“ETH - Approach to Quantum Mechanics”

*where “E” stands for Events, “T” for Trees, and “H” for Histories. This approach enables us to introduce a precise notion of “events” into Quantum Mechanics (as first emphasized by R. Haag), explain what it means to observe an event by recording the value of an appropriate physical quantity, and to exhibit the **stochastic dynamics of states** of **isolated open systems** featuring events.*

*I will then focus on explaining how quantum theory might be reconciled with relativity theory, and what it may tell us about the fabric of **space-time** and its **causal structure**.*

*The “ETH - Approach to QM” results in a “**Quantum Theory without observers**”. It does away with “extensions of Quantum Mechanics”, all of which have remained unacceptably vague.*

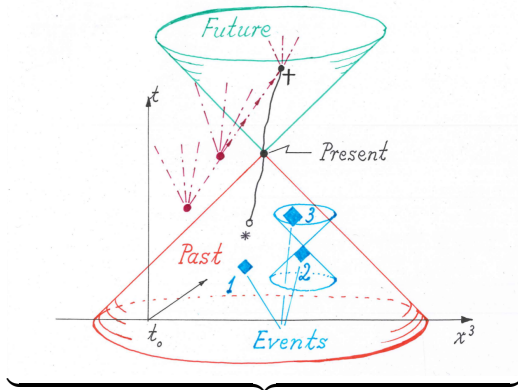
Specific topics to be addressed

- I. **Foundations of Quantum Theory**. Why are physical theories never fully predictive? Why is quantum theory intrinsically probabilistic? What are “*events*” in quantum theory? How do we measure physical quantities and detect “events”? What is the role of time in quantum theory, and why does it distinguish between *past* and *future*? What is the fundamental significance of “*locality*” and *Einstein causality*?
- II. **Quantum Theory and Relativity Theory**. What are some of the basic problems in coming up with a framework unifying Quantum Theory with a theory of space and time? Could it be that a consistent “*Quantum Theory of Events*” must necessarily be relativistic? What does such a theory tell us about the fabric of space-time; does it explain why space-time is even-dimensional and curved?
- III. Can we view “**physical space**” and the **causal structure of space-time** as “emerging” from Quantum Theory?

Etc. ...

2. What prevents theories from being (fully) predictive?

Space-time with an *event horizon*. (Observer sits at “Present”; is unaware of dangers lurking from outside his past light-cone; he might get killed at †. Events 1 & 2 are space-like separated; event 3 is in the *future* of 2)



t_0 : time right after inflation \rightarrow event horizon \Rightarrow initial conditions not fully accessible!

Past = History of Events / Future = Ensemble of Potentialities

This fundamental structure should be retained in Quantum Mechanics!

... Quantum theory is fundamentally probabilistic –

in spite of the *deterministic nature of the Schrödinger Eq.*! –

Temporary assumptions (leading to a contradiction):

- ▶ P and P' : Spin- $\frac{1}{2}$ particles prepared in a *spin-singlet initial state*; spin filter prepared in a poorly known initial state not entangled with initial state of P' and P .
- ▶ *Dynamics of state of total system fully determined by Schrödinger equation*. In particular, initial state of spin filter determines whether P' will pass through it or not, (given that the initial state of $P' \vee P$ is a spin-singlet state, with P' and P moving into opposite cones).
- ▶ *Correlations between outcomes of spin measurements of P' and of P are as predicted by standard quantum mechanics*, (relying on the “Copenhagen interpretation”).

Fact: Heisenberg-picture dynamics of observables (spin, etc.) referring to P is ess. *independent* of dynamics of $Q := \{P' \vee \text{spin filter}\}$. This follows from our choice of initial conditions & cluster properties of time evolution! Hence *spin of P ess. conserved before measurement* \Rightarrow

Expectation value of spin of $P \approx 0, \forall$ times!

But *this contradicts the third (last) assumption stated above!*

3. Locality in relativistic theories

Thus, if the usual correlations between two “independent” measurements (here of spins of P' and of P), predicted on the basis of the projection postulate of “Copenhagen”, are valid² then it follows that the Schrödinger equation *cannot* describe the evolution of states of systems, and hence that Quantum Mechanics is *fundamentally probabilistic*. It turns out that one may safely assume the validity of the *Heisenberg-picture evolution* of “observables” for isolated systems; it is perfectly *deterministic*. But, in Quantum Mechanics, the *evolution of states is stochastic*. \Rightarrow Equivalence of the *Heisenberg picture* and the *Schrödinger picture* is an erroneous claim!

Let us assume that “Copenhagen” is correct, in the sense that if the spin of P' has been measured to be $\sigma' \in \{+, -\}$, along the z-axis, and the spin of P has been measured to be $\sigma \in \{+, -\}$, along an axis \vec{n} , then the state of the system, right after these measurements, is a simultaneous eigenstate of the two projections, $\Pi_{\sigma', \vec{e}_z}^{P'}$ and $\Pi_{\sigma, \vec{n}}^P$, corresponding to these measurement outcomes, with eigenvalue +1.

²as suggested by the experiments of Aspect, Gisin, and others 

Commutativity of operators localized in space-like separated regions

It is possible that these two spin measurements are made in *space-like separated regions* of space-time, so that the localization regions of the operators $\Pi_{\sigma', \vec{e}_z}^{P'}$ and $\Pi_{\sigma, \vec{n}}^P$ are space-like separated. The order in which these two measurements occur then depends on the rest frame of the observer who records the data of both measurements. This implies that the operators $\Pi_{\sigma', \vec{e}_z}^{P'} \cdot \Pi_{\sigma, \vec{n}}^P$ and $\Pi_{\sigma, \vec{n}}^P \cdot \Pi_{\sigma', \vec{e}_z}^{P'}$ must have the *same effect* when applied on the state of the system. The most general way in which this can be guaranteed is to require that

$$\Pi_{\sigma', \vec{e}_z}^{P'} \cdot \Pi_{\sigma, \vec{n}}^P = \Pi_{\sigma, \vec{n}}^P \cdot \Pi_{\sigma', \vec{e}_z}^{P'} \quad (1)$$

This is *locality* (in the sense of RQFT) or *Einstein causality*!

[*Remark:*

It might suffice to require a weaker form of locality by only requiring Eq. (1) to hold on all those states that actually admit measurements of the spins of P' and P in the prescribed local regions of space-time; (“*weak locality*” – compare to Jost’s proof of the CPT theorem!.)]

4. The “ETH Approach” to Quantum Theory

Next, we address the question of what is meant by “*events*” featured by isolated systems, and of how they can be recorded (in *direct/projective measurements/observations*). I sketch what I call the “ETH Approach” to QM. I first consider *non-relativistic theories*:

Let S be an isolated physical system. Pure states of S are given by unit rays in a separable Hilbert space \mathcal{H}_S ; general states by density operators, ω , acting on \mathcal{H}_S , with $\omega(A) := \text{Tr}(\omega \cdot A)$, for any bd. operator A on \mathcal{H}_S .

Time is a fundamental quantity in n.r. physics. The time axis is given by \mathbb{R} . Let's suppose the *present time* is t_0 , and let I be an arbitrary interval of *future times*, i.e., $I \subset [t_0, \infty)$.

Definition: Let S be an isolated physical system. “*Potential future events*” in S – “*potentialities*” – are described by certain *orthogonal projections* acting on \mathcal{H}_S associated with time intervals. The *algebra generated by all “potential future events” assoc. with a future interval, I , of times is denoted by \mathcal{E}_I , and we define

$$\mathcal{E}_{\geq t} := \overline{\bigvee_{I \subset [t, \infty)} \mathcal{E}_I}, \quad \text{and} \quad \mathcal{E} := \overline{\bigvee_{t \in \mathbb{R}} \mathcal{E}_{\geq t}}^{\|\cdot\|}, \quad (2)$$

"Principle of Diminishing Potentialities"

where the algebras $\mathcal{E}_{\geq t}$, $t \in \mathbb{R}$, are assumed to be weakly closed!³

By definition,


$$\mathcal{E}_I \supseteq \mathcal{E}_{I'} \text{ if } I \supseteq I', \quad \mathcal{E}_{\geq t} \supseteq \mathcal{E}_{\geq t'} \text{ if } t' > t.$$

In Quantum Mechanics, an *isolated open system* S is defined by a filtration $\{\mathcal{E}_{\geq t} | t \in \mathbb{R}\}$ of algebras of potential future events (potentialities). The "Principle of Diminishing Potentialities" (PDP) is the statement that

$$\boxed{\mathcal{E}_{\geq t} \supsetneq \mathcal{E}_{\geq t'}, \text{ whenever } t' > t \geq t_0} \quad (3)$$

Given a state ω of S , we set

$$\omega_t := \omega|_{\mathcal{E}_{\geq t}}, \quad \text{i.e.,} \quad \omega_t(A) = \omega(A), \quad \forall A \in \mathcal{E}_{\geq t}. \quad (4)$$

³Passing to von Neumann algebras is convenient, because the spectral projections of any element of the algebra will then also belong to the algebra! 

Events

Note that ω might be a *pure* state on \mathcal{E} . But, since $\mathcal{E}_{\geq t} \subsetneq \mathcal{E}$, $\forall t < \infty$, ω_t will generally be a *mixed* state on $\mathcal{E}_{\geq t}$; (entanglement!). This observation opens the door towards a clear notion of what might be meant by “*events*” and to a theory of direct/projective measurements and observations (of “events”).

To render the above *definition* more precise, we say that a “*potential future event*” is given by a family, $\{\pi_\xi | \xi \in \mathcal{X}\}$, of disjoint orthogonal projections contained in an algebra $\mathcal{E}_{\geq t}$, for some $t \geq t_0$, (t_0 = time of “present”), with $\sum_{\xi \in \mathcal{X}} \pi_\xi = \mathbf{1}$.

In accordance with the “Copenhagen interpretation” of QM, it appears natural to say that a potential future event $\{\pi_\xi | \xi \in \mathcal{X}\} \subset \mathcal{E}_{\geq t}$ *actually happens* in the interval $[t, \infty)$ of times iff

$$\boxed{\omega_t(A) = \sum_{\xi \in \mathcal{X}} \omega(\pi_\xi A \pi_\xi), \quad \forall A \in \mathcal{E}_{\geq t},} \quad (5)$$

i.e., no off-diagonal elements appear on the R.S. of (5)!

The centralizer of a state and its center

Next, we render the meaning of Eq. (5) more precise.

Let \mathcal{M} be a von Neumann algebra, and let ω be a state on \mathcal{M} . Given an operator $X \in \mathcal{M}$, we set

$$ad_X(\omega)(A) := \omega([A, X]), \quad \forall A \in \mathcal{M}.$$

We define the *centralizer* of a state ω on \mathcal{M} by

$$\mathcal{C}_\omega(\mathcal{M}) := \{X \in \mathcal{M} | ad_X(\omega) = 0\}$$

Note that ω is a *normalized trace* on $\mathcal{C}_\omega(\mathcal{M})$... ! The *center*, $\mathcal{Z}_\omega(\mathcal{M})$, of $\mathcal{C}_\omega(\mathcal{M})$ is defined by

$$\mathcal{Z}_\omega(\mathcal{M}) := \{X \in \mathcal{C}_\omega(\mathcal{M}) | [X, A] = 0, \quad \forall A \in \mathcal{C}_\omega(\mathcal{M})\}. \quad (6)$$

We are now prepared to introduce the notion of (actual) “events”.

Events happening at time $\geq t$

Let S be an **isolated** physical system. We set $\mathcal{M} =: \mathcal{E}_{\geq t}$, with $\omega =: \omega_t$.

Definition: Given that ω_t is the state of S on the algebra $\mathcal{E}_{\geq t}$, an “**event**” is happening at time t iff $\mathcal{Z}_{\omega_t}(\mathcal{E}_{\geq t})$ contains at least two non-zero orthogonal projections, $\pi^{(1)}, \pi^{(2)}$, which are disjoint, i.e., $\pi^{(1)} \cdot \pi^{(2)} = 0$, and

$$0 < \omega_t(\pi^{(i)}) < 1, \quad \text{for } i = 1, 2. \quad \square$$

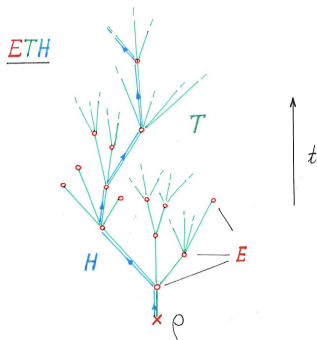
Let us suppose for simplicity that $\mathcal{Z}_{\omega_t}(\mathcal{E}_{\geq t})$ is generated by a family of orthogonal projections $\{\pi_\xi | \xi \in \mathcal{X}_{\omega_t}\}$, where $\mathcal{X}_{\omega_t} = \text{spec}[\mathcal{Z}_{\omega_t}(\mathcal{E}_{\geq t})]$ is a countable set.

“Axiom”: If $\text{card}(\mathcal{X}_{\omega_t}) \geq 2$ and $\omega_t(\pi_\xi) \neq 0$, for **at least two** different points $\xi \in \mathcal{X}_{\omega_t}$, then the state ω_t must be replaced by one of the states $\omega_{t,\xi} := \omega_t(\pi_\xi)^{-1} \cdot \omega_t(\pi_\xi(\cdot)\pi_\xi)$, for some $\xi \in \mathcal{X}_{\omega_t}$ with $\omega_t(\pi_\xi) \neq 0$. The probability, $\text{prob}_t(\xi)$, for the state $\omega_{t,\xi}$ to be selected as the state of S right after time t is given by

$$\text{prob}_t(\xi) = \omega_t(\pi_\xi) \quad - \quad \text{Born's Rule} \quad \square \quad (7)$$

A metaphoric picture of the time evolution of states in QM – “ETH”

Apparently, the time-evolution of states of a phys. system S is described by a *stochastic branching process*, with branching rules as determined by the above “Axiom”. Illustration:



E : “Events”, T : “Trees” of possible states, H : “Histories” of states.

This is different from and supercedes the “decoherence mumbo-jumbo”!

5. Events and their detection

We have characterised an *isolated open system* S in terms of a filtration of algebras

$$\{\mathcal{E}_{\geq t}\}_{t \in \mathbb{R}},$$

with

$$\boxed{\mathcal{E}_{\geq t} \supsetneq \mathcal{E}_{\geq t'}, \quad \text{whenever } t' > t} \quad (8)$$

The flow of time in S , (i.e., the time evolution of S in the Heisenberg picture) is encoded in the *proper* embeddings (8), which, in an *autonomous* system S , are completely determined by its *Hamiltonian*.

However, the characterisation of S given in (8) is incomplete! To retrieve physical information from (8) and from our definition of *events*, we must specify operators that represent “*physical quantities*” characteristic of S and – when observed/measured – may *signal the occurrence of events*. Let

$$\mathcal{O}_S := \{\hat{X}_\iota | \iota \in \mathcal{I}_S\} \quad (9)$$

be a list/set of abstract linear operators representing physical quantities characteristic of S ; (usually, \mathcal{O}_S is not a linear space, let alone an alg.).

Measurements of physical quantities

For any operator $\hat{Y} \in \mathcal{O}_S$ and any time t , we specify a concrete self-adjoint operator $Y(t) \in \mathcal{E}_{\geq t}$ representing \hat{Y} at time t ; (i.e., \exists a repr. of \mathcal{O}_S by operators on \mathcal{H}_S , $\forall t \in \mathbb{R}$). For an *autonomous* system S , the operators $Y(t)$ and $Y(t')$ are conjugated to one another by the *propagator* of S .

Suppose that, at some time t , an *event* happens; i.e., \exists a partition of unity, $\{\pi_\xi | \xi \in \mathcal{X}_{\omega_t}\} \subseteq \mathcal{Z}_{\omega_t} \subset \mathcal{E}_{\geq t}$, by disjoint (commuting) orthogonal projections, as above, containing ≥ 2 elements with positive probability of occurrence representing *possible events* (one of which *actually happens*). Let $\hat{Y} \in \mathcal{O}_S$, and let $Y(t) = \sum_{\eta \in \text{spec}(\hat{Y})} \eta \Pi_\eta(t)$ (spectral dec. of $Y(t)$) be the operator representing \hat{Y} at time t . If the “distance”⁴

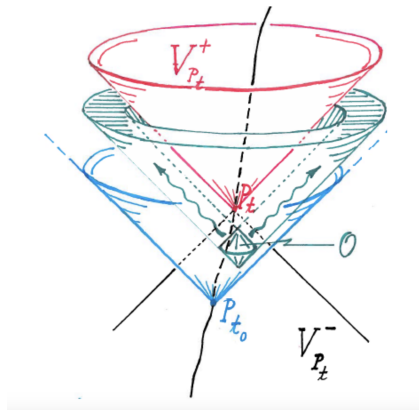
$$\text{dist}(\Pi_\eta(t), \langle \pi_\xi | \xi \in \mathcal{X}_{\rho_t} \rangle) \text{ is “very small” , } \forall \eta \in \text{spec}(\hat{Y}), \quad (10)$$

then we say that the *physical quantity* $\hat{Y} \in \mathcal{O}_S$ is *observed/measured* after time t , because the state of S just after time t is then an approximate eigenstate of $Y(t)$. The measurement of \hat{Y} is a *signal of an event happening* at time t

⁴defined in terms of conditional expectations

6. Local Relativistic Quantum Theory

To begin with, I assume that space-time is Minkowski space, \mathbb{M}^4 , and, immodestly, that my own proper time is the time of the Universe.



Worldline of JF \uparrow

A Theorem of Buchholz

Theorem

In an RQFT in dim. $2n$, $n \geq 2$ with massless particles, such as photons, the algebra, $\mathcal{E}_{\geq P_t}$, of all physical quantities (“observables”) potentially measurable in the future of the space-time point P_t is of type III_1 , and $\mathcal{E}'_{\geq P_t} \cap \mathcal{E}_{\geq P_{t_0}}$ is of type III_1 , too, for arbitrary times $t_0 < t$. \square

This result is a consequence of “*Huygens’ Principle*” (in the jargon of Buchholz): Photons from the region \mathcal{O} will asymptotically escape along lightcones in the future, $V_{P_{t_0}}^+$, of P_{t_0} but below $V_{P_t}^+$. We cannot catch up with them, anymore, if we have missed them just after they have been emitted. Thus, the “*Principle of Diminishing Potentialities*” (PDP) holds in the form proposed in Eq. (3) of the last Section:

$$\mathcal{E}_{\geq P_{t_0}} \supsetneq \mathcal{E}_{\geq P_t}, \quad \text{for } t > t_0, \quad (11)$$

and we could now follow the arguments outlined in Sect. 5. However, I don’t like to be in the center of the Universe; so, let’s take JF out of the picture! Before knowing better I propose a formulation of *relativistic local Quantum Theory* with roughly the following features:

A tentative formulation of relativistic local quantum theory

Let \mathcal{M} be some (Hausdorff) topological space. We consider a *fibre bundle*, ${}^q\mathcal{F}$, with base space given by \mathcal{M} and fibre above a point $P \in \mathcal{M}$ given by an ∞ -dimensional von Neumann algebra $\mathcal{E}_{\geq P}$. All the algebras $\{\mathcal{E}_{\geq P}\}_{P \in \mathcal{M}}$ are assumed to be *isomorphic* to one another.⁵


Definition:

We say that a point $P_0 \in \mathcal{M}$ is in the *past* of a point $P \in \mathcal{M}$, written as $P_0 \prec P$, iff $\mathcal{E}_{\geq P_0} \supseteq \mathcal{E}_{\geq P}$, and

$$(\mathcal{E}_{\geq P})' \cap \mathcal{E}_{\geq P_0}$$

is an infinite-dimensional n.c. algebra. □

The relation \prec introduces a *partial order* on \mathcal{M} . If $P_0 \not\prec P$ and $P \not\prec P_0$ then we say that P_0 and P are *space-like separated*, written as $P_0 \text{ X } P$. The relations “ \prec ” and “X” determine a “*causal structure*” on \mathcal{M} .

⁵This framework could be generalized by first considering C^* -algebras, rather than von Neumann algebras, and introducing sheaves of algebras 

What are “events”?

Let ω be a state on the algebra

$$\mathcal{E}_{\geq \Sigma} := \bigvee_{P' \in \Sigma} \mathcal{E}_{\geq P'},$$

where Σ is a space-like hypersurface contained in \mathcal{M} containing a point $P \in \mathcal{M}$.

Definition: We say that an “event” happens in P iff the center $\mathcal{Z}_{\omega_{\Sigma}}(\mathcal{E}_{\geq P}) \equiv \mathcal{Z}_{\omega_{\Sigma}}^P$ of the centralizer, $\mathcal{C}_{\omega_{\Sigma}}(\mathcal{E}_{\geq P})$, is *non-trivial* and contains at least *two* projections, Π_1^P and Π_2^P with the property that

$$0 < \omega_{\Sigma}(\Pi_i^P) < 1, \quad \text{for } i = 1, 2. \quad \square$$

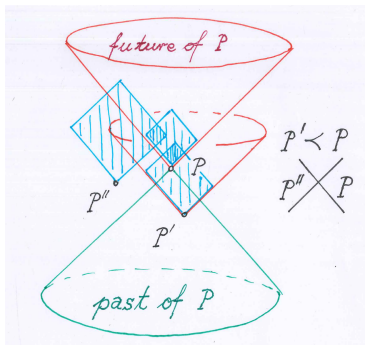
Let $\mathcal{X}_{\omega_{\Sigma}}^P$ denote the *spectrum* of $\mathcal{Z}_{\omega_{\Sigma}}^P$.

“Axiom” (compatibility – locality): If two points, P and P'' , of \mathcal{M} are *space-like separated*, and “events”, Π_{ξ}^P and $\Pi_{\eta}^{P''}$, actually happen in P and P'' then

$$[\Pi_{\xi}^P, \Pi_{\eta}^{P''}] = 0, \quad \forall \xi \in \mathcal{X}_{\omega_{\Sigma}}^P \text{ and all } \eta \in \mathcal{X}_{\omega_{\Sigma}}^{P''}. \quad \square \quad (12)$$

The compatibility axiom

→ introduces “geometrical structure” on \mathcal{M} !

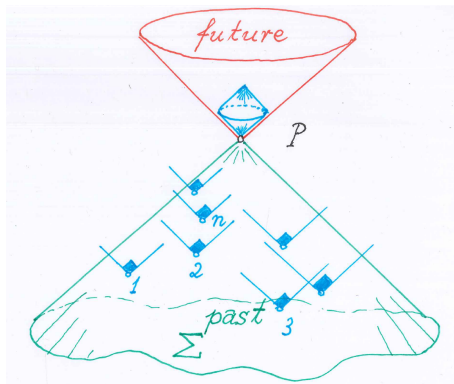


Graphical illustration of the axiom

However, projections describing events happening in P' and P do *not* commute in general, since P' is in the *past* of P .

Next, we describe *histories of events*. We choose a space-like surface Σ in \mathcal{M} with the property that some bounded subset of Σ lies in the past of a point $P \in \mathcal{M}$, as shown in the following figure.

Histories of events



We suppose that a state ω_Σ associated with a space-like surface Σ is prescribed; (choice of “initial conditions”). Our task is to find out whether all events in the *past* of P but in the future of Σ (so-called “histories”), together with the state ω_Σ , uniquely determine a state, ω_P , on the algebra $\mathcal{E}_{\geq P}$ and, given ω_P , to find out whether an *event* happens at P .

Probabilities of histories of events

Inductive hypothesis: Let P_1, P_2, \dots , be all points in the **past** of P but **not** in the past of **any** point on Σ with the property that, given initial conditions corresponding to ω_Σ , an event has happened at $P_i, i = 1, 2, \dots$. With any of these points we can then associate an orthogonal projection $\Pi_{\xi_i}^{P_i}$, $\xi_i \in \mathcal{X}_{\omega_{P_i}}^{P_i} = \text{spec}(\mathcal{Z}_{\omega_{P_i}}^{P_i})$. We define “*history operators*”

$$H(P|\omega_\Sigma) := \prod_{i=1,2,\dots}^{\rightarrow} \Pi_{\xi_i}^{P_i}, \quad (10)$$

where P_i is either in the past of P_{i+1} , or P_i and P_{i+1} are space-like, $\forall i = 1, 2, \dots$. Thanks to the **compatibility-locality axiom** the operator $H(P|\omega_\Sigma)$ is well-defined! We then set

$$\omega_P(A) := \text{prob}(H(P|\omega_\Sigma))^{-1} \omega_\Sigma(H(P|\omega_\Sigma) A H(P|\omega_\Sigma)^*), \quad (11)$$

$\forall A \in \mathcal{E}_{\geq P}$, where

Events and the fabric of space-time

$$\text{prob}(H(P|\omega_\Sigma)) := \omega_\Sigma(H(P|\omega_\Sigma) \cdot H(P|\omega_\Sigma)^*)$$

Generalized Born Rule

Induction step: We are now able to answer the question whether an event happens in the space-time point P :

An **event** happens in P iff the center $\mathcal{Z}_{\omega_P}(\mathcal{E}_{\geq P})$ of the centraliser of ω_P is non-trivial and contains ≥ 2 disjoint orthogonal projections with strictly positive probabilities in ω_P , as predicted by **Born's Rule**.

Note: The “compatibility-locality axiom” is expected to yield non-trivial constraints on the geometry of space-time in the vicinity of two space-like separated points, P and P'' , if it is known that \exists **events** in P and P'' localised in explicitly known regions in the **future** of P and of P'' , respectively, which are represented by projections commuting with one another; ... But these matters remain to be investigated more thoroughly in the **future**.

7. Summary and conclusions

- ▶ As in the genesis of Special Relativity, the e.m. field, as well as Huygens' Principle (\leftrightarrow even-dim. of space-time) play key roles in the genesis of a Quantum Theory solving the “measurement problem” – *not properly appreciated, so far!*
- ▶ As in the genesis of General Relativity, the causal structure of space-time plays a key role in the functioning of Relativistic Quantum Theory.
- ▶ The non-commutative nature of Quantum Theory and the “*compatibility-locality axiom*” governing the relations between events determine a “*causal structure*” on space-time. *Events weave the fabric of space-time!*
- ▶ Thanks to the “*Principle of Diminishing Potentialities*” (PDP) and the natural presence of an “*arrow of time*” in the “ETH approach” to Quantum Theory, the “*Information Paradox*” and the “*Unitarity Paradox*” appear to dissolve. ... The end of time: ...

I thank you for your attention !