Quantum space-time, higher spin and gravity from the IKKT matrix model

Harold Steinacker

Department of Physics, University of Vienna

Bures-sur-Yvette, february 2019

Action MP 1405
Quantum Structure of Spacetime

how to formulate quantum theory of spacetime & gravity?

guidelines:

- simple
- gauge theory (Minkowski signature!)
- finite dof per volume (Planck scale!)
  $\rightarrow$ underlying d.o.f. non-geometric
- GR established only in IR regime
  space-time & gravity may emerge from other d.o.f.

  (cf. Navier-Stokes)

Matrix Models (of Yang-Mills type)

$S = Tr([X^\mu, X^\nu][X_\mu, X_\nu] + ...)$ provide such models!

- simple
- describe dynamical NC / fuzzy spaces, gauge theory

  $$X^a \to U^{-1} X^a U$$

- well suited for quantization: $\int dX\, e^{-S[X]}$
  - generic models: serious UV/IR mixing problem
  - preferred model: maximal SUSY = IKKT model

    shares features of string theory, cut the “landscape”
- gravity?

summary of recent results to be discussed:

- (3+1)-dim. covariant quantum space-time solution
  (FLRW cosmology, Big Bounce)
- tower of higher-spin modes, truncated at n.

  $\rightarrow$ higher-spin gauge theory, all d.o.f. required for gravity
- propagation governed by universal dynamical metric
  (Lorentz invar. only partially manifest)
- metric perturbations $\rightarrow$ massless graviton & scalar
- analog of the linearized Einstein-Hilbert action
  (expected to be induced upon quantization)

HS, arXiv:1606.00769
HS, arXiv:1710.11495
M. Sperling, HS arXiv:1806.05907
M. Sperling, HS arXiv:1901.03522

outline:

- the IKKT matrix model & NC gauge theory
- *4D covariant quantum spaces*:
  fuzzy H_n^4, cosmological space-time $\mathcal{M}_n^{3,1}$
- fluctuations $\rightarrow$ higher spin gauge theory
- metric perturbations
  linearized Einstein-Hilbert action & gravity

The IKKT model

IKKT or IIB model Ishibashi, Kawai, Kitazawa, Tsuchiya 1996

$$S[Y,\Psi] = -Tr\left([Y^a,Y^b][Y^{a'},Y^{b'}]\eta_{aa'}\eta_{bb'} \; + m^2 Y^a Y_a \; + \bar\Psi\gamma_a[Y^a,\Psi]\right)$$

$$Y^a = Y^{a\dagger} \in Mat(N,\mathbb{C}), \qquad a = 0,...,9, \qquad N \to \infty$$

gauge invariance $Y^a \to UY^aU^{-1}$, $SO(9,1)$, SUSY

- quantized Schild action for IIB superstring
- reduction of $10D$ SYM to point, N large
- equations of motion:

  $\Box Y^a + m^2 Y^a = 0, \qquad \Box \equiv \eta_{ab}[Y^a,[Y^b,.]]$
- quantization: $Z = \int dY d\Psi\, e^{iS[Y]}$, SUSY essential

strategy:

- look for solutions $\rightarrow$ space(time)
- fluctuations $\rightarrow$ gauge theory, dynamical geometry, gravity ?!
- matrix integral = (Feynman) path integral, incl. geometry

class of solutions: fuzzy spaces = quantized symplectic manifolds

$$X^a \sim x^a : \quad \mathcal{M} \hookrightarrow \mathbb{R}^{9,1}$$

$[X^a, X^b] \sim i\theta^{ab}(x)$ ... (quantized) Poisson tensor

algebra of functions on fuzzy space: $\mathrm{End}(\mathcal{H})$

- Moyal-Weyl quantum plane $\mathbb{R}^4_\theta$:

$$[X^a, X^b] = i\theta^{ab}\,\mathbb{1}$$

quantized symplectic space $(\mathbb{R}^4, \omega)$

admits translations $X^a \to X^a + c^a\mathbb{1}$, no rotation invariance

- fuzzy 2-sphere S^2_N

$$X_1^2 + X_2^2 + X_3^2 = R_N^2, \qquad [X_i, X_j] = i\epsilon_{ijk}X_k$$

fully covariant under $SO(3)$ (Hoppe; Madore)

fluctuations in M.M. on fuzzy space $\rightarrow$ NC gauge theory

derivations:

$$[X^\mu, \phi] =: i\theta^{\mu\nu}\partial_\nu\phi$$

consider fluctuations around background X^a

$$Y^a = X^a + \mathcal{A}^a$$

$$\begin{aligned}
[Y^\mu, \phi] &= i\theta^{\mu\nu} D_\nu\phi, \qquad D_\mu = \partial_\mu + i[A_\mu, .] \\
[Y^\mu, Y^\nu] &= i\theta^{\mu\nu} + i\theta^{\mu\mu'}\theta^{\nu\nu'} \underbrace{(\partial_{\mu'}A_{\nu'} - \partial_{\nu'}A_{\mu'} + [A_{\mu'}, A_{\nu'}])}_{F_{\mu'\nu'}} \\
S &= Tr([Y^\mu, Y^\nu][Y_\mu, Y_\nu]) \sim \int F_{\mu\nu}F^{\mu\nu} + c.
\end{aligned}$$

$\rightarrow$ YM gauge theory

$$A_\mu \rightarrow U^{-1}A_\mu U + U^{-1}\partial_\mu U$$

$\leftrightarrow$ dynamical geometry ("emergent gravity") emergent gravity on deformed $\mathcal{M}_\theta^4$?

H.S. arXiv:1003.4134 ff

cf. H.Yang, hep-th/0611174 ff

eff. metric encoded in $\Box = [X_a, [X^a, .]] \sim -e^\sigma(x)\Delta_G$

$$G^{\mu\nu} = e^{-\sigma}\theta^{\mu\mu'}\theta^{\mu\mu'}g_{\mu'\nu'}$$

fluctuations $X^a + \mathcal{A}^a(X) \to$ dynamical metric

cf. Rivelles hep-th/0212262

bare M.M action: 2 Ricci-flat metric fluctuations, not full Einstein eq.

quantum effects $\to$ induced gravity (Sakharov)

problems:

- $\theta^{\mu\nu}$ breaks Lorentz invariance $\to$ other terms possible
  (e.g. $R_{\mu\nu\alpha\beta}\theta^{\mu\nu}\theta^{\alpha\beta}$ D. Klammer, H.S. arXiv:0909.5298)
- huge cosm. constant

issues seem resolved for covariant quantum spaces:

4D covariant quantum spaces

- in 4D: symplectic form ω breaks local (Lorentz/Euclid.) invar.
- avoided on covariant quantum spaces

  example: fuzzy four-sphere S_N^4

  Grosse-Klimcik-Presnajder; Castelino-Lee-Taylor; Medina-o'Connor; Ramgoolam; Kimura; Karabail-Nair; Zhang-Hu 2001 (QHE) ...

  $\rightarrow$ higher-spin gauge theory in IKKT HS arXiv:1606.00769
- noncompact H_n^4 $\rightarrow$ higher-spin gauge theory

  M. Sperling, HS arXiv:1806.05907
- projection of $H_n^4 \rightarrow$ cosmological space-time $\mathcal{M}_n^{3,1}$

  HS, arXiv:1710.11495, arXiv:1709.10480
- spin 2 modes on $\mathcal{M}_n^{3,1}$ $\rightarrow$ gravity M. Sperling, HS arXiv:1901.03522

Euclidean fuzzy hyperboloid H_n^4 $(=EAdS_n^4)$

Hasebe arXiv:1207.1968

$\mathcal{M}^{ab}$ ... hermitian generators of $\mathfrak{so}(4,2)$,

$$[\mathcal{M}_{ab}, \mathcal{M}_{cd}] = i(\eta_{ac}\mathcal{M}_{bd} - \eta_{ad}\mathcal{M}_{bc} - \eta_{bc}\mathcal{M}_{ad} + \eta_{bd}\mathcal{M}_{ac}) \ .$$

$\eta^{ab} = \mathrm{diag}(-1,1,1,1,1,-1)$

choose "short" discrete unitary irreps $\mathcal{H}_n$ ("minireps", doubletons)

special properties:

- irreps under $\mathfrak{so}(4,1)$, multiplicities one, minimal oscillator rep.
- positive discrete spectrum

$$\mathrm{spec}(\mathcal{M}^{05}) = \{E_0, E_0+1, ...\}, \qquad E_0 = 1 + \frac{n}{2}$$

lowest eigenspace is $n+1$-dim. irrep of $SU(2)_L$: fuzzy S_n^2

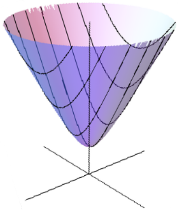
fuzzy hyperboloid H_n^4

def.

$$\begin{aligned} X^a &:= r\mathcal{M}^{a5}, \qquad a = 0, \dots, 4 \\ [X^a, X^b] &= ir^2\mathcal{M}^{ab} =: i\Theta^{ab} \end{aligned}$$

5 hermitian generators X^a satisfy (cf. Snyder)

$$\eta_{ab}X^aX^b = X^iX^i - X^0X^0 = -R^2\mathbb{1}, \qquad R^2 = r^2(n^2-4)$$



one-sided hyperboloid in $\mathbb{R}^{1,4}$, covariant under $SO(4,1)$

note: induced metric = Euclidean AdS^4

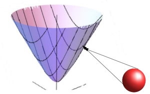
oscillator construction: 4 bosonic oscillators $[\psi_\alpha, \bar\psi^\beta] = \delta_\alpha^\beta$

$\mathcal{H}_n$ = suitable irrep in Fock space

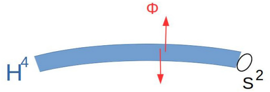
Then

$$\mathcal{M}_{ab} = \bar\psi \Sigma_{ab} \psi, \qquad \gamma_0 = diag(1,1,-1,-1)$$
$$X^a = r\bar\psi\gamma^a\psi$$

fact: $H_n^4 \;=\;$ quantized $\mathbb{C}P^{1,2} = S^2$ bundle over H^4, selfdual $\theta^{\mu\nu}$



functions on $H_n^4 \overset{loc}{\cong} S^2 \times H^4$ = harmonics on S^2 $\times$ functions on H^4



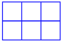
local stabilizer acts on S^2 $\rightarrow$ harmonics = higher spin modes

fuzzy "functions" on H_n^4:

$$(End(\mathcal{H}_n) \rightsquigarrow) \qquad HS(\mathcal{H}_n) = \int_{\mathbb{C}P^{1,2}} f(m)\, |m\rangle\langle m| \;\cong\; \bigoplus_{s=0}^{n} \mathcal{C}^s$$

$\mathcal{C}^0$ = scalar functions on H^4: $\qquad \phi(X)$

$\mathcal{C}^1$ = selfdual 2-forms on H^4: $\qquad \phi_{\mu\nu}(X)\theta^{\mu\nu} \;=$ 

$End(\mathcal{H}_n) \cong$ fields on H^4 taking values in $\mathfrak{hs} = \oplus$  $\ni \theta^{\mu_1\nu_1}\ldots\theta^{\mu_s\nu_s}$

higher spin modes = would-be KK modes on S^2

i.e. higher spin theory, truncated at n

M. Sperling, HS arXiv:1806.05907

H_n^4 is starting point for cosmological quantum space-times $\mathcal{M}_n^{3,1}$:

- exactly homogeneous & isotropic, Big Bounce
- on-shell higher-spin fluctuations obtained
- spin 2 metric fluctuations $\to$ gravity (linearized)

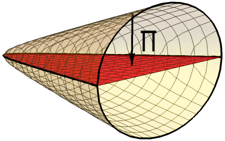
open FRW universe from H_n^4

HS arXiv:1710.11495

$\mathcal{M}_n^{3,1}$ = H_n^4 projected to $\mathbb{R}^{1,3}$ via

$$Y^\mu \sim y^\mu : \mathbb{C}P^{1,2} \to H^4 \subset \mathbb{R}^{1,4} \xrightarrow{\Pi} \mathbb{R}^{1,3} .$$

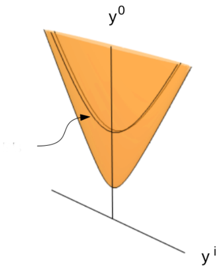
induced metric has Minkowski signature!



algebraically: $\mathcal{M}_n^{3,1}$ generated by

$$\boxed{Y^\mu := X^\mu, \quad \text{for } \mu = 0,1,2,3 \qquad \text{(drop } X^4 \text{)}}$$

geometric properties:



- $SO(3,1)$ manifest $\Rightarrow$ foliation into $SO(3,1)$-invariant space-like 3-hyperboloids H^3_τ
- double-covered FRW space-time with hyperbolic ($k=-1$) spatial geometries

$$ds^2 \quad = d\tau^2 - a(\tau)^2 d\Sigma^2,$$

$d\Sigma^2$ ... $SO(3,1)$-invariant metric on space-like H^3

functions on $\mathcal{M}^{3,1}$:

generated by $X^\mu = r\mathcal{M}^{\mu 5} \sim x^\mu$ and $T^\mu = \frac{1}{R}\mathcal{M}^{\mu 4} \sim t^\mu$, with CR

$$\begin{aligned} \{t^\mu, x^\nu\} &= \sinh(\eta)\eta^{\mu\nu} \\ \{x^\mu, x^\nu\} &= \theta^{\mu\nu} \\ \{t^\mu, t^\nu\} &= -\frac{1}{r^2 R^2}\theta^{\mu\nu} \end{aligned}$$

constraints

$$\begin{aligned} x_\mu x^\mu &= -R^2\cosh^2(\eta), \qquad x^4 = R\sinh(\eta) \\ t_\mu t^\mu &= r^{-2}\,\cosh^2(\eta)\,, \\ t_\mu x^\mu &= 0, \\ t_\mu \theta^{\mu\alpha} &= -\sinh(\eta) x^\alpha, \\ x_\mu \theta^{\mu\alpha} &= -r^2 R^2 \sinh(\eta) t^\alpha, \\ \eta_{\mu\nu}\theta^{\mu\alpha}\theta^{\nu\beta} &= R^2 r^2 \eta^{\alpha\beta} - R^2 r^4 t^\alpha t^\beta - r^2 x^\alpha x^\beta \\ \theta^{\mu\nu}\theta_{\mu\nu} &= 2R^2 r^2\left(2 - \cosh^2(\eta)\right) \end{aligned}$$

hence: t^μ ... space-like S^2

self-duality on H^4 $\Rightarrow$ $\theta^{\mu\nu} = c(x^\mu t^\nu - x^\nu t^\mu) + b\epsilon^{\mu\nu\alpha\beta}x_\alpha t_\beta$

functions as higher-spin modes:

$$\phi = \sum_{s=0}^{n} \phi^{(s)} \quad \in \mathrm{End}(\mathcal{H}_n), \qquad \phi^{(s)} \in \mathcal{C}^s$$

2 points of view:

- functions on H_n^4: full $SO(4,1)$ covariance

  represent $\phi^{(s)}$ as

$$\begin{aligned} \phi^H_{a_1\ldots a_s} &\propto \{x^{a_1}, \ldots \{x^{a_s}, \phi^{(s)}\} \ldots\}_0 \\ \phi^{(s)} &= \{x^{a_1}, \ldots \{x^{a_s}, \phi^H_{a_1\ldots a_s}\} \ldots\} \end{aligned}$$

- functions on $\mathcal{M}_n^{3,1}$: reduced $SO(3,1)$ covariance

$$\phi^{(s)} = \phi^{(s)}_{\mu_1\ldots\mu_s}(x) t^{\mu_1} \ldots t^{\mu_s}$$

$t_\mu x^\mu = 0 \;\Rightarrow\;$ "space-like gauge" $\quad \boxed{x^{\mu_i}\phi^{(s)}_{\mu_1\ldots\mu_s} = 0}$

($\rightarrow$ no ghosts!)

$\mathcal{M}^{3,1}$ realization in IKKT model:

background solution:

$$T^\mu := \frac{1}{R}\mathcal{M}^{\mu 4}$$

satisfies

$$\Box T^\mu = 3R^{-2}T^\mu, \qquad \Box = [T^\mu,[T_\mu,.]]$$

- $[\Box,\mathcal{S}^2] = 0, \quad \mathcal{S}^2 = [\mathcal{M}^{ab},[\mathcal{M}_{ab},\cdot]] + r^{-2}[X_a,[X^a,\cdot]]$

  ... spin Casimir, selects spin s sectors $\mathcal{C}^s$

  $\Rightarrow$ higher-spin expansion $\phi = \phi(X) + \phi_\mu(X)T^\mu + \ldots$ on $\mathcal{M}^{3,1}$
- $\Box \sim \alpha^{-1}\Box_G$ encodes eff. FRW metric $ds_G^2 = -dt^2 + a(t)^2 d\Sigma^2$, asymptotically coasting $a(t) \propto t$
- Big Bounce, initial $a(t) \sim t^{1/5}$ singularity

fluctuations & higher spin gauge theory

$$S[Y] = Tr(-[Y^a, Y^b][Y_a, Y_b] + m^2 Y^a Y_a) = S[U^{-1} Y U]$$

background solution: $S_N^4,\ H_n^4,\ \mathcal{M}_n^{3,1}$

add fluctuations $Y^a = \overline{Y}^a + \mathcal{A}^a,$

gauge trafos $\mathcal{A}^a \to [\Lambda, \mathcal{A}^a] + [\Lambda, \overline{Y}^a], \qquad \Lambda \in End(\mathcal{H})$

expand action to second oder in $\mathcal{A}^a$

$$S[Y] = S[\overline{Y}] + \frac{2}{g^2} \mathrm{Tr} \mathcal{A}_a \Big(\underbrace{(\Box + \frac{1}{2} m^2)\delta^a_b + 2[[\overline{Y}^a, \overline{Y}^b], .]}_{\mathcal{D}^2} - \underbrace{[\overline{Y}^a, [\overline{Y}^b, .]]}_{g.f.} \Big) \mathcal{A}_b$$

$\mathcal{A}_a$ ... $\mathfrak{hs}$-valued field on $\mathcal{M}$, incl. spin 2

diagonalization & eigenmodes on $\mathcal{M}_n^{3,1}$: background $\overline{Y}^\mu = T^\mu$

M. Sperling, HS: arXiv:1901.03522

symmetry: only space-like $SO(3,1)$
underlying $SO(4,1)$ & $SO(4,2)$ extremely useful

- ansatz:

$$
\begin{aligned}
\mathcal{A}_\mu^{(+)}[\phi^{(s)}] &:= \{x_\mu, \phi^{(s)}\}_+ \\
\mathcal{A}_\mu^{(-)}[\phi^{(s)}] &:= \{x_\mu, \phi^{(s)}\}_- \ .
\end{aligned}
$$

can show using $\mathfrak{so}(4,2)$ structure: are eigenmodes

$$
\begin{aligned}
\mathcal{D}^2 \mathcal{A}_\mu^{(+)}[\phi^{(s)}] &= \mathcal{A}_\mu^{(+)}\left[\left(\Box + \tfrac{2s+5}{R^2}\right)\phi^{(s)}\right] \\
\mathcal{D}^2 \mathcal{A}_\mu^{(-)}[\phi^{(s)}] &= \mathcal{A}_\mu^{(-)}\left[\left(\Box + \tfrac{-2s+3}{R^2}\right)\phi^{(s)}\right] \ .
\end{aligned}
$$

$\rightarrow$ 2 physical modes $(\Box + \frac{c_s}{R^2})\phi^{(s)} = 0$ in

$\mathcal{H}_{\rm phys} = \{\mathcal{D}^2\mathcal{A} = 0, \ \ \mathcal{A}$ gauge fixed$\}/\{$pure gauge$\}$

- pure gauge mode

$$
\mathcal{A}_\mu^{(g)}[\phi^{(s)}] = \{t_\mu, \phi^{(s)}\}
$$

- time-like mode $\mathcal{A}_\mu^{(\tau)}[\phi^{(s)}] := x_\mu \phi^{(s)}$
  ... off-shell d.o.f. , not in $\mathcal{H}_{\rm phys}$ (no proof)
  conjecture: no ghosts (cf. YM !)
- all physical on-shell modes found,
  2 physical propagating modes for each spin $s \leq n$
- same propagation for all physical modes

  even though $SO(3,1)$ only space-like

  $\exists$ Lorentz-violating structures: $x_\mu\{t^\mu, .\} \sim \frac{\partial}{\partial\tau}$ ... time-like VF
  (cosmic background!)

vielbein, metric & dynamical geometry

consider scalar field $\phi = \phi(x)$ (e.g. transversal fluctuation)

kinetic term $-Tr[T^\alpha,\phi][T_\alpha,\phi] \sim \int e^\alpha\phi e_\alpha\phi = \int \gamma^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$

vielbein

$$\begin{aligned} e^\alpha &:= \{T^\alpha, .\} = e^{\alpha\mu}\partial_\mu \\ e^{\alpha\mu} &= \sinh(\eta)\eta^{\alpha\mu} \end{aligned}$$

metric $\gamma^{\mu\nu} = \eta_{\alpha\beta} e^{\alpha\mu}\, e^{\beta\nu}$

perturbed vielbein: $Y^\alpha = T^\alpha + \mathcal{A}^\alpha$

$$\begin{aligned} e^\alpha &= \{T^\alpha + \mathcal{A}^\alpha, .\} = e^{\alpha\mu}[\mathcal{A}]\partial_\mu \\ \delta_\mathcal{A}\gamma^{\mu\nu} &\sim \{\mathcal{A}^\mu, x^\nu\} + (\mu \leftrightarrow \nu) \end{aligned}$$

linearize & average over fiber $\rightarrow$ $h^{\mu\nu} = [\delta_\mathcal{A}\gamma^{\mu\nu}]_0$
coupling to matter:

$$S[\text{matter}] \sim \int_\mathcal{M} d^4x\, h^{\mu\nu}\, T_{\mu\nu}$$

vielbein, metric & dynamical geometry

consider scalar field $\phi = \phi(x)$ (e.g. transversal fluctuation)

kinetic term $-Tr[T^\alpha, \phi][T_\alpha, \phi] \sim \int e^\alpha \phi e_\alpha \phi = \int \gamma^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$

vielbein

$$e^\alpha := \{T^\alpha, .\} = e^{\alpha\mu} \partial_\mu$$
$$e^{\alpha\mu} = \sinh(\eta) \eta^{\alpha\mu}$$

metric $\gamma^{\mu\nu} = \eta_{\alpha\beta} e^{\alpha\mu} e^{\beta\nu}$

perturbed vielbein: $Y^\alpha = T^\alpha + \mathcal{A}^\alpha$

$$e^\alpha = \{T^\alpha + \mathcal{A}^\alpha, .\} = e^{\alpha\mu}[\mathcal{A}] \partial_\mu$$
$$\delta_\mathcal{A} \gamma^{\mu\nu} \sim \{\mathcal{A}^\mu, x^\nu\} + (\mu \leftrightarrow \nu)$$

linearize & average over fiber $\rightarrow$ $h^{\mu\nu} = [\delta_\mathcal{A} \gamma^{\mu\nu}]_0$

coupling to matter:

$$S[\text{matter}] \sim \int_\mathcal{M} d^4x\, h^{\mu\nu} T_{\mu\nu}$$

effective metric $G^{\mu\nu}$ & conformal factor:

encoded in Laplacian $\Box_Y = [Y_\mu, [Y^\mu, .]] \sim \frac{1}{\sqrt{|G|}} \partial_\mu (\sqrt{|G|} G^{\mu\nu} \partial_\nu .)$:

$$\begin{aligned} G^{\mu\nu} &= \alpha\, \gamma^{\mu\nu}\,, \qquad \alpha = \sqrt{\frac{|\theta^{\mu\nu}|}{|\gamma^{\mu\nu}|}}\,, \\ \gamma^{\mu\nu} &= g_{\mu'\nu'} [\theta^{\mu'\mu} \theta^{\nu'\nu}]_{S^2} \end{aligned}$$

$[.]_{S^2}$ ... averaging over the internal S^2.

$\rightarrow$ scale factor of FLRW background:

- late times:

$$a(t) \approx \frac{3}{2} t, \qquad t \to \infty\,.$$

... coasting universe (no too bad !)

- big bounce:

$$a(t) \propto (t - t_0)^{\frac{1}{5}}$$

towards gravity on $\mathcal{M}^{3,1}$

linearized metric: $h^{\mu\nu} \propto \{\mathcal{A}^\mu, x^\nu\} + (\mu \leftrightarrow \nu)$

contains all dof required for gravity

5+1 off-shell dof from $\mathcal{A}^{(-)}[\phi^{(2)}]$ and $\mathcal{A}^{(+)}[\phi^{(0)}]$, 3 pure gauge (!)

lin. Ricci:

$$\mathcal{R}^{\mu\nu}_{(\text{lin})}[h[\mathcal{A}]] \approx \frac{1}{2} \underbrace{\Box h_{\mu\nu}[\mathcal{A}]}_{h_{\mu\nu}[\mathcal{D}^2\mathcal{A}]\approx 0} - \frac{1}{4}\left(\{t_\mu, \{t_\nu, h\}\} + (\mu \leftrightarrow \nu)\right)$$

(up to cosm. scales)

on-shell (vacuum) in M.M.:

$\mathcal{A}^-[\phi^{(2,0)}]$: $h \approx 0 \Rightarrow \mathcal{R}^{\mu\nu}_{(\text{lin})} \approx 0$ ... 2 graviton modes (massless !)

$\mathcal{A}^-[\phi^{(2,1)}]$: $h \approx 0 \Rightarrow \mathcal{R}^{\mu\nu}_{(\text{lin})} \approx 0$ ... trivial (on-shell)

extra scalar mode(s): $\mathcal{A}^-[\phi^{(2,2)}],\ \mathcal{A}^+[\phi^{(0)}]$

(approx. identical $\mathcal{R}^{\mu\nu}_{(\text{lin})} \neq 0$)

gauge transformations

-of functions: $\phi \mapsto \{\Lambda, \phi\}$

spin 1 trafos: $\Lambda = v^\mu(x) t_\mu \ \in \mathcal{C}^1$:

$$\{v^\mu t_\mu, \phi\}_0 \quad = \frac{1}{3}\left(\sinh(\eta)\left(3 v^\mu \partial_\mu + (\mathrm{div} v)\tau - \tau v^\mu \partial_\mu\right) + x_\gamma \varepsilon^{\gamma\mu\alpha\beta}\partial_\alpha v_\mu \partial_\beta\right)\phi$$

3 (rather than 4) diffeomorphisms !

due to invar. symplectic volume on $\mathbb{C}P^{1,2}$

-of gauge fields: $\mathcal{A}^\mu \mapsto \{\Lambda, T^\mu + \mathcal{A}^\mu\}$

-of gravitons:

$$\boxed{\delta G_{\mu\nu} = \nabla_\mu \mathcal{A}_\nu + \nabla_\mu \mathcal{A}_\nu,} \qquad \mathcal{A}_\mu = \{x_\mu, \Lambda\}_- \quad \text{... VF}$$

∇ ... covariant w.r.t. FLRW background

$$\nabla_\alpha \mathcal{A}^\alpha = \frac{1}{x_4^2}(x \cdot \mathcal{A}) \qquad \text{...(almost) volume preserving}$$

linearized Einstein-Hilbert action

- determined by gauge invariance $\delta_\phi S_{\rm EH} = 0$
  result:

$$
\begin{aligned}
S_{\rm EH} &:= S_1 - \frac{1}{24} S_h - \frac{1}{R^2} S_3 - \frac{1}{2R^2} S_4 + \frac{3}{R^2}\left(\frac{1}{2} S_{M3} - 2 S_{gf2} + \frac{3}{2} S_{gf3}\right) \\
&\approx \text{ linearized Einstein-Hilbert for traceless modes}
\end{aligned}
$$

where

$$
\begin{aligned}
S_1 &= \int \left(h_{\mu\nu} - \tfrac{1}{3} h\,\eta_{\mu\nu}\right) \mathcal{G}^{\mu\nu}_{\rm (lin)}\left[h_{\alpha\beta} - \tfrac{1}{3} h\,\eta_{\alpha\beta}\right] \\
S_h &= \int h \Box h\,, \\
S_3 &= \int \delta g^0_{\mu\nu} \delta g_0^{\mu\nu}\,, \\
S_4 &= \int \delta g^0_{\mu\nu} \{\mathcal{M}^{\mu\rho}, \delta g_0^{\rho\nu}\}\,, \\
S_{M3} &= \int f_{\mu\nu} \{\mathcal{M}^{\nu\rho}, h^{\rho\mu}\}\,, \\
S_{gf2} &= \int \{x_\mu, A^\mu\}_-\, D^- \{t_\rho, A^\rho\}\,, \\
S_{gf3} &= \int (D^- \{t_\nu, A^\nu\}) \Box^{-1} (D^- \{t_\nu, A^\nu\})\,,
\end{aligned}
$$

- expected to be induced by quantum effects (cf. Sakharov 1967)

- Ricci-flat vacuum solutions for both bare M.M. and induced S_{EH}

  S_{EH} needed to recover inhomogeneous Einstein eq. $G_{\mu\nu} \propto T_{\mu\nu}$

  $\Rightarrow$ expect $\approx$ linearized GR at intermediate scale,
  good agreement with solar system tests

- model is fully non-linear (to be understood)

- no cosm. const. $\int d^4x \sqrt{g}$
  replaced by YM-action, stabilizes $\mathcal{M}^{3,1}$

  $\rightarrow$ no cosm. const. problem ?!

- significant differences at cosmic scales,

  reasonable (coasting) cosmology without any fine-tuning !!

bare YM action & gravitons:

can rewrite

$$S_2[\mathcal{A}^{(-)}[\phi^{(2,0)}]] \propto -\int h^{\mu\nu}[\phi^{(2,0)}](\Box - \frac{2}{R^2})(\Box_H - 2r^2)^{-1} h_{\mu\nu}[\phi^{(2,0)}]$$

leading to

$$(\Box - \frac{2}{R^2})h_{\mu\nu} \sim -(\Box_H - 2r^2)T_{\mu\nu}$$

summary

- matrix models:
  natural framework for quantum theory of space-time & matter
- 4D covariant quantum spaces $\rightarrow$ higher spin theories
- $\exists$ nice cosmological FRW space-time solutions
  reg. BB, finite density of microstates
- fluct. $\rightarrow$ all ingredients for (lin.) gravity
- Yang-Mills structure $\rightarrow$ emergent gravity rather than GR
- good UV behavior (SUSY), well suited for quantization

... seems to work! needs to be elaborated

breaking $SO(4,1) \to SO(3,1)$ and sub-structure

consider

$$D\phi := -i[X^4, \phi], \qquad \text{respects } SO(3,1)$$

acts on spin s modes as follows

$$D = \underbrace{div^{(3)}\phi}_{D^-\phi} + \underbrace{t^\mu \nabla^{(3)}_\mu \phi}_{D^+\phi} : \quad \mathcal{C}^s \to \mathcal{C}^{s+1} \oplus \mathcal{C}^{s-1}$$

decomposition into $SO(3,1)$ irreps on $H^3 \subset H^4$

$$\mathcal{C}^{(s)} = \mathcal{C}^{(s,0)} \oplus \mathcal{C}^{(s,1)} \oplus \ldots \oplus \mathcal{C}^{(s,s)}$$

D^- resp. D^+ act as

$$D^- : \mathcal{C}^{(s,k)} \to \mathcal{C}^{(s-1,k-1)}, \qquad D^+ : \mathcal{C}^{(s,k)} \to \mathcal{C}^{(s+1,k+1)} .$$