Non-perturbative Studies of Membrane Matrix Models

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Space Time Matrices

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Membrane Actions

Nambu Goto for p-brane

$S_{NG} = \int_{\mathcal{M}} d^{p+1}x\sqrt{-detG} \qquad G_{\mu\nu} = \partial_\mu x^M \partial_\nu x^N g_{MN}(x)$

Higher form gauge field on the world volume

$$S_{p-form} = -\int_{\mathcal{M}} \frac{1}{(p+1)!}\epsilon^{\mu_1\ldots\mu_{p+1}} C_{\mu_1\ldots\mu_{p+1}}$$

$$C_{\mu_1\ldots\mu_{p+1}} = \partial_{\mu_1}x^{M_1}\ldots\partial_{\mu_{p+1}}x^{M_{p+1}} C_{M_1\ldots M_{p+1}}$$

We could add

- an anti-symmetric part to $G_{\mu\nu}$ to get a Dirac-Born-Infeld action.
- extrinsic curvature terms.

Supersymmetric S_{NG} exist only in 4, 5, 7 and 11 dim-spacetime.

Polyakov action

The Membrane action á la Polyakov

$$S_{NG} = -\frac{T}{2} \int_{\mathcal{M}} d^3\sigma \sqrt{-h} \left(h^{\alpha\beta} \partial_\alpha x^M \partial_\beta x^N g_{MN} - (p-1) \right)$$

Eliminating $h_{\mu\nu}$

$$h_{\alpha\beta} = \partial_\alpha x^M \partial_\beta x^N g_{MN} = G_{\alpha\beta}$$

returns us to Nambu-Goto.

Membranes in flat spacetime, $g_{MN} = \eta_{MN}$ and $C_3 = 0$

$$dS^2_{\mathcal{M}} = \dot{x}^M \dot{x}_M d\tau^2 + 2\dot{x}^M \partial_j x_M d\tau d\sigma^j + \partial_i x^M \partial_j x_M d\sigma^i d\sigma^j$$

In lightcone cooridnates, $x^{\pm} = (x^0 \pm x^D)/\sqrt{2}$

$$ds^2 = \eta_{MN} dx^M dx^N = -2dx^+ dx^- + dx^a dx^a$$

Noting $\partial_i x^+ = 0$ and with $\tau = x^+$

$$dS^2_{\mathcal{M}} = (-2\dot{x}^- + \dot{x}^a \dot{x}_a) d\tau^2 + 2N_j d\tau d\sigma^j + \partial_i x^a \partial_j x_a d\sigma^i d\sigma^j \ .$$

Gauge fixing by setting the shift $N_j = (-\partial_j x^- + \dot{x}^a \partial_j x^a) = 0$ yields

$$S_{NG} = -T\sqrt{-2\dot{x}^- - \dot{x}^a \dot{x}^a}\sqrt{\det G_{ij}}.$$

N.B. $\dot{x}^-$ linear in square root! And $\partial_j x^-$ only via the constraint.

On shell P^- is constant

$$P^- = \frac{\partial L_{NG}}{\partial \dot{x}^-} \quad \text{is a constant of the motion}$$

In 2-dim $det(G_{ij})$ can be rewritten using $\{x, y\} = \epsilon^{ij}\partial_i x \partial_j y$

Flat space Hamiltonian

$$S = -T\int\sqrt{-G} \longrightarrow H = \int_\Sigma (\frac{1}{\rho T} P^a P^a + \frac{T}{2\rho}\{X^a, X^b\}^2)$$

With the remaining constraint $\{P^a, X^a\} = 0$.

For higher p-branes the procedure works the same and using

$$\det(\partial_i X^a \partial_j X^b h_{ab}) =$$
$$\frac{1}{p!}\{X^{a_1}, X^{a_2} \ldots, X^{a_p}\}\{X^{b_1}, X^{b_2} \ldots, X^{b_p}\} h_{a_1 b_1} h_{a_2 b_2} \ldots h_{a_p b_p}$$
$$\{X^{a_1}, X^{a_2} \ldots, X^{a_p}\} := \epsilon^{j_1, j_2, \ldots, j_p} \partial_{j_1} X^{a_1} \partial_{j_2} X^{a_2} \ldots \partial_{j_p} X^{a_p}$$

and the Hamiltonian becomes

$$H = \int_\Sigma d^p\sigma \left(\frac{1}{\rho T} P^a P^a + \frac{4}{p! \rho^2} \{X^{a_1}, X^{a_2} \ldots, X^{a_p}\}^2 \right)$$

The residual symmetry is that of area preserving diffeomorphisms.

Quantisation

A direct approach, either Hamiltonian or path integral, has not yet been successful.

Matrix membranes

Functions are approximated by $N \times N$ matrices, $f \to \mathrm{F}$, and $\int_\Sigma f \to \mathrm{TrF}$.

The Hamiltonian becomes

$$\mathrm{H} = -\frac{1}{2}\nabla^2 - \frac{1}{4}\sum_{a,b=1}^{D} \mathrm{Tr}[X^a, X^b]^2$$

restricted to $U(N)$ singlet "physical" states.

- H describes a matrix membrane (or "fuzzy" membrane) in $D+1$ spacetime.
- At low energy—the bottom of the potential the coordinates commmute $[X^a, X^b] = 0$.
- Saddle points of the potential satisfy $[X^a[X^a, X^b]] = 0$.

Once we have the Hamiltonian H we can consider thermal ensembles of membranes whose partition function is given by

$$Z = \mathrm{Tr}_{Phys}(\mathrm{e}^{-\beta H})$$

where the physical constraint means the states are $U(N)$ invariant.

Path Integral version

$$Z = \int [dX] \mathrm{e}^{-\int_0^\beta d\tau \, \mathrm{Tr}(\frac{1}{2}(D_\tau X^a)^2 - \frac{1}{4}[X^a, X^b]^2)}$$

Gauss law constraint

The projection onto physical states — the Gauss law constraint is implemented by the gauge field, A, with

$$D_\tau X^a = \partial_\tau X^a - i[A, X^a].$$

Matrix membrane models are the zero volume limit of Yang-Mills compactified on a torus.

pp-wave backgrounds

Instead of membranes propagating on flat space we could have considered membranes propagating on different spacetimes. A very nice example is the pp-wave background

$$ds^2 = -2dx^+ dx_- + 2V(x)(dx^+)^2 + dx^a dx_a$$

$V(x)$ adds as a potential to the Hamiltonian.

$$\mathrm{H} = -\frac{1}{2}\nabla^2 + V(X) - \frac{1}{4}\sum_{a,b=1}^{D} \mathrm{Tr}[X^a, X^b]^2$$

The BMN model

The BMN action

$$S_{BMN} = N\int_0^\beta d\tau\,\mathrm{Tr}\left\{\frac{1}{2}(\mathcal{D}_\tau X^i)^2 + \frac{1}{2}(\frac{\mu}{3})^2(X^i)^2 + \frac{\mu}{3}i\epsilon_{ijk}X^iX^jX^k - \frac{1}{4}[X^i,X^j]^2 + \frac{1}{2}\Psi^T D_\tau\Psi + \frac{1}{2}(\frac{\mu}{4})\Psi^T i\gamma^{123}\Psi + \frac{1}{2}\Psi^T\Gamma^i[X^i,\Psi] + \frac{1}{2}(\mathcal{D}_\tau X^a)^2 + \frac{1}{2}(\frac{\mu}{6})^2(X^a)^2 + \frac{1}{2}\Psi^T\Gamma^a[X^a,\Psi] - \frac{1}{2}[X^a,X^j]^2 - \frac{1}{4}[X^a,X^b]^2\right\}\ .$$

The $SO(3)$ X^i shown as red give the previous model while together with the blue term combine as

$$\frac{1}{4}\mathrm{Tr}\left(i[X^i,X^j] + \frac{\mu}{3}\epsilon^{ijk}X^k\right)^2 .$$

The model has non-trivial fuzzy sphere vacua

$X^i = -\frac{\mu}{3}L^i$, with L^i $su(2)$ generators.

Fermions and the Dirac operator

Fermions on the fuzzy sphere

$$N\int_0^\beta d\tau\,\mathrm{Tr}\left\{\frac{1}{2}\Psi^T D_\tau\Psi-\frac{\mu}{6}\Psi^T\left(\Gamma^i\mathcal{L}_i-\frac{3}{4}i\gamma^{123}\right)\Psi\right\}$$

To be compared with

Standard Dirac operator on the fuzzy sphere

$$N\int_0^\beta d\tau\,\mathrm{Tr}\left\{\frac{1}{2}\Psi^T D_\tau\Psi+A\Psi^T\left(\Gamma^i\mathcal{L}_i+i\gamma^{123}\right)\Psi\right\}$$

where A is some coefficient and $\mathcal{L}_i\Psi=[L_i,\Psi]$.

The fermions on these fuzzy spheres are massless but Spin-C.

Understanding gauged quantum matrix models

The simplest example of a quantum mechanical model with Gauss Law constraint is a set of p gauged Gaussians. Their Euclidean actions are

$$N\int_0^\beta \mathrm{Tr}(\frac{1}{2}(\mathcal{D}_\tau X^i)^2+\frac{1}{2}m^2(X^i)^2)$$

$\mathcal{D}_\tau X^i = \partial_\tau X^i - i[A, X^i]$.

Properties of gauge Gaussian models

- The eigenvalues of X^i have a Wigner semi-circle distribution.
- At $T = 0$, we can gauged A away, while for large T we get a pure matrix model with A one of the matrices.
- The entry of A as an additional matrix in the dynamics signals a phase transition. In the Gaussian case with p scalars it occurs at

$$T_c = \frac{m}{\ln p}$$

  The transition can be observed as centre symmetry breaking in the Polyakov loop.

Bosonic matrix membranes are approximately gauge gaussian models V. Filev and D.O'C. [1506.01366 and 1512.02536]. They have however two phase transitions, very close in temperature.

The simplest model of an emergent geometry.

Consider the Matrix Energy Functional

$$E = \frac{Tr}{N}\left(-\frac{1}{4}[D_a, D_b]^2 + \frac{2i}{3}\epsilon_{abc}D_aD_bD_c\right)$$

Partition Function

$$Z(\beta, g, b, c) = \int [dD_a] \mathrm{e}^{-S(D)} \qquad \text{where} \qquad S(D) = -\beta E(D)$$

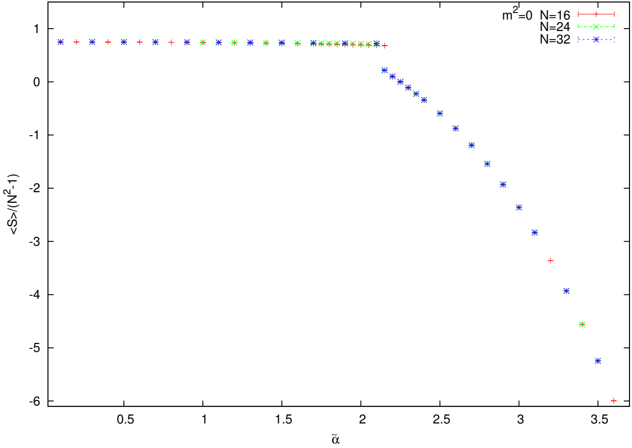
The minimum energy configuration is

$D_a = L_a$ with $[L_a, L_b] = i\epsilon_{abc}L_c$ and $L_aL_a = \frac{N^2-1}{4}\mathbf{1}$.
This configuration has $E_0 = -\frac{N^2-1}{48}$.

The ground state is a fuzzy sphere.
But this is the picture in the absence of fluctuations

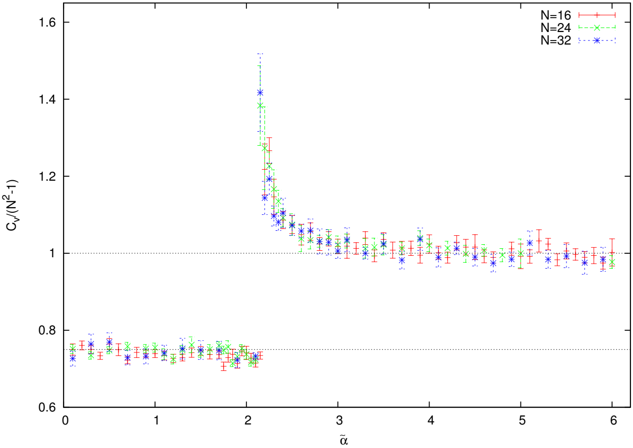
Monte Carlo Simulations

The singular part of the entropy is given by $\mathcal{S}/N^2$ where $\mathcal{S} =< S >$ and $\beta = \tilde{\alpha}^4$



Specific Heat

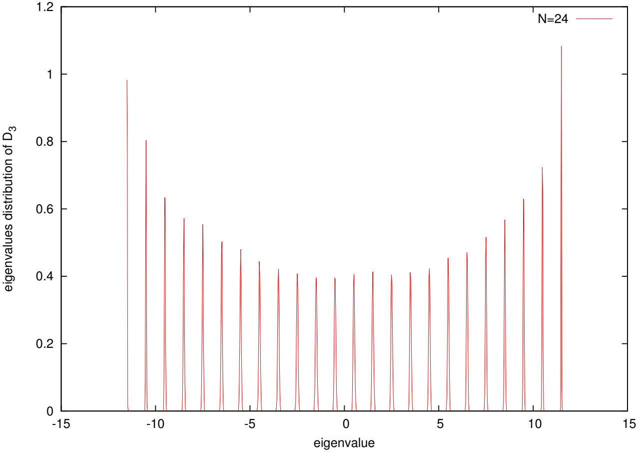
The specific heat C_v/N^2 where $C_v = <S^2> - <S>^2$ and



$\beta = \tilde{\alpha}^4$

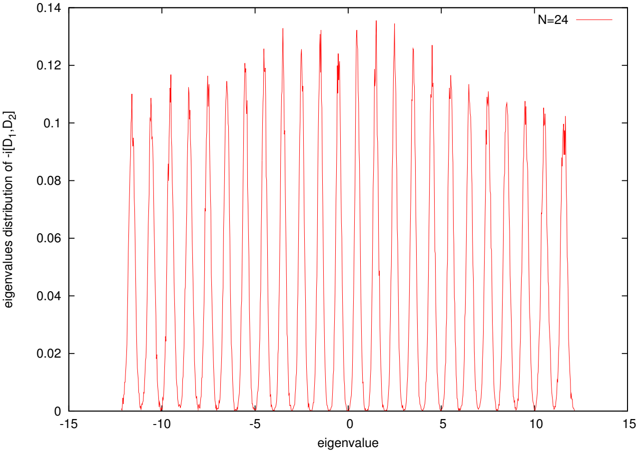
Eigenvalues in the low temperature phase

Eigenvalue distribution of D_3 for $N = 24$.



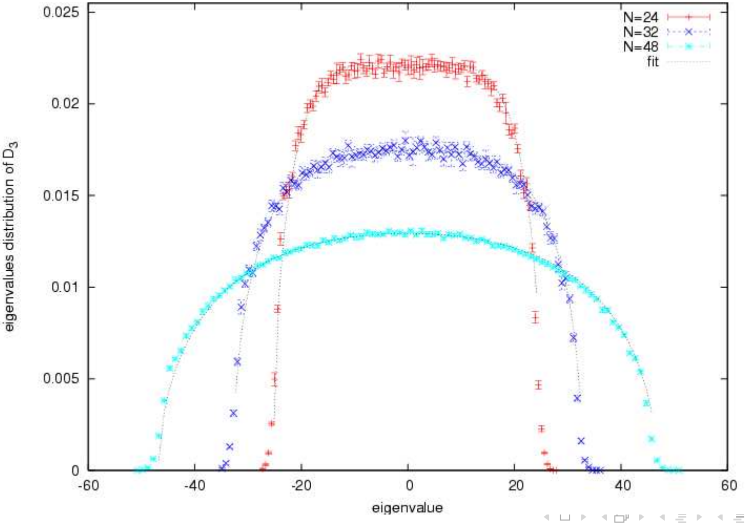
Eigenvalues in the low temperature phase

Eigenvalue distribution of $[D1, D_2]$ for $N = 24$.



Eigenvalues in the high temperature phase

Eigenvalue distribution of D_3 for $N = 24$.

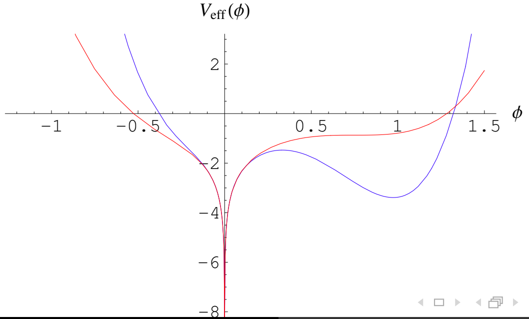


Effective potential

The effective potential, $V_{eff}(\phi)$, for ϕ where $D_a = \phi L_a$.

$$V_{eff} = \beta(\tfrac{1}{4}\phi^4 - \tfrac{1}{3}\phi^3) + \ln \phi^2$$

The location of the minimum gives predictions in excellent agreement with numerical data for the entropy and specific heat. It predicts the critical point as $\beta_c = (\frac{8}{3})^3$ and a critical exponent $\alpha = \frac{1}{2}$ for the divergence of the specific heat.



Predictions from $V_{eff}(\phi)$

$\mathcal{S} = \frac{5}{12}$ as the transition is approached from the fuzzy sphere side,

and jumps to $\mathcal{S} = \frac{3}{4}$ in the high temperature phase.

Entropy Jump

The transition is unusual in that it has a jump in the entropy. $\Delta\mathcal{S} = \frac{1}{3}$ indicating a 1st order transition.

Divergent Specific Heat

But it has a divergent specific heat $C = A_-(T_c - T)^{-\alpha}$ typical of a continuous (or second order) transition. We find the specific heat exponent $\alpha = \frac{1}{2}$.

When we add fermionic coordinates and demand supersymmetry S_{NG} with susy exist only in 4, 5, 7 and 11 dim-spacetime. These models are the susy models on flat backgrounds are toroidal dimensional reductions of Super-Yang-Mills.

κ-symmetry.

When we consider the models in non-trivial backgrounds consistency requires the backgrounds are solutions to supergravity.

This is reminiscent of the string σ-model β-functions being zero giving supergravity in strings.

The BFSS model

$$S_{SMembrane} = \int \sqrt{-G} - \int C + \quad \text{Fermionic terms}$$

The susy version only exists in $4, 5, 7$ and 11 spacetime dimensions.

BFSS Model — The supersymmetric membrane à la Hoppe

$$\mathrm{H} = \mathrm{Tr}\Big(\frac{1}{2}\sum_{a=1}^{9} P^a P^a - \frac{1}{4}\sum_{a,b=1}^{9}[X^a, X^b][X^a, X^b] + \frac{1}{2}\Theta^T \gamma^a [X^a, \Theta]\Big)$$

The model is claimed to be a non-perturbative 2nd quantised formulation of M-theory.

A system of N interacting D0 branes.

Note the flat directions.

Finite Temperature Model

The partition function and Energy of the model at finite temperature is

$$Z = Tr_{Phys}(\mathrm{e}^{-\beta\mathcal{H}}) \qquad \text{and} \qquad E = \frac{Tr_{Phys}(\mathcal{H}\mathrm{e}^{-\beta\mathcal{H}})}{Z} = \langle\mathcal{H}\rangle$$

The 16 fermionic matrices $\Theta_\alpha = \Theta_{\alpha A} t^A$ are quantised as

$$\{\Theta_{\alpha A}, \Theta_{\beta B}\} = 2\delta_{\alpha\beta}\delta_{AB}$$

The $\Theta_{\alpha A}$ are $2^{8(N^2-1)}$ and the Fermionic Hilbert space is

$$\mathcal{H}^F = \mathcal{H}_{256} \otimes \cdots \otimes \mathcal{H}_{256}$$

with $\mathcal{H}_{256} = \mathbf{44} \oplus \mathbf{84} \oplus \mathbf{128}$ suggestive of the graviton (**44**), anti-symmetric tensor (**84**) and gravitino (**128**) of $11 - d$ SUGRA.

For an attempt to find the ground state see: J. Hoppe et al arXiv:0809.5270

Lagrangian formulation

The BFSS matrix model is also the dimensional reduction of ten dimensional supersymmetric Yang-Mills theory down to one dimension:

$$
\begin{aligned}
S_{BFSS} \;&=\; \int d\tau \,\mathrm{Tr}\left\{\frac{1}{2}(\mathcal{D}_\tau X^i)^2 - \frac{1}{4}[X^i, X^j]^2 \right. \\
&\qquad \left. + \frac{1}{2}\Psi^T D_\tau \Psi + \frac{1}{2}\Psi^T \Gamma^i [X^i, \Psi]\right\} ,
\end{aligned}
$$

where Ψ is a sixteen component Majorana–Weyl spinor, Γ^i are gamma matrices of $Spin(9)$ in a basis for which the charge conjugation matrix $C = \mathbf{1}$.

The gravity dual and its geometry

Gauge/gravity duality predicts that the strong coupling regime of the theory is described by II_A supergravity, which lifts to 11-dimensional supergravity.

The bosonic action for eleven-dimensional supergravity is given by

$$S_{11D} = \frac{1}{2\kappa_{11}^2} \int [\sqrt{-g}R - \frac{1}{2}F_4 \wedge *F_4 - \frac{1}{6}A_3 \wedge F_4 \wedge F_4]$$

where $2\kappa_{11}^2 = 16\pi G_N^{11} = \frac{(2\pi l_p)^9}{2\pi}$.

The relevant solution to eleven dimensional supergravity for the dual geometry to the BFSS model corresponds to N coincident $D0$ branes in the IIA theory. It is given by

$$ds^2 = -H^{-1}dt^2 + dr^2 + r^2 d\Omega_8^2 + H(dx_{10} - Cdt)^2$$

with $A_3 = 0$

The one-form is given by $C = H^{-1} - 1$ and $H = 1 + \frac{\alpha_0 N}{r^7}$ where $\alpha_0 = (2\pi)^2 14\pi g_s l_s^7$.

Including temperature

The idea is to include a **black hole** in the gravitational system.

The Hawking temperature is matched with the temperature of the system.

Hawking radiation

We expect difficulties at low temperatures, as the system should Hawking radiate. It is argued (Hanada et al arXiv:1311.5607) that this is related to the flat directions and the propensity of the system to leak into these regions.

The black hole geometry

$$ds_{11}^2 = -H^{-1}Fdt^2 + F^{-1}dr^2 + r^2 d\Omega_8^2 + H(dx_{10} - Cdt)^2$$

Set $U = r/\alpha'$ and we are interested in $\alpha' \to \infty$
$H(U) = \frac{240\pi^5\lambda}{U^7}$ and the black hole time dilation factor
$F(U) = 1 - \frac{U_0^7}{U^7}$ with $U_0 = 240\pi^5\alpha'^5\lambda$. The temperature

$$\frac{T}{\lambda^{1/3}} = \frac{1}{4\pi\lambda^{1/3}} H^{-1/2} F'(U_0) = \frac{7}{2^4 15^{1/2} \pi^{7/2}} (\frac{U_0}{\lambda^{1/3}})^{5/2}.$$

From black hole entropy we obtain the prediction for the Energy

$$S = \frac{A}{4G_N} \sim \left(\frac{T}{\lambda^{1/3}}\right)^{9/2} \implies \frac{E}{\lambda N^2} \sim \left(\frac{T}{\lambda^{1/3}}\right)^{14/5}$$

Checks of the predictions

We found excellent agreement with this prediction V. Filev and D.O'C. [1506.01366 and 1512.02536].
The best current results (Berkowitz et al 2016) consistent with gauge gravity give

$$\frac{1}{N^2}\frac{E}{\lambda^{1/3}} = 7.41\left(\frac{T}{\lambda^{1/3}}\right)^{\frac{14}{5}} - (10.0 \pm 0.4)\left(\frac{T}{\lambda^{1/3}}\right)^{\frac{23}{5}} + (5.8 \pm 0.5)T^{\frac{29}{5}} + \dots - \frac{5.77T^{\frac{2}{5}} + (3.5 \pm 2.0)T^{\frac{11}{5}}}{N^2} + \dots$$

Checking the geometry with D4-brane probes

Berkooz and Douglas added new degrees of freedom to the BFSS model to describe the membrane in the presence of N_f longitudional M5-branes. When reduced to the 10-dim IIA string setting this means D4-branes.

Berkooz-Douglas model

The Berkooz-Douglas model
("Five-branes in M(atrix) theory," [hep-th/9610236])
is $\mathcal{N} = 1$ Susy in 6-dim, or $\mathcal{N} = 2$ in 4-dim reduced to 1-dim i.e. time.
The system describes a D0/D4 intersection.

The more general framework involves $Dp/D(p+4)$ systems.

Add new bosonic degrees of freedom Φ_α as two complex $N \times N_f$ matrices and their super partners λ_α so that the matrix model

BD-matrix model

The full model is

$$S_{BD} = S_{BFSS} + S_\Phi + S_\chi \,.$$

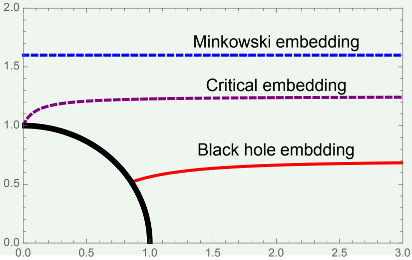
$$S_{\rm bos} = N \int_0^\beta d\tau \left[\mathrm{Tr} \left(\frac{1}{2} D_\tau X^a D_\tau X^a + \frac{1}{2} D_\tau \bar{X}^{\rho\dot{\rho}} D_\tau X_{\rho\dot{\rho}} - \frac{1}{4}[X^a, X^b]^2 + \frac{1}{2}[X^a, \bar{X}^{\rho\dot{\rho}}][X^a, X_{\rho\dot{\rho}}] \right) + \mathrm{tr} \left(D_\tau \bar{\Phi}^\rho D_\tau \Phi_\rho + \bar{\Phi}^\rho (X^a - m^a)^2 \Phi_\rho \right) + \frac{1}{2} \mathrm{Tr} \sum_{A=1}^{3} \mathcal{D}^A \mathcal{D}^A \right] .$$

Blackhole embeddings

The location of the D4-branes can be varied, relative to the central axis of the black hole by tuning the mass parameter of the fundamental multiplet.

Topologically distinct options

Topologically inequivalent embeddings correspond to a phase transition in the matrix model. The transition occurs when the mass of the fundamental fields is increased so that the D4-brane no longer intersects the blackhole.



The geometry can therefore be probed in some detail.

The Condensate

$$\langle \mathcal{O}_m^a \rangle \equiv \frac{\delta F}{\delta m^a} = \frac{1}{\beta} \left\langle \frac{\delta S_E}{\delta m^a} \right\rangle \ ,$$

$\tilde{U} = U/U_0$, (recall U_0 was the blackhole radius r_0/α')

$$\tilde{u} \sin\theta = \tilde{m} + \frac{\tilde{c}}{u^2} + \ldots . \tag{1}$$

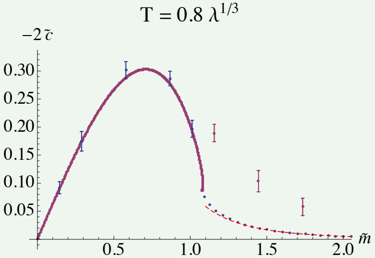
Using a Born-Infeld action (Nambu-Goto in this case) and solving for the embedding into the dual geometry, the holographic prediction relates to the BD-model parameters via:

$$\begin{aligned} m^a &= \left(\frac{120\,\pi^2}{49}\right)^{1/5} \tilde{T}^{\,2/5} \tilde{m}\, n^a \ , \\ \langle \mathcal{O}_m^a \rangle &= \left(\frac{2^4\, 15^3\, \pi^6}{7^6}\right)^{1/5} N_f\, N_c\, \tilde{T}^{\,6/5} (-2\,\tilde{c})\, n^a \ , \end{aligned} \tag{2}$$

The condensate senses the transition

The location of the D4-branes can be varied, relative to the central axis of the black hole by tuning the mass parameter of the fundamental multiplet.

Note the non-trivial scaling! The transition occurs when the mass of the fundamental fields is increased so that the D4-brane no longer intersects the blackhole.



The D4-brane can probe near the black hole surface.

Membranes on other backgrounds

There are many options for background geometries:

PP-Wave backgrounds

Two options that lead to massive deformations of the BFSS model

N=1*

Breaks susy down to 4 remaining.

BMN model

Preserves all 16 susys and has $SU(4|2)$ symmetry.

The BMN or PWMM

The supermembrane on the maximally supersymmetric plane wave spacetime

$$ds^2 = -2dx^+dx^- + dx^a dx^a + dx^i dx^i - dx^+dx^+((\frac{\mu}{6})^2(x^i)^2 + (\frac{\mu}{3})^2(x^a)^2)$$

with

$$dC = \mu dx^1 \wedge dx^2 \wedge dX^3 \wedge dx^+$$

so that $F_{123+} = \mu$. This leads to the additional contribution to the Hamiltonian

$$\begin{aligned} \Delta \mathrm{H}_\mu &= \frac{N}{2}\mathbf{Tr}\Big((\frac{\mu}{6})^2(X^a)^2 + (\frac{\mu}{3})^2(X^i)^2 \\ &\qquad + \frac{2\mu}{3} i\epsilon_{ijk}X^iX^jX^k + \frac{\mu}{4}\Theta^T i\gamma^{123}\Theta\Big) \end{aligned}$$

The BMN model

The BMN action

$$\begin{aligned}
S_{BMN} = \quad & N\int_0^\beta d\tau\, \mathrm{Tr}\left\{\frac{1}{2}(\mathcal{D}_\tau X^i)^2 + \frac{1}{2}(\frac{\mu}{3})^2(X^i)^2\right.\\
& +\frac{\mu}{3} i\epsilon_{ijk}X^iX^jX^k - \frac{1}{4}[X^i,X^j]^2\\
& +\frac{1}{2}\Psi^T D_\tau\Psi + \frac{1}{2}(\frac{\mu}{4})\Psi^T i\gamma^{123}\Psi + \frac{1}{2}\Psi^T\Gamma^i[X^i,\Psi]\\
& +\frac{1}{2}(\mathcal{D}_\tau X^a)^2 + \frac{1}{2}(\frac{\mu}{6})^2(X^a)^2\\
& \left.+\frac{1}{2}\Psi^T\Gamma^a[X^a,\Psi] - \frac{1}{2}[X^a,X^j]^2 - \frac{1}{4}[X^a,X^b]^2\right\} .
\end{aligned}$$

Large mass expansion

For large μ the model becomes the supersymmetric Gaussian model

Finite temperature Euclidean Action

$$S_{BMN} = \frac{1}{2g^2}\int_0^\beta d\tau \, \mathrm{Tr}\Big\{(\mathcal{D}_\tau X^i)^2 + (\frac{\mu}{6})^2(X^a)^2 + (\frac{\mu}{3})^2(X^i)^2 \\ \Psi^T D_\tau \Psi + \frac{\mu}{4}\Psi^T\gamma^{123}\Psi\Big\}$$

This model has a phase transition at $T_c = \frac{\mu}{12\ln 3}$

Diffeomorphism invariant Gaussian model.

If one goes back to the membrane action is of the form

$$S[\mathcal{X}] = \int dt \int d^2\sigma \mathcal{W}(\sigma) \left(\frac{1}{2}(D_0\mathcal{X}^a)D_0\mathcal{X}^a + \frac{1}{2}m^2\mathcal{X}^a(\sigma)\mathcal{X}^a(\sigma) \right)$$

where $a = 1, \ldots, p$, $D_0\mathcal{X}^a = \partial_0\mathcal{X}^a - \{\omega, \mathcal{X}^a\}$ and $\{A, B\} - \frac{\varepsilon^{rs}}{\mathcal{W}}\partial_r A \partial_s B$ and normalised so that $\int d^2\sigma\mathcal{W}(\sigma) = 1$. This is just a Gaussian version of the membrane action that arises in the large mass deformation limit on a particular *pp*-wave background.

Perturbative expansion in large μ.

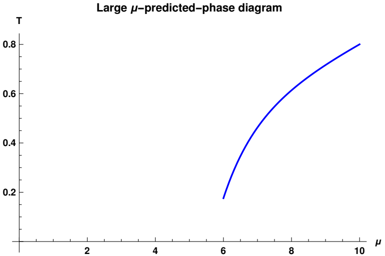
Three loop result of *Hadizadeh, Ramadanovic, Semenoff and Young* [hep-th/0409318]

$$T_c = \frac{\mu}{12 \ln 3} \left\{ 1 + \frac{2^6 \times 5}{3^4} \frac{\lambda}{\mu^3} - \left(\frac{23 \times 19927}{2^2 \times 3^7} + \frac{1765769 \ln 3}{2^4 \times 3^8} \right) \frac{\lambda^2}{\mu^6} + \cdots \right\}$$

Perturbative expansion in large μ.

Three loop result of *Hadizadeh, Ramadanovic, Semenoff and Young* [hep-th/0409318]

$$T_c = \frac{\mu}{12 \ln 3} \left\{ 1 + \frac{2^6 \times 5}{3^4} \frac{\lambda}{\mu^3} - \left(\frac{23 \times 19927}{2^2 \times 3^7} + \frac{1765769 \ln 3}{2^4 \times 3^8} \right) \frac{\lambda^2}{\mu^6} + \cdots \right\}$$



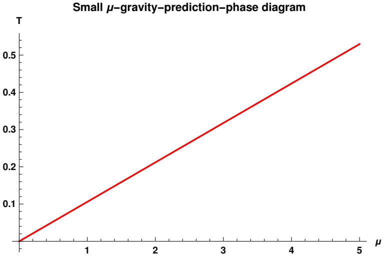
Passes through zero at $\mu = 5.65$.

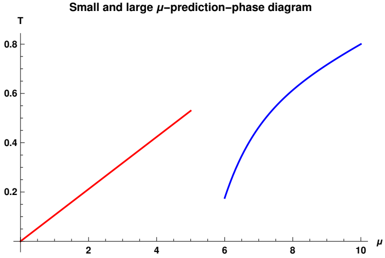
Gravity prediction at small μ

Costa, Greenspan, Penedones and Santos, [arXiv:1411.5541]

$$\lim_{\frac{\lambda}{\mu^2}\to\infty} \frac{T_c^{\text{SUGRA}}}{\mu} = 0.105905(57)\,.$$

The prediction is for low temperatures and small μ the transition temperature approaches zero linearly in μ.





Padé approximant prediction of T_c

$$T_c = \frac{\mu}{12\ln 3}\left\{1 + r_1\frac{\lambda}{\mu^3} + r_2\frac{\lambda^2}{\mu^6} + \cdots\right\}$$

with $r_1 = \frac{2^6\times 5}{3}$ and $r_2 = -(\frac{23\times 19927}{2^2\times 3} + \frac{1765769\ln 3}{2^4\times 3^2})$

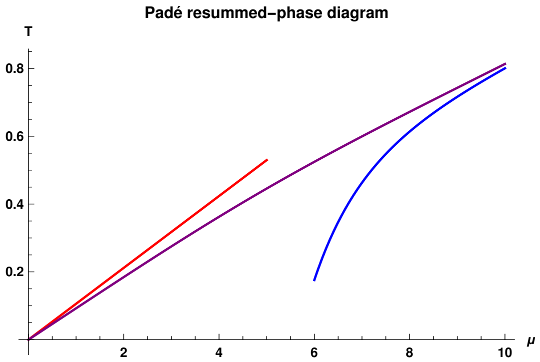
Using a Padé Approximant: $1 + r_1 g + r_2 g^2 + \cdots \to 1 + \frac{1+r_1 g}{1-\frac{r_2}{r_1}g}$

$$\implies T_c^{\text{Padé}} = \frac{\mu}{12\ln 3}\left\{1 + \frac{r_1\frac{\lambda}{\mu^3}}{1 - \frac{r_2}{r_1}\frac{\lambda}{\mu^3}}\right\}$$

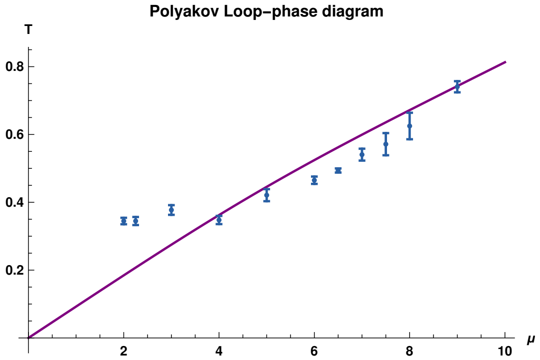
Now we can take the small μ limit

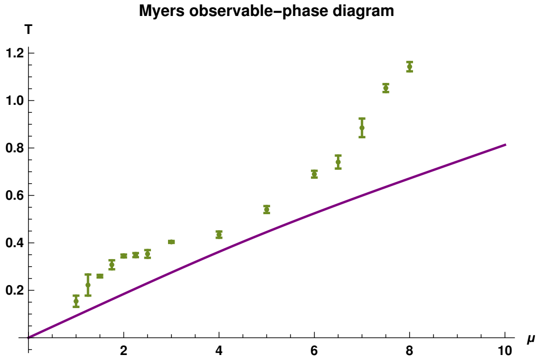
$$\lim_{\frac{\lambda}{\mu^2}\to\infty} \frac{T_c^{\text{Padé}}}{\mu} \simeq \frac{1}{12\ln 3}\left(1-\frac{r_1^2}{r_2}\right) = 0.0925579$$

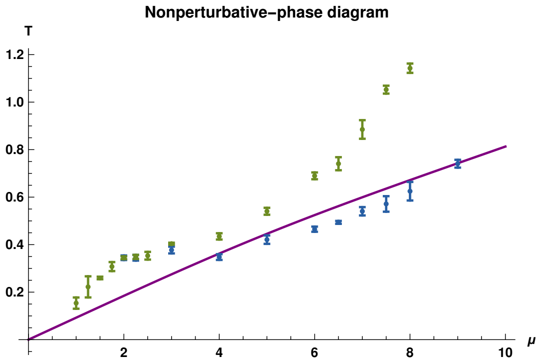
$$\lim_{\frac{\lambda}{\mu^2}\to\infty} \frac{T_c^{\text{SUGRA}}}{\mu} = 0.105905(57)\,.$$



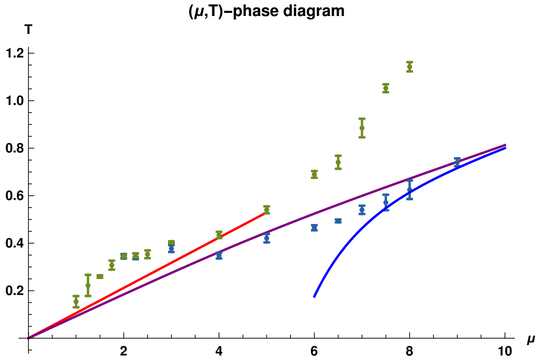
A non-perturbative phase diagram from the Polyalov loop.




Myers observable-phase diagram
T
1.2
1.0
0.8
0.6
0.4
0.2
2
4
6
8
10
μ

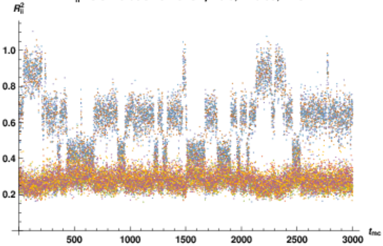


Green Myers transition
Blue Polyakov loop transition
Purple Padé prediction for the transition

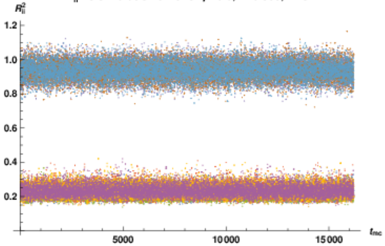

(μ,T)–phase diagram
T
1.2
1.0
0.8
0.6
0.4
0.2
2
4
6
8
10
μ

Observables

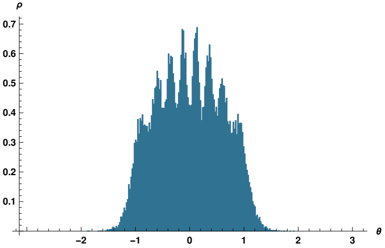
R^2_{ii} vs simulation time for μ=6.0, T=0.69, N=8



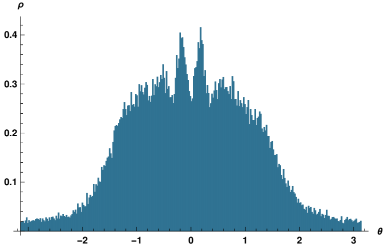
R^2_{ii} vs simulation time for μ=6.0, T=0.365, N=8

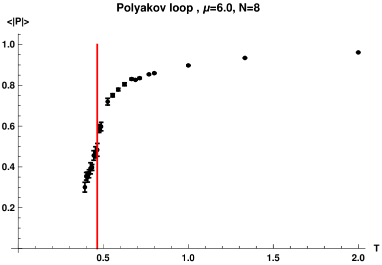


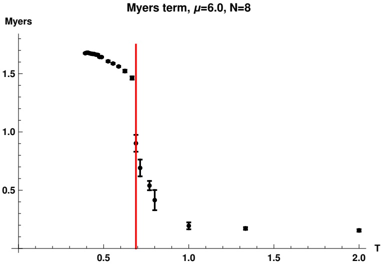
Eigenvalue density of A for μ=6.0, T=0.69, N=8, Λ=24

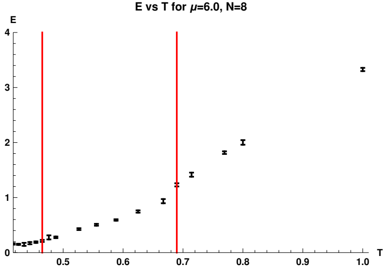


Eigenvalue density of A for μ=6.0, T=0.465, N=8, Λ=24

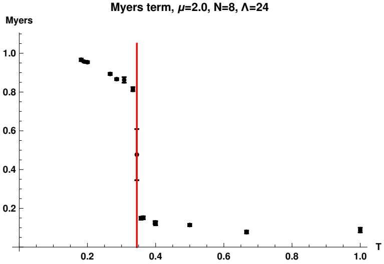


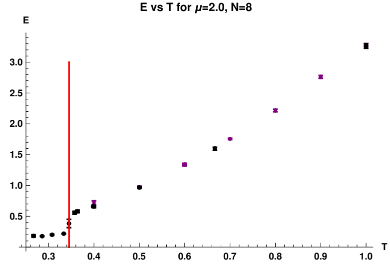

Polyakov loop , μ=6.0, N=8
<|P|>
1.0
0.8
0.6
0.4
0.2
0.5
1.0
1.5
2.0
T

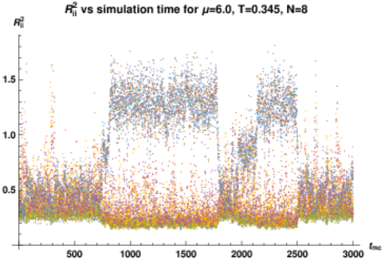

Myers term, μ=6.0, N=8
Myers
1.5
1.0
0.5
0.5
1.0
1.5
2.0
T

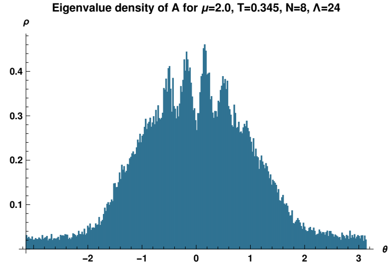

E vs T for μ=6.0, N=8
E
4
3
2
1
0
0.5
0.6
0.7
0.8
0.9
1.0
T

Small μ

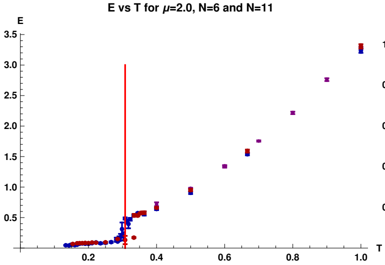

Myers term, μ=2.0, N=8, Λ=24
Myers
1.0
0.8
0.6
0.4
0.2
0.2
0.4
0.6
0.8
1.0
T

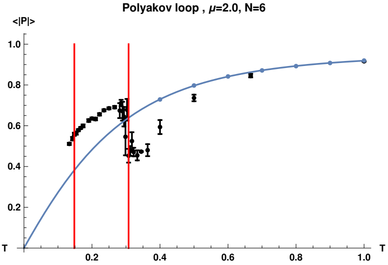

E vs T for μ=2.0, N=8
E
3.0
2.5
2.0
1.5
1.0
0.5
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
T


R_{ii}^2 vs simulation time for μ=6.0, T=0.345, N=8
R_{ii}^2
1.5
1.0
0.5
500
1000
1500
2000
2500
3000
t_{mc}


Eigenvalue density of A for μ=2.0, T=0.345, N=8, Λ=24
ρ
0.4
0.3
0.2
0.1
−2
−1
0
1
2
3
θ

Non-monotonic Polyakov loop





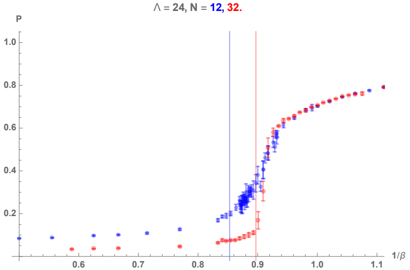
The Bosonic BMN model

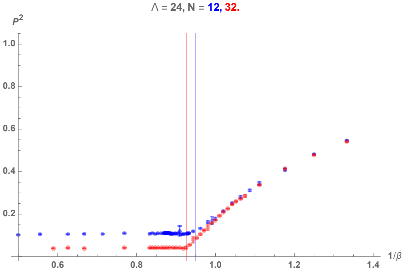
We have recently studied the bosonic BMN model.

Bosonic BMN

The Bosonic BMN model has two phase transitions very close together. Both are visible in the eigenvalues of A. It does not have a fuzzy sphere phase.

With Y. Asano and S. Kováčik (to appear)





Conclusions

- We have seen membrane matrix models in action. We can probe the conjecture dual geometry using probe D4 branes.
- The BMN – plane wave matrix model is very rich with both emergent geometry in the form of fuzzy spheres and confining-deconfining phase transitions.
- The models have gravity dual geometries that predict their strong coupling behaviour.
- Much overlap with large N reduction (Kawai).
- Also saddle points of the bosonic parts of the actions are equations for non-commutative minimal surfaces (Arnlind). It would be nice to make contact here with "resurgence".
- Questions such as "Can one find a background independence formulation?' arise just as in string theory.
- These models have a countable number of degrees of freedom.
- They cut the multiverse from string theory and yet are very closely related!

Thank you for your attention!