# Space-Time Structure in the Type IIB Matrix Model

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Space Time Matrices
@IHES, February 26th, 2019

Based on collaboration with
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Jun Nishimura (Sokendai, KEK), Atis Yosprakob (Sokendai)

# Introduction

# Type IIB matrix model

Ishibashi-Kawai-Kitazawa-A.T. ('96)

A proposal for nonperturbative formulation of superstring theory

$$S = S_b + S_f$$

$$S_b = -\frac{1}{4g^2}\mathrm{Tr}\left([A^\mu, A^\nu][A_\mu, A_\nu]\right)$$

$$S_f = -\frac{1}{2g^2}\mathrm{Tr}\left(\bar{\psi}\Gamma^\mu[A_\mu, \psi]\right)$$

$N \times N$ Hermitian matrices

$A_\mu$ : 10D Lorentz vector $(\mu = 0, 1, \ldots, 9)$

$\psi$ : 10D Majorana-Weyl spinor

Space-time does not exist a priori, but is generated dynamically from degrees of freedom of matrices

$A_\mu$ : coordinates, whole universe

We can ask whether (3+1)-dim. space-time emerges or not.

# Evidences for nonperturbative formulation

(1) Manifest SO(9,1) symmetry and manifest 10D N=2 SUSY

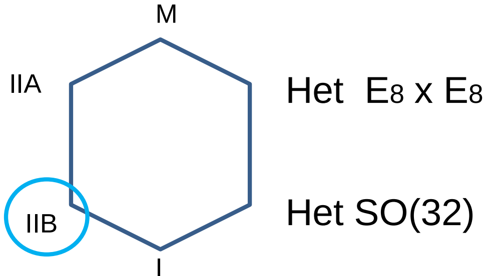
(2) Matrix regularization of Green-Schwarz action of Schild-type for type IIB superstring with κ symmetry fixed

(3) Long distance behavior of interaction between D-branes is reproduced

(4) Light-cone string field theory for type IIB superstring is reproduced from SD equations for Wilson loops under some assumptions

Fukuma-Kawai-Kitazawa-A.T. ('97)

(5) Believing string duality, one can start from anywhere with nonperturbative formulation to tract strong coupling regime



# Euclidean vs Lorentzian

- Lorentzian model

$$S_b \propto \mathrm{Tr}(F^{\mu\nu}F_{\mu\nu}) = \boxed{-2\mathrm{Tr}F_{0i}^2} + \boxed{\mathrm{Tr}F_{ij}^2}$$

$$F_{\mu\nu} = -i[A_\mu, A_\nu]$$

opposite sign
extremely unstable system!
No one dared to study
the Lorentzian model

- Euclidean model

Wick rotation $\quad A_0 = iA_{10} \quad \Gamma^0 = -i\Gamma^{10}$

$\Rightarrow \quad S_b \propto \mathrm{Tr}(F_{\mu\nu})^2$ positive definite

The flat direction $[A_\mu, A_\nu] \sim 0$ is lifted due to quantum effects

Aoki-Iso-Kawai-Kitazawa-Tada ('99)

The Euclidean model is well defined without any cutoffs

Krauth-Nicolai-Staudacher ('98),
Austing-Wheater ('01)

# Why we study the Lorentzian model?

- see time evolution of the Universe

  ~ need to study real time dynamics

- Wick rotation in gravitational theory is more subtle than field theory on flat space-time

  ex.) Causal dynamical triangulation（CDT) Ambjorn-Jurkiewicz-Loll ('05)

  Coleman mechanism in space-time with Lorentzian signature

  Kawai-Okada ('11)

  The Lorentzian model can be totally different from the Euclidean one.

- Here we study Lorentzian version of the type IIB matrix model

# Our claim

- The definition of the Lorentzian model is not straightforward.
- Monte Carlo studies suffer from the sign problem (complex action problem).
- We have to use the complex Langevin method.
- (3+1)-dim. Expanding universe emerges dynamically.
- The mechanism suggests a singular space-time structure.
- We discuss the emergence of a smooth space-time.
- A classical solution should dominate the path integral.
  There are infinitely many classical solutions which have
  (3+1)-dim. expanding behavior with a smooth space-time structure,
  which supports the emergence of a smooth space-time.

# Plan of the present talk

✓ 1. Introduction

2. Defining the Lorentzian model

3. Emergence of (3+1)-dim. expanding behavior

4. Emergence of a smooth space-time

Nishimura-
A.T.,
to appear

5. Analysis of classical EOM

6. Space-time structure in classical solutions

Hatakeyama-
Matsumoto-
Nishimura-
A.T.-
Yosprakob,
to appear

7. Summary and discussion

# Defining the Lorentzian model

# Partition function of the Lorentzian model

$$Z = \int dA d\psi \, e^{i(S_b + S_f)} = \int dA \; e^{iS_b} \; \mathrm{Pf}\mathcal{M}(A)$$

This seems to be natural from the connection to the worldsheet theory.

$$S = \int d^2\xi \sqrt{g} \left( \frac{1}{4}\{X^\mu, X^\nu\}^2 + \frac{1}{2}\bar{\psi}\Gamma^\mu\{X^\mu, \psi\} \right)$$

$$\xi_0 \equiv -i\xi_2$$

The worldsheet coordinates should also be Wick-rotated.

# Regularizing the Lorentzian model

Unlike the Euclidean model,
the Lorentzian model is NOT well defined as it is.

$$Z = \int dA d\psi \; e^{i(S_b + S_f)} = \int dA \, e^{iS_b} \underbrace{\mathrm{Pf}\mathcal{M}(A)}_{\text{polynomial in } A}$$

pure phase factor

Introduce the IR cutoffs so that
the extent in temporal and spatial directions become finite.

$$\frac{1}{N}\mathrm{Tr}(A_0)^2 = \kappa L^2$$

$$\frac{1}{N}\mathrm{Tr}(A_i)^2 = L^2$$

In what follows, we set $L = 1$ without loss of generality.

# Introducing two more parameters

Pure imaginary action is hard to deal with numerically.
We introduce two more deformation parameters.

$$Z = \int dA \; e^{-S(A)} \; \mathrm{Pf}\mathcal{M}(A)$$

$$S(A) = N\beta e^{-i\frac{\pi}{2}(1-s)} \left( \frac{1}{2} e^{-ik\pi} \mathrm{Tr}[A_0, A_i]^2 - \frac{1}{4} \mathrm{Tr}[A_i, A_j]^2 \right)$$

Wick rotation on the worldsheet

Wick rotation in the target space

$$A_0 \to e^{-ik\pi/2} A_0$$

$(s,k) = (0,0)$ corresponds to the Lorentzian model.

The first term can be made real positive by choosing $e^{-i\frac{\pi}{2}(1-s)} e^{-ik\pi} = -1$

$\Rightarrow$ $k = \dfrac{1+s}{2}$ We focus on this case for the moment.

# Summary of the definition of the model

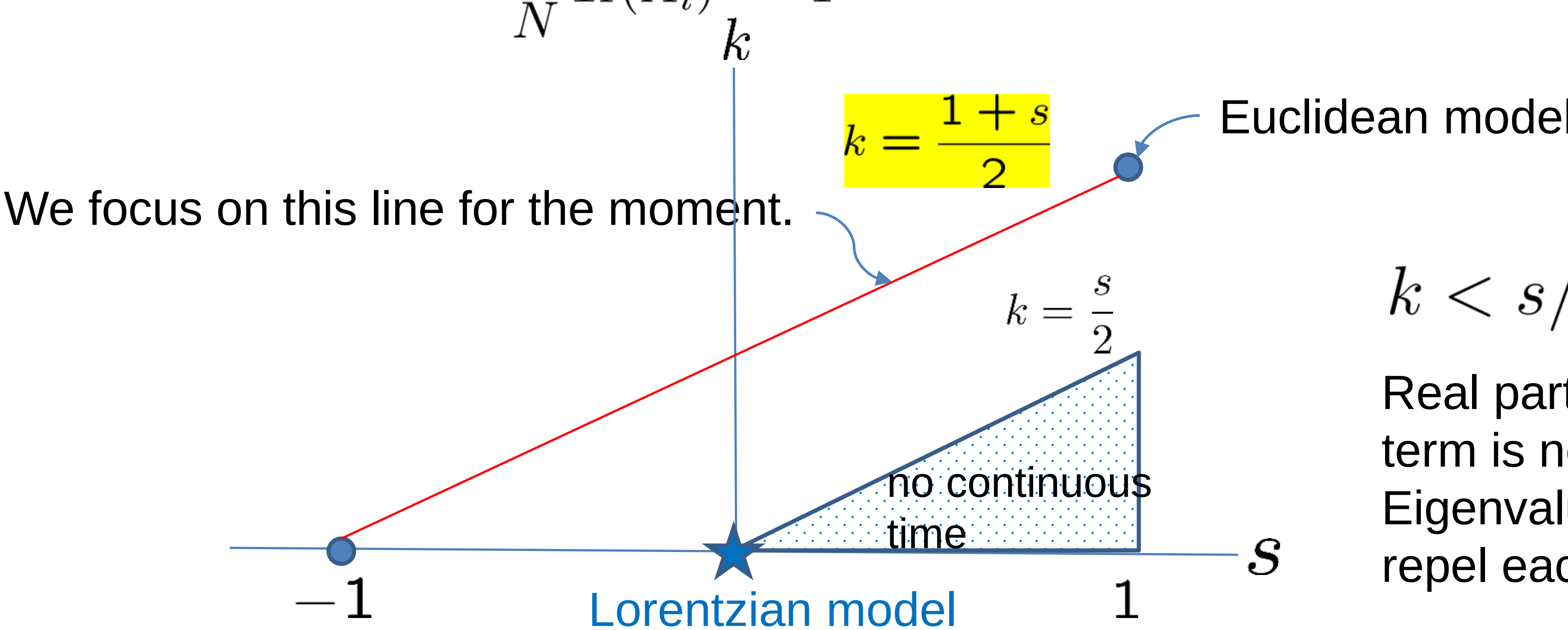
$$Z = \int dA\ e^{-S(A)}\ \mathrm{Pf}\mathcal{M}(A)$$

$$S = N\beta e^{-i\frac{\pi}{2}(1-s)} \left( \frac{1}{2} e^{-ik\pi} \mathrm{Tr}[A_0, A_i]^2 - \frac{1}{4} \mathrm{Tr}[A_i, A_j]^2 \right)$$

IR cutoffs :

$$\frac{1}{N}\mathrm{Tr}(A_0)^2 = \kappa > 0$$

$$\frac{1}{N}\mathrm{Tr}(A_i)^2 = 1$$




$$k < s/2$$

Real part of the 1st term is negative.
Eigenvalues of $A_0$ repel each other.

# Complex Langevin method

$A_\mu$ : Hermitian matrix ⇒ complex matrix

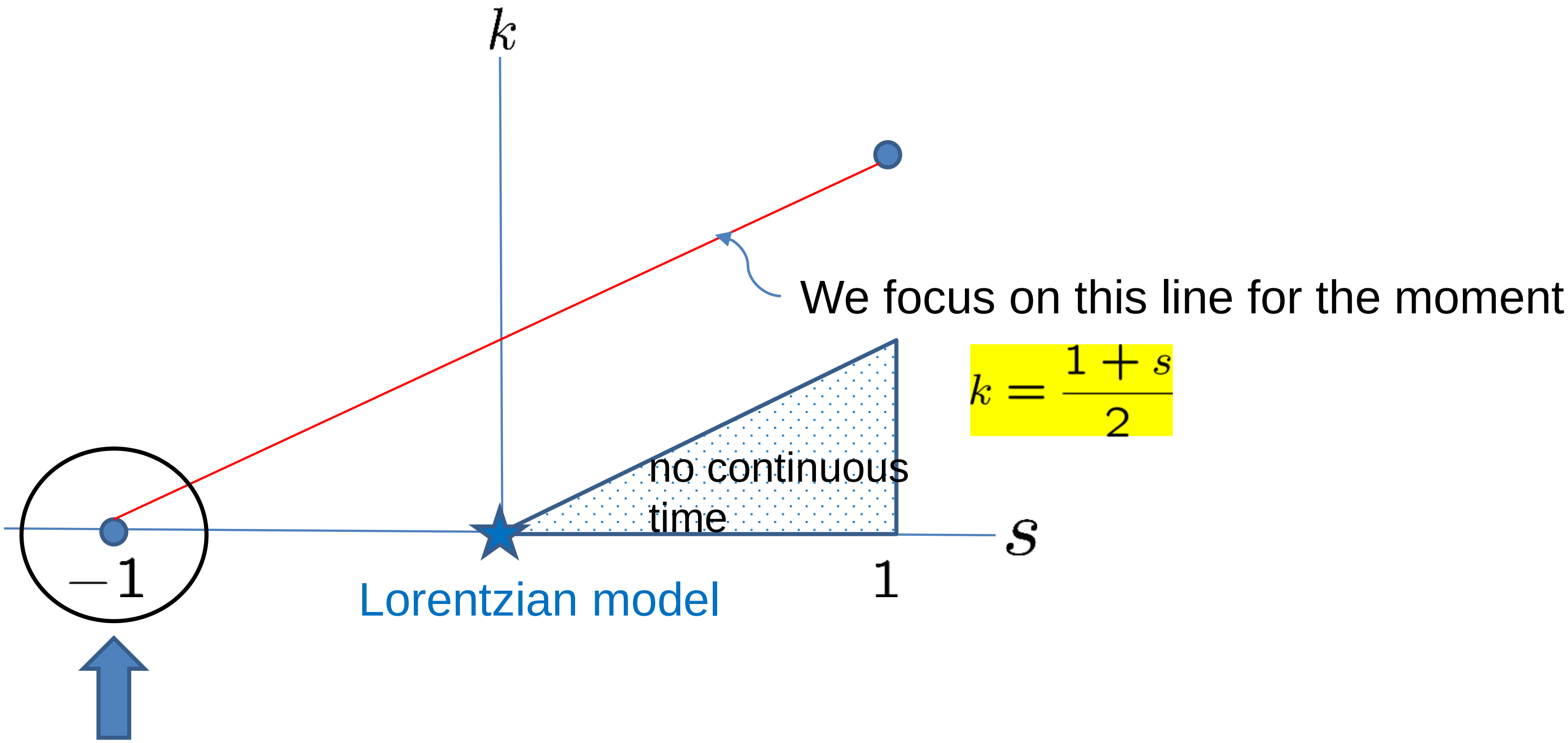
$$\frac{d(A_\mu)_{ij}}{d\tau} = \underbrace{-\frac{\partial S_{eff}}{\partial (A_\mu)_{ji}}}_{\text{drift term}} + \underbrace{(\eta_\mu)_{ij}(\tau)}_{\text{white noise}} \qquad S_{eff} = S - \log \mathrm{Pf}\mathcal{M}$$

$\tau$ : fictinal time

Whether CLM works well depends on (the parameter region of) the system.

# Emergence of (3+1)-dim. expanding behavior

# Results at (s,k)=(-1,0) in the 6D bosonic model



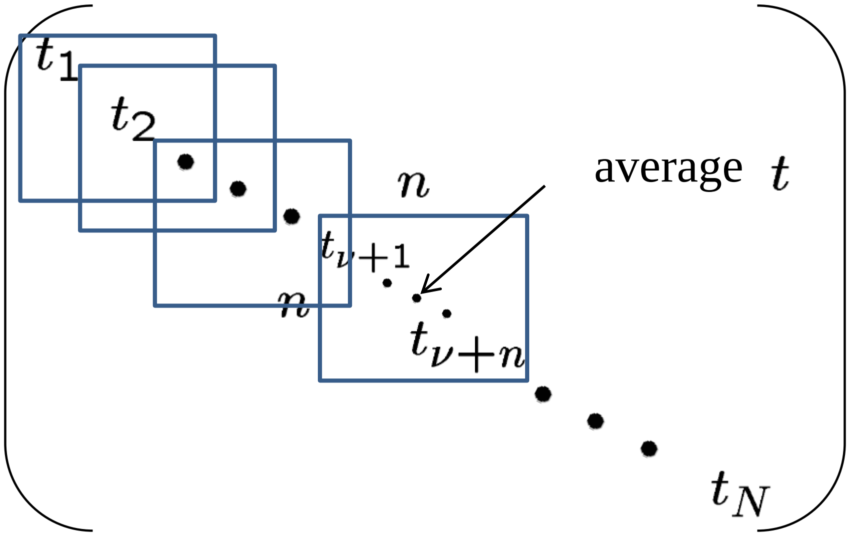

$$S = N\beta\left(-\frac{1}{2}\mathrm{Tr}[A_0, A_i]^2 + \frac{1}{4}\mathrm{Tr}[A_i, A_j]^2\right)$$

favors $A_j$ close to diagonal

favors maximal non-commutativity between $A_j$

# Emergence of concept of ``time evolution”

$$A_0 =$$



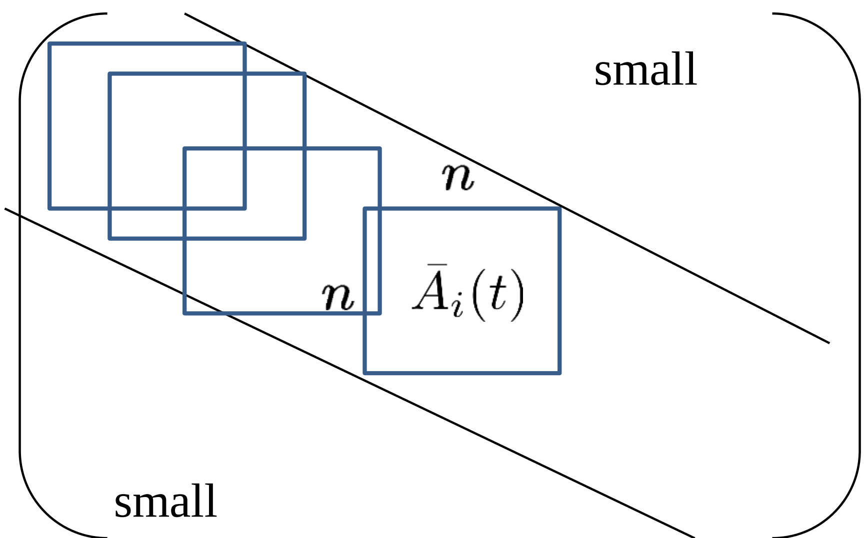

By SU(N) symmetry, we diagonalize $A_0$

$$t_1 < t_2 < \cdots < t_N$$

These values are dynamically determined

$$A_i =$$

$$(i = 1, \ldots, 5)$$




Band-diagonal structure is observed, which is nontrivial

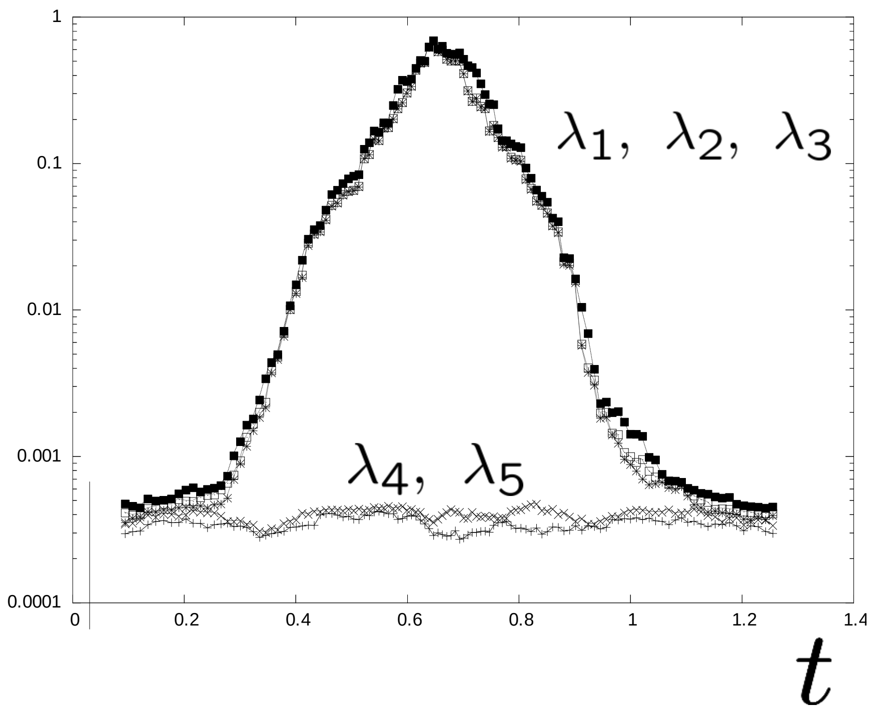
$\bar{A}_i(t)$ represents space structure at fixed time t

concept of “time evolution” emerges

# Emergence of (3+1)-dim. expanding behavior

$$T_{ij}(t) = \frac{1}{n}\mathrm{tr}(\bar{A}_i(t)\bar{A}_j(t)) \quad i,j = 1,\ldots,5$$ ~ moment of inertia tensor

Eigenvalues $\lambda_1(t),\ldots,\lambda_5(t)$ represent the spatial extent in each directions in 5 directions




$N = 128 \quad \kappa = 0.13 \quad \beta = 2$

$(s,k) = (-1,0)$

SSB : SO(5) → SO(3)
occurs at some point in time

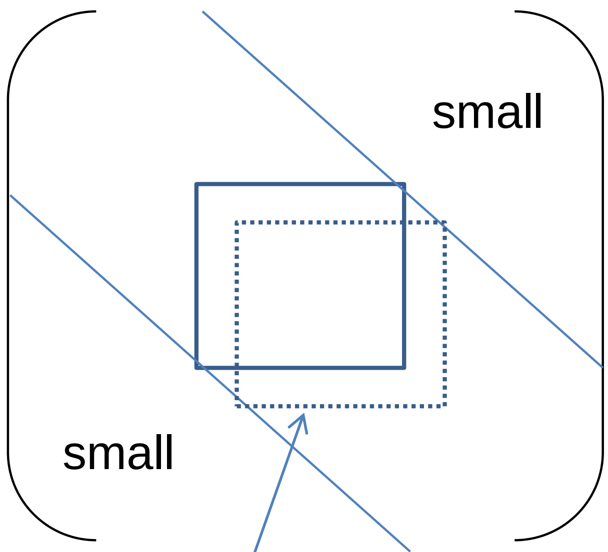
# The mechanism of SSB

$$S = N\beta\left(-\frac{1}{2}\mathrm{Tr}[A_0, A_i]^2 + \frac{1}{4}\mathrm{Tr}[A_i, A_j]^2\right)$$

favors $A_j$ close to diagonal

favors maximal non-commutativity between $A_j$

$A_i =$




$\bar{A}_i(t)$

maximize NC $= -\mathrm{tr}\,[\bar{A}_i(t), \bar{A}_j(t)]^2$
for $\mathrm{tr}\,(\bar{A}_i(t))^2 = \mathrm{const.}$

$$\begin{array}{lcll} \bar{A}_i(t) & \propto & \sigma_i & \text{for } i = 1, 2, 3 \\ \bar{A}_i(t) & = & 0 & \text{for } i \geq 4 \end{array}$$

up to SO(5) rotation

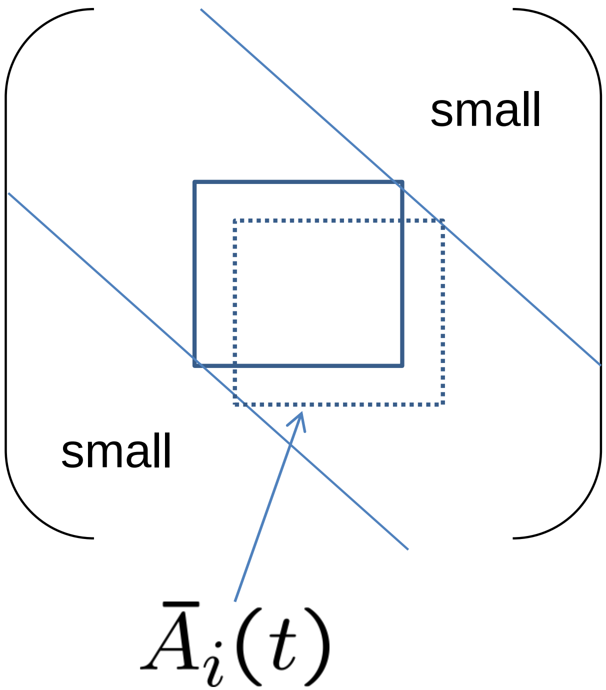
Kim-Nishimura-A.T., PRL 108 (2012) 011601

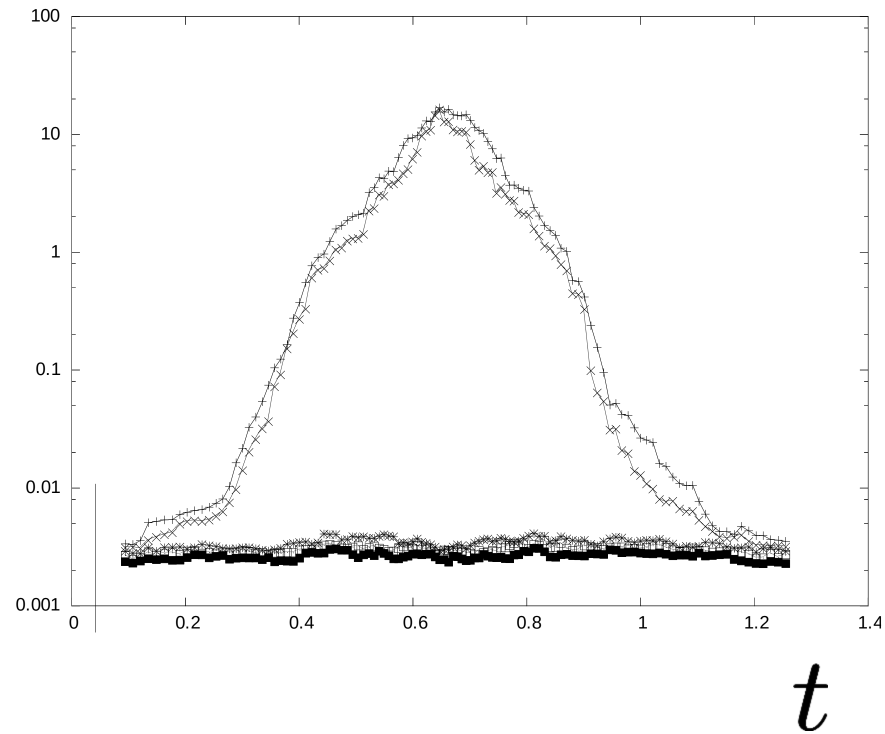
# Confirmation of the mechanism

$$N = 128\ , \quad \kappa = 0.13\ , \quad \beta = 2\ , \quad (s,k) = (-1,0)$$

eigenvalues of $Q = \sum_{i=1}^{5} \left\{ \bar{A}_i(t) \right\}^2$

$A_i =$

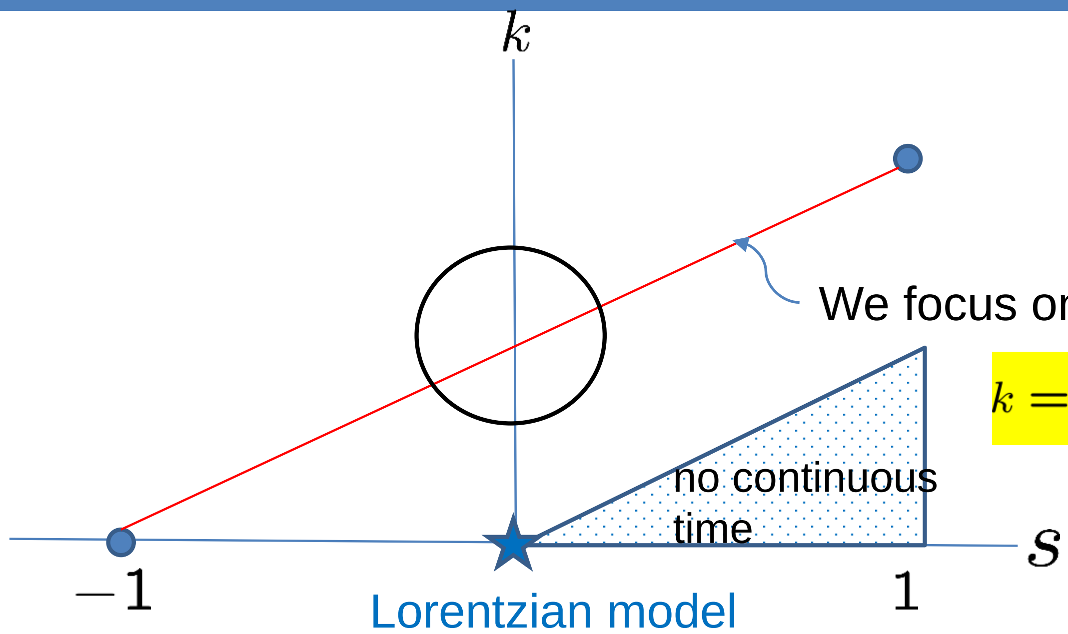



Only 2 eigenvalues of Q become large.

# Emergence of a smooth space-time

# Exploring the phase diagram




$$S = -N\beta \left( \frac{1}{2} \mathrm{Tr}[A_0, A_i]^2 + \frac{1}{4} e^{-i\frac{\pi}{2}(1-s)} \mathrm{Tr}[A_i, A_j]^2 \right)$$

Real part changes sign at $s = 0$.

Noncommutativity between $A_i$

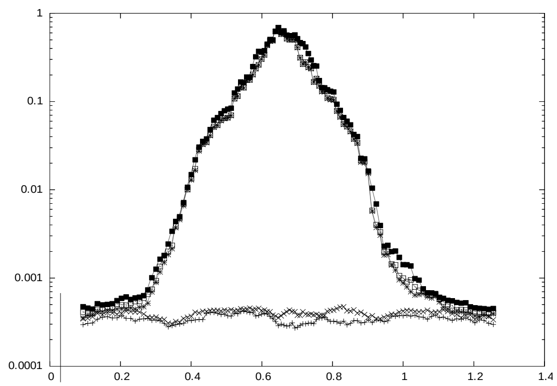
maximize $\longrightarrow$ minimize

Can we obtain (3+1)-dim. expanding behavior

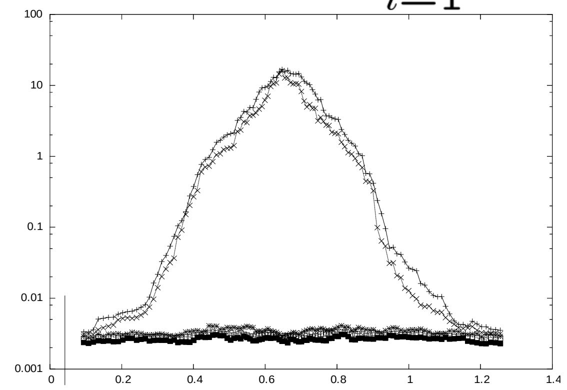
with a smooth space-time structure ?

# Comparing s=-1 and s~0

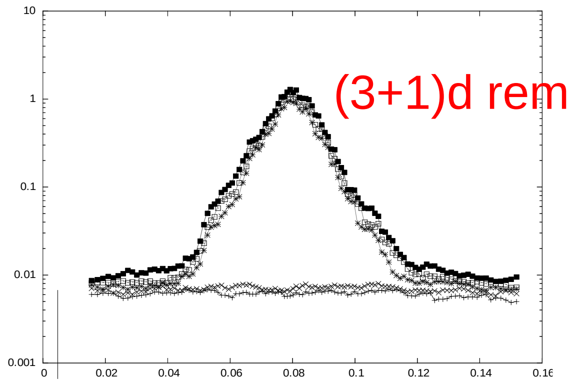
eigenvalues of $T_{ij}(t) = \frac{1}{n}\mathrm{tr}\left\{\bar{A}_i(t)\bar{A}_j(t)\right\}$ eigenvalues of $Q = \sum_{i=1}^{5}\left\{\bar{A}_i(t)\right\}^2$




$s = -1$

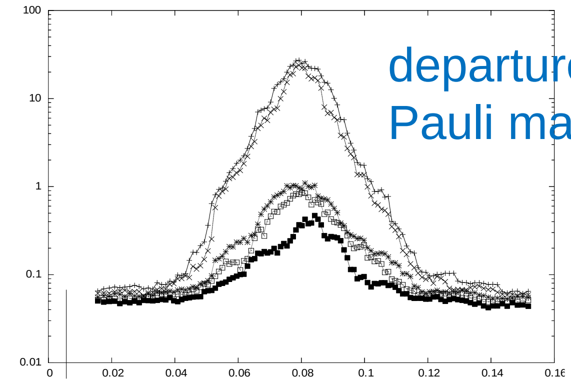



$N = 128\ ,\quad \kappa = 0.13\ ,\quad \beta = 2\ ,\quad (s, k) = (-1, 0)$




(3+1)d remains

$s \sim 0$



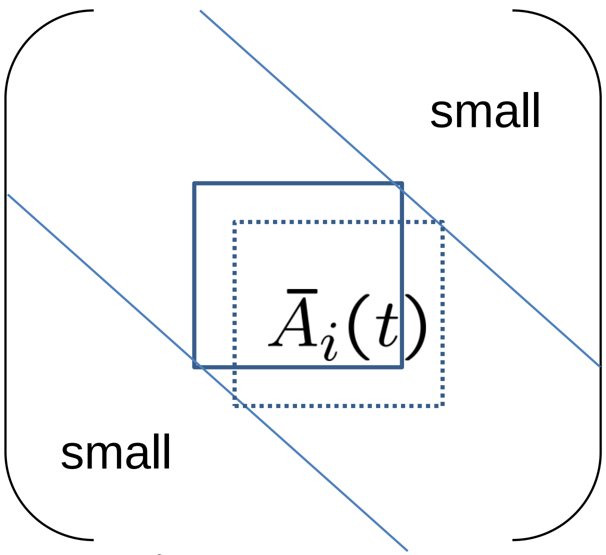

departure from Pauli matrices

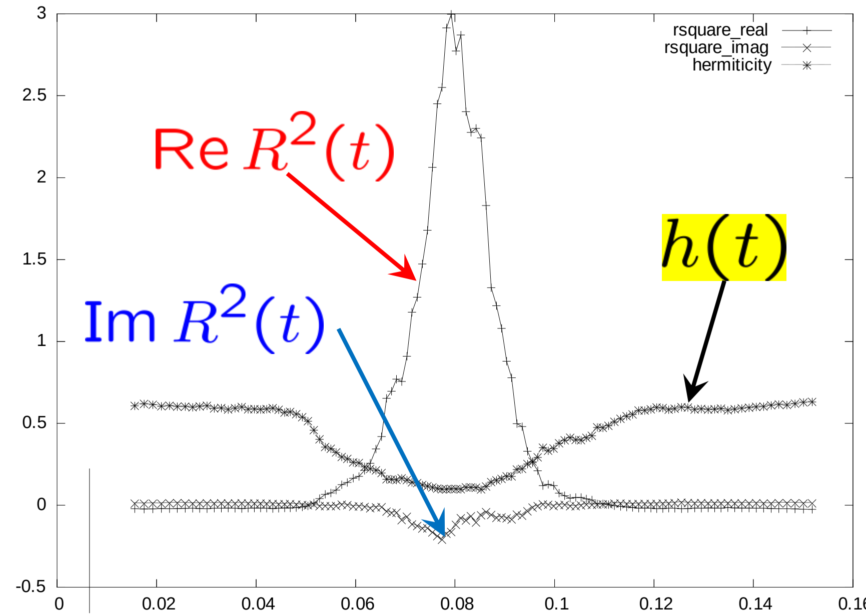
$N = 128\ ,\quad \kappa = 0.0037\ ,\quad \beta = 32\ ,\quad (s, k) = (0.0076, 0.5038)$

# Hermiticity of the spatial matrices

$N = 128\ ,\quad \kappa = 0.0037\ ,\quad \beta = 32\ ,\quad (s,k) = (0.0076, 0.5038)$

$A_i =$




$$R^2(t) = \frac{1}{n}\mathrm{tr}\,(\bar{A}(t)^2)$$

$$h(t) = \frac{-\mathrm{tr}\,(\bar{A}_i(t) - \bar{A}_i(t)^\dagger)^2}{4\mathrm{tr}\,(\bar{A}_i(t)^\dagger \bar{A}_i(t))}$$

$$0 \le h(t) \le 1$$

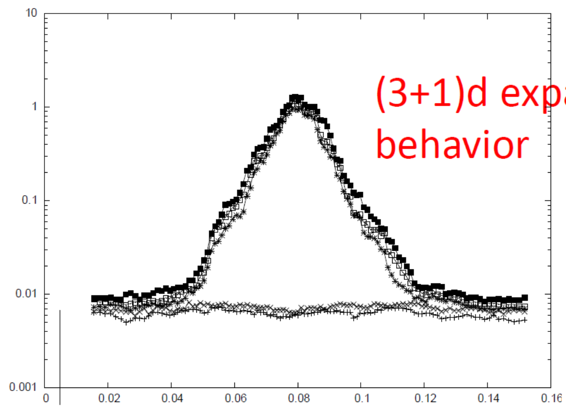
Hermitian — anti-Hermitian

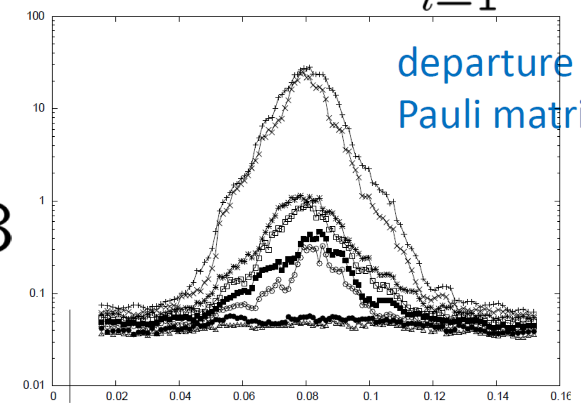
Spatial matrices become close to Hermitian near the peak of Re $R^2(t)$.

Classical solution seems to be dominating in this region.

# N=128 vs N=192

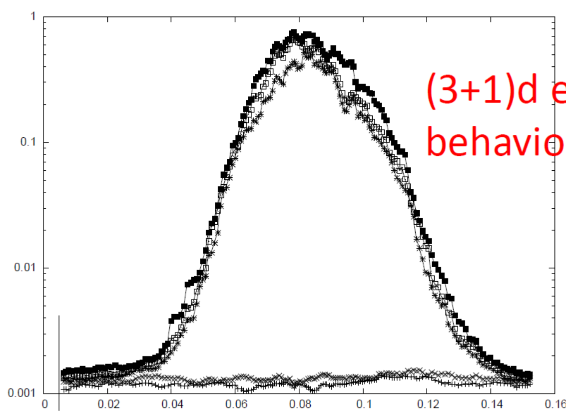
eigenvalues of $T_{ij}(t) = \frac{1}{n}\mathrm{tr}\left\{X_i(t)X_j(t)\right\}$ eigenvalues of $Q = \sum_{i=1}^{5}\left\{X_i(t)\right\}^2$

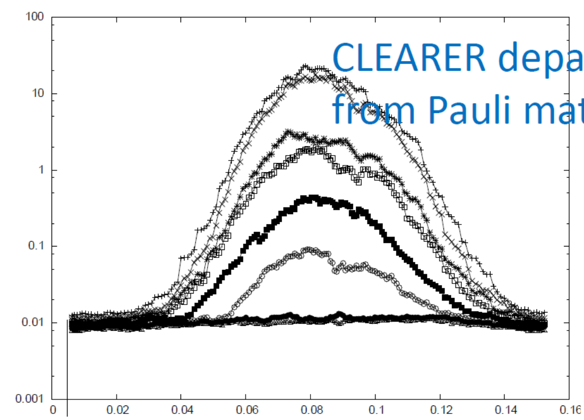



$$N = 128$$

$$N = 128\ , \quad \kappa = 0.0037\ , \quad \beta = 32\ , \quad (s,k) = (0.0076, 0.5038)\ , \quad n = 16$$




$$N = 192$$

$$N = 192\ , \quad \kappa = 0.0044\ , \quad \beta = 64\ , \quad (s,k) = (0.0118, 0.5059)\ , \quad n = 24$$

# Analysis of classical EOM

# Motivation for studying classical solutions

- The results in numerical simulation suggest that some classical solution is dominating the path integral in the time region near the peak.

- We also expect that the classical equations of motion are expected to become more and more valid at later times which are seen at larger N, since the value of the action increases with the cosmic expansion.

- It is worth while studying classical solutions. We find that there are infinitely many classical solutions which have (3+1)-dim. expanding behavior with a smooth space-time structure.

- This supports that we obtain (3+1)-dim. expanding behavior with a smooth space-time structure.

# Motivation for studying classical solutions

- The late-time behaviors are difficult to study by direct numerical simulation, since larger matrix sizes are required. Fortunately, we can solve classical EOM with larger matrix size much easier.

- We develop a numerical algorithm for searching for classical solutions satisfying the most general ansatz with "quasi direct product structure"

- If some classical solution indeed dominates the path integral at later times, we can discuss a possibility that the Standard model appears by examining various classical solutions.
  I will not discuss this point in this talk.

A. Chatzistavrakidis, H. Steinacker and G. Zoupanos ('11)

# Equation of motion

$$S = -\frac{1}{4}\mathrm{Tr}([A^M, A^N][A_M, A_N]$$

Arnlind-Hoppe,
Arnlind-Choe-Hoppe-Huisken-Kontsevich,
Steinacker,
Stern,…

$$[A^M, [A_M, A_0]] + \alpha A_0 = 0$$

$$[A^M, [A_M, A_i]] - \beta A_i = 0 \quad (i = 1, \ldots, 9)$$

$\alpha,\ \beta$ : Lagrange multiplier

constraints

$$\frac{1}{N}\mathrm{Tr}(A_0^2) = \kappa$$

$$\frac{1}{N}\mathrm{Tr}(A_i^2) = 1$$

corresponding to IR cutoffs

# Configuration with "quasi direct product structure"

Nishimura-A.T.('13)

$$A_\mu = \boxed{X_\mu} \otimes \boxed{M} \quad (\mu = 0, \ldots, 3)$$

$$A_a = \boxed{1_{N_X}} \otimes \boxed{Y_a} \quad (a = 4, \ldots, 9)$$

$M = 1$ : direct product space-time

$N_X \times N_X \quad N_Y \times N_Y \qquad N = N_X \times N_Y$

Each point on (3+1)d space-time has the same structure in the extra dimensions

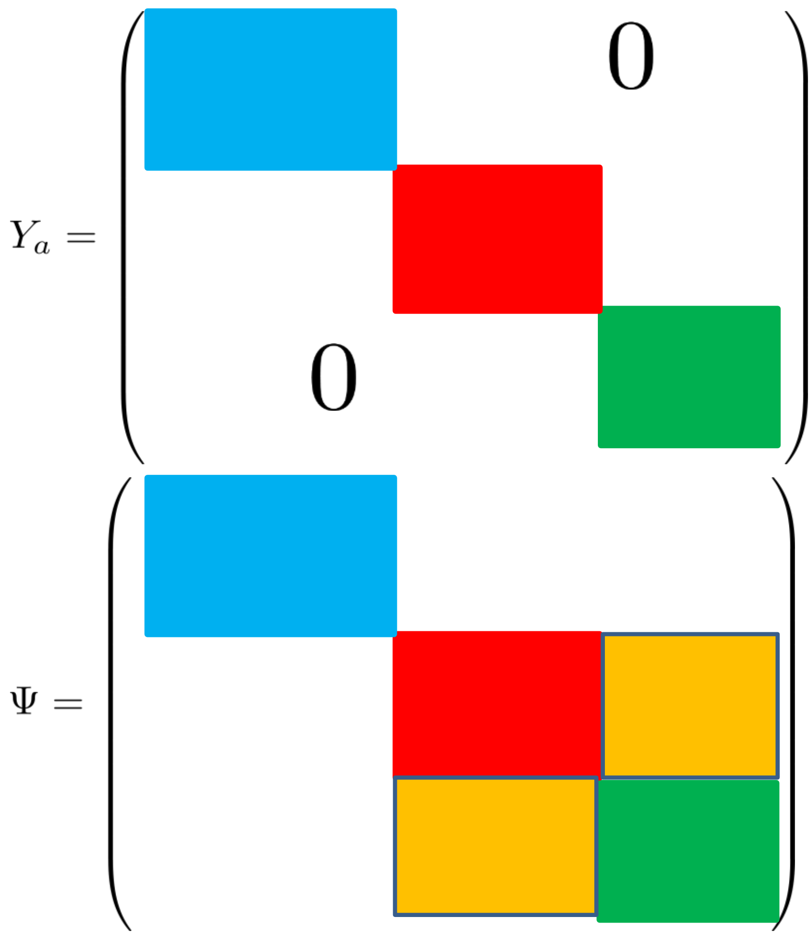
This ansatz is compatible with Lorentz symmetry to be expected at late time

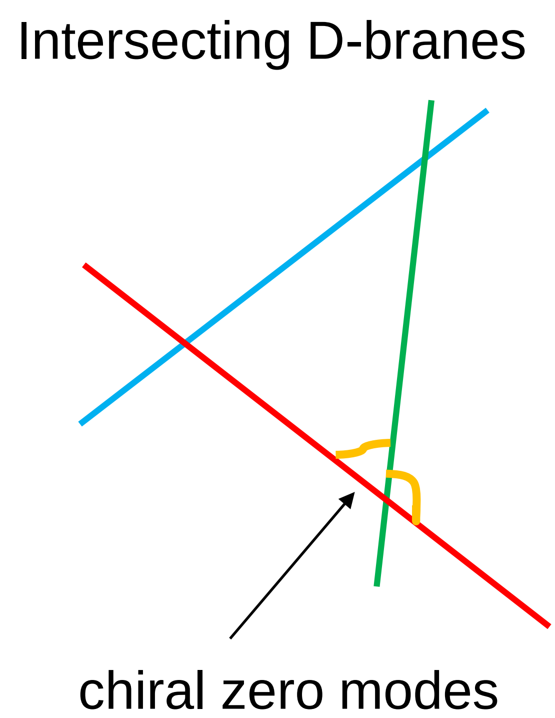
$$O_{\mu\nu} X_\nu = g[O]\, X_\mu\, g[O]^\dagger$$

$$O \in \mathrm{SO}(3,1) \qquad g[O] \in SU(N_X)$$

# Structure of Ya and chiral zero modes

$Y_a$ and $M$ should determine matter contents and gauge interactions.
For instance, block diagonal structures of $Y_a$ can give chiral zero modes



# Algorithm for finding solutions

$$I = \mathrm{Tr}([A^M, [A_M, A_0]] + \alpha A_0)^2 + \mathrm{Tr}([A^M, [A_M, A_i]] - \beta A_i)^2$$

$$A_\mu = X_\mu \otimes M \quad (\mu = 0, \ldots, 3)$$

$$A_a = 1_{N_X} \otimes Y_a \quad (a = 4, \ldots, 9)$$

We search for configurations that gives $I = 0$

gradient descent algorithm

update configurations following

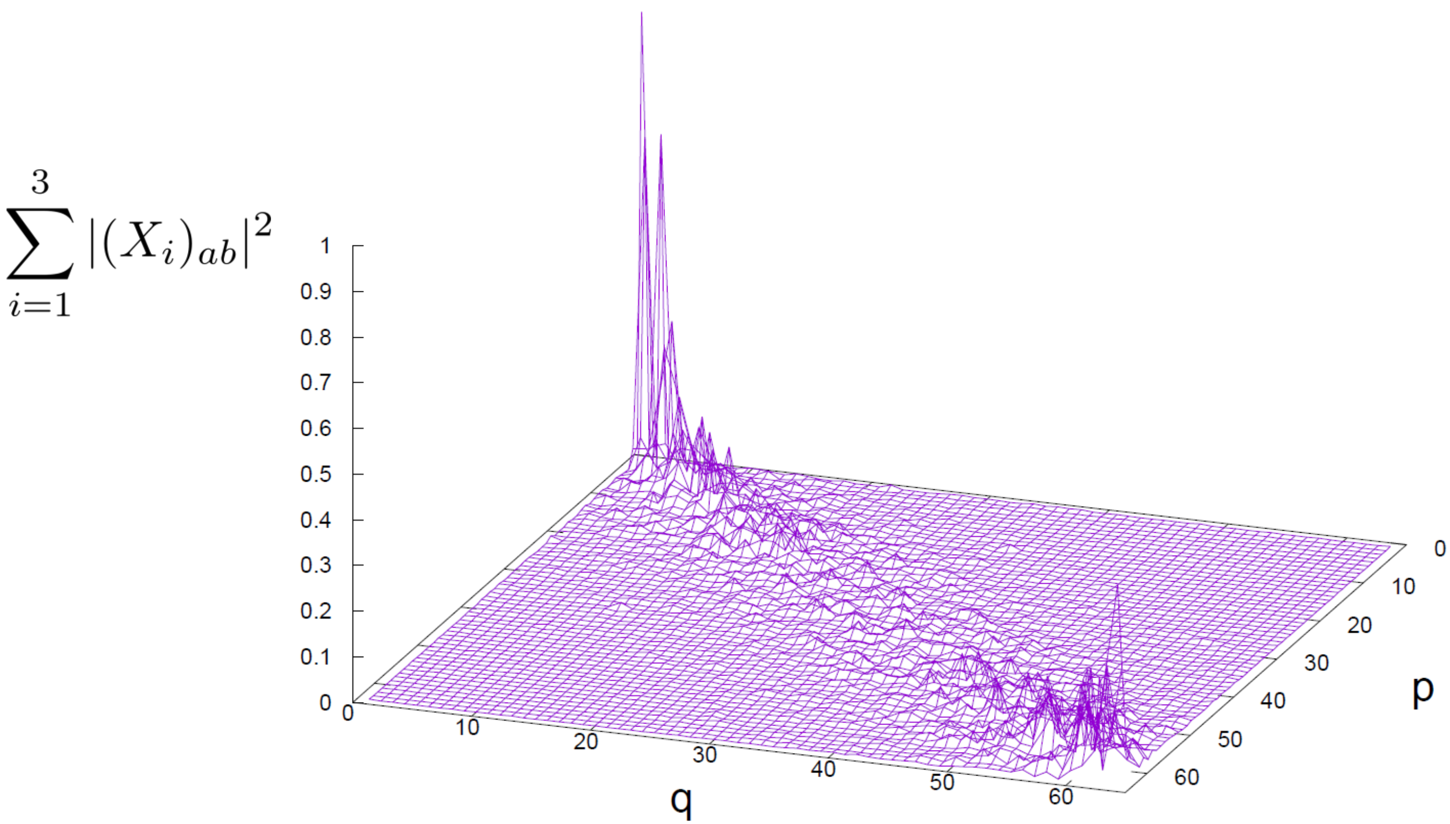
$$\delta X_\mu = -\epsilon \frac{\partial I}{\partial X_\mu^\dagger} \qquad \delta Y_a = -\epsilon \frac{\partial I}{\partial Y_a^\dagger} \qquad \delta M = -\epsilon \frac{\partial I}{\partial M^\dagger}$$

$\Rightarrow$ $\delta I \leq 0$

# Space-time structure in classical solutions

# Band diagonal structure of Xi

A typical solution at $N_X = 64$

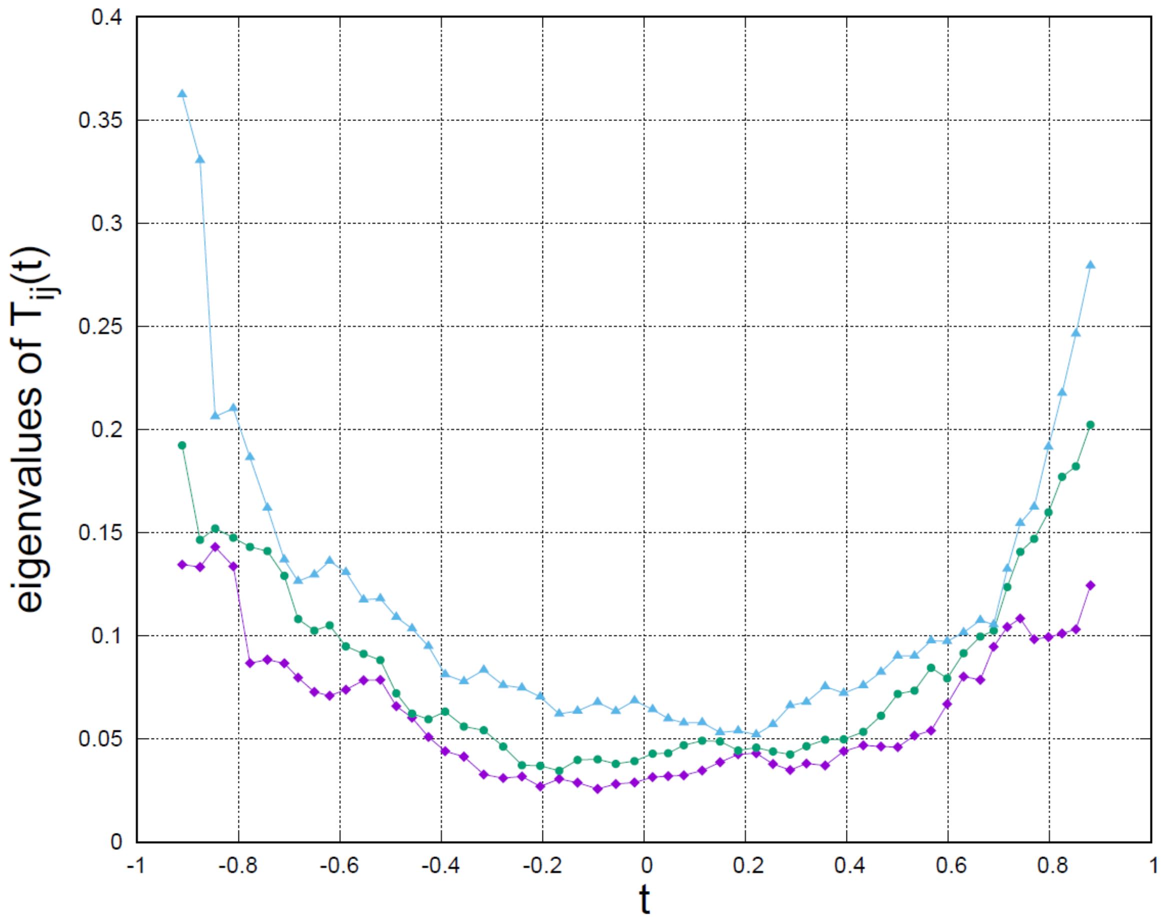



$n = 10$

# Eigenvalues of Tij

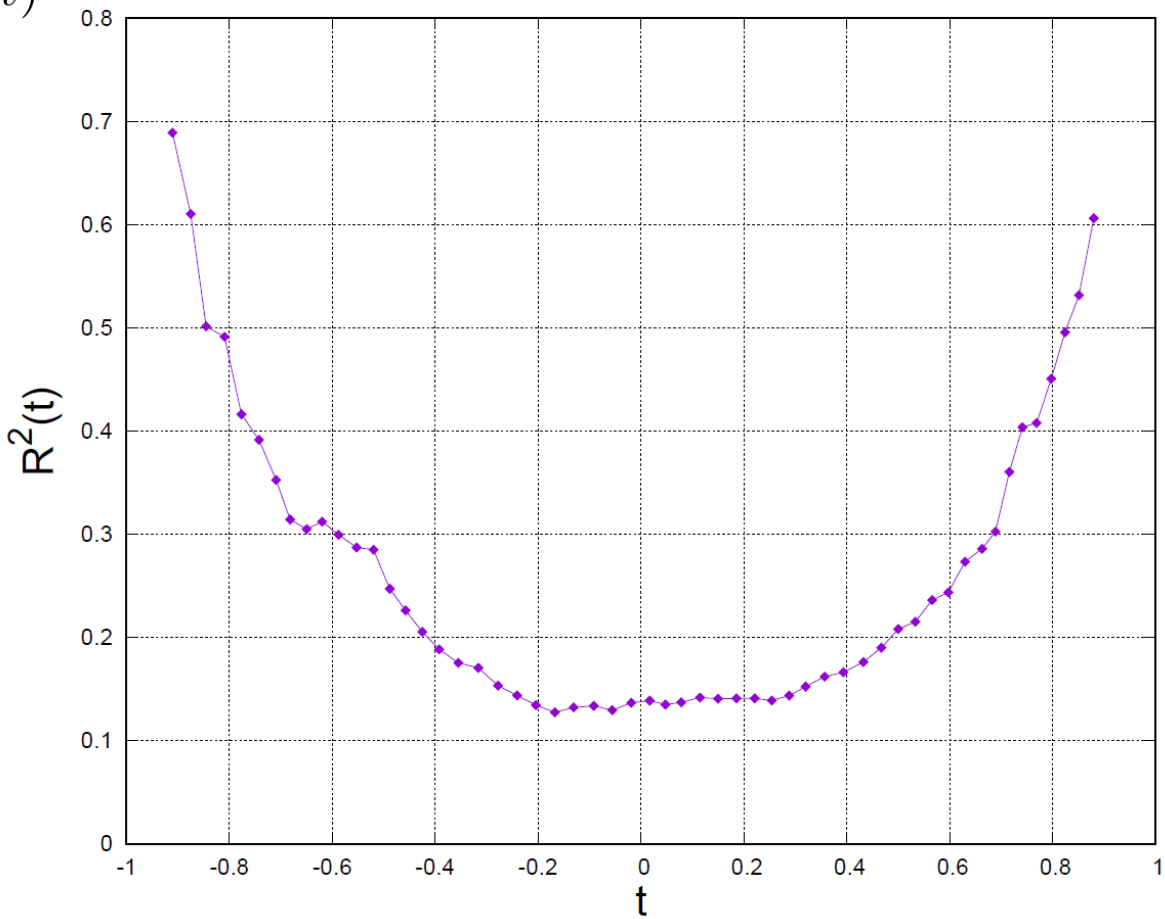
$$T_{ij}(t) = \frac{1}{n}\mathrm{tr}(\bar{X}_i(t)\bar{X}_j(t))$$

SO(3) symmetric



# R^2(t)

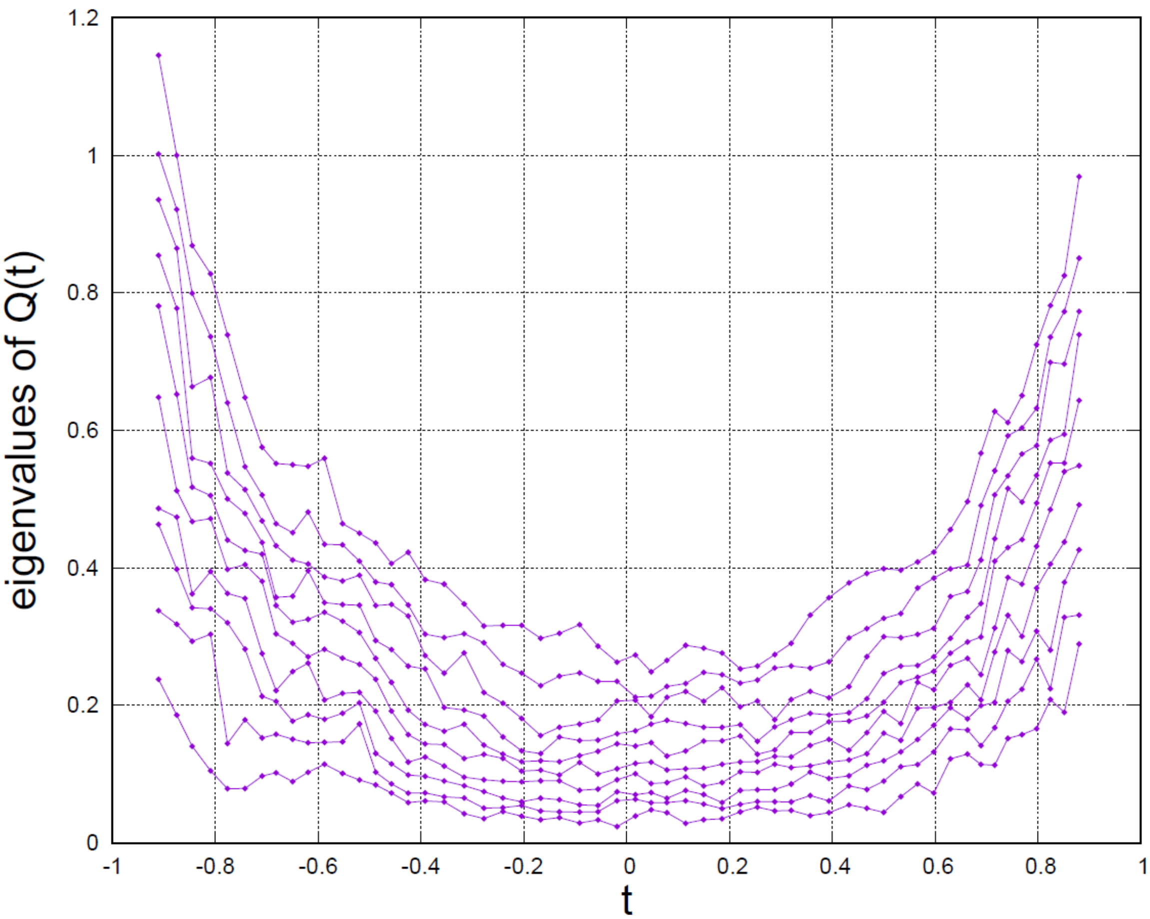
$$R^2(t) = \frac{1}{n}\mathrm{Tr}\bar{X}_i^2(t)$$
$$= T_{ii}(t)$$



# Space-time structure

$$Q(t) = \sum_{i=1}^{3} \bar{X}_i(t)^2$$

dense distribution
⇒ smooth structure



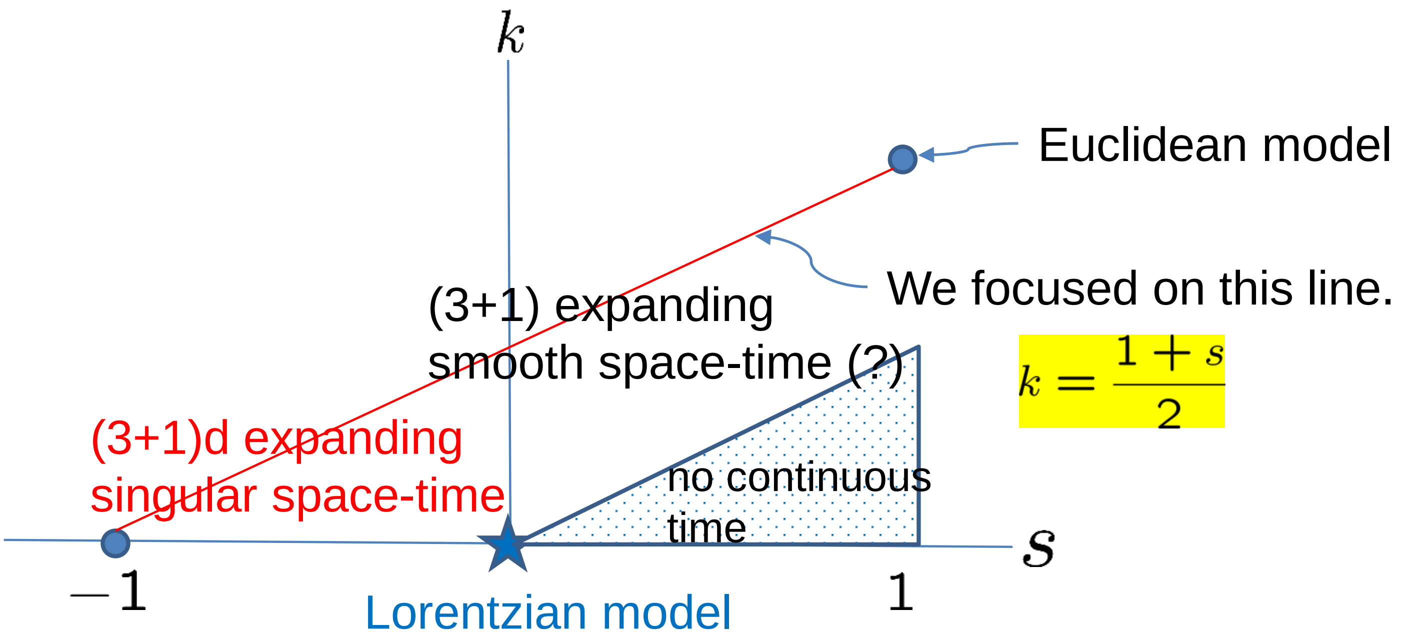
# Summary and discussion

# Summary

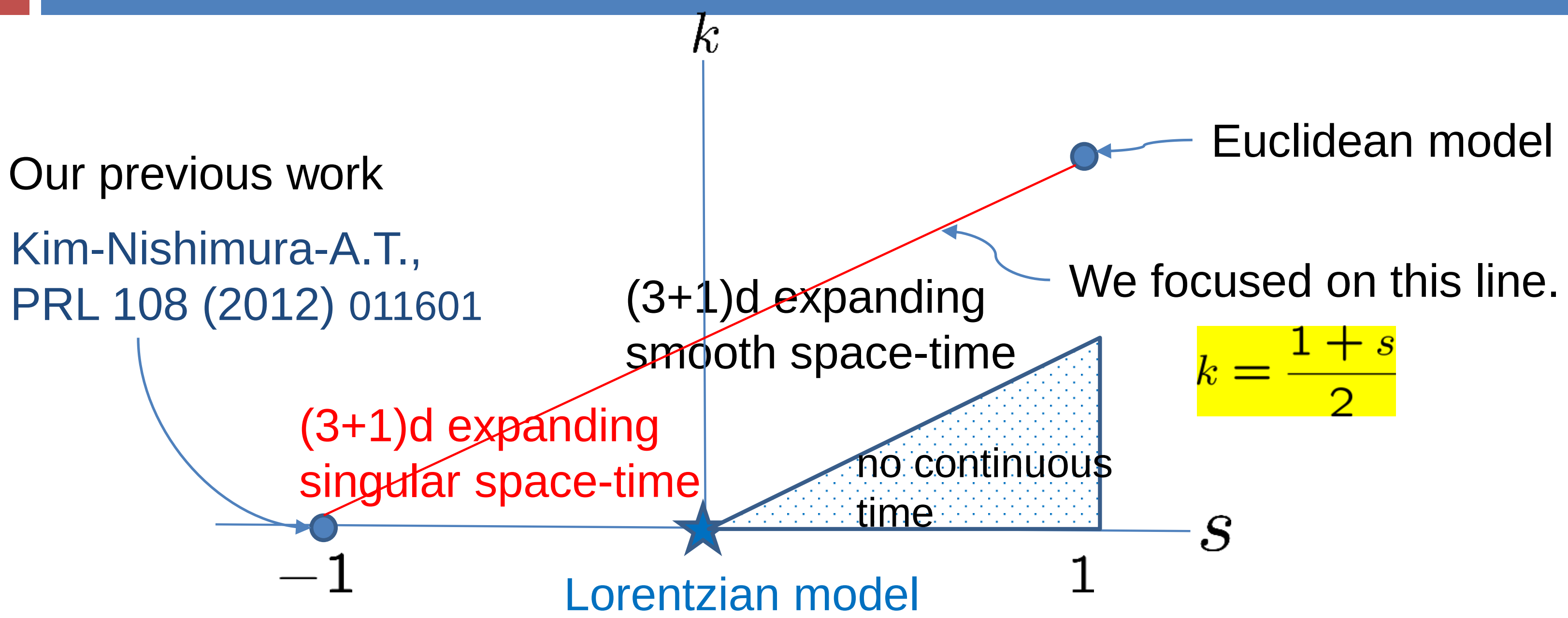
The Lorentzian version of the type IIB matrix model with certain generalization

$$S = N\beta e^{-i\frac{\pi}{2}(1-s)}\left(\frac{1}{2}e^{-ik\pi}\mathrm{Tr}[A_0,A_i]^2 - \frac{1}{4}\mathrm{Tr}[A_i,A_j]^2\right)$$

IR cutoffs : $\frac{1}{N}\mathrm{tr}\,(A_0)^2 = \kappa\ ,\quad \frac{1}{N}\mathrm{tr}\,(A_i)^2 = 1$



# Summary




- Transition from the Pauli matrices to a smooth space-time occurs at slightly positive *s* for N=128, 192.

  Does the transition point approaches *s=0* at larger *N* ?

- Complex Langevin simulation becomes unreliable due to growing non-hermiticity when we decrease *k* from *k*=(1+*s*)/2 too much.

Can we approach the target (s,*k*)=(0,0) at larger N ?
Does the (3+1)-dim. expanding smooth space-time survive there ?

# Summary

- Hermiticity of spatial matrices emerges as the space expands.

  This suggests that a classical solution is dominating there.
  This is also expected from the fact that the action becomes large there due to the space expansion.

- There are infinitely many classical solutions which have (3+1)-dim. expanding behavior with a smooth space-time structure.
  This supports that we obtain (3+1)-dim expanding smooth space-time in the large N limit.

- If some classical solution indeed dominates at late times, we can discuss a possibility that the Standard model appears by examining various classical solutions.

# Discussion

- Effects of the fermionic matrices ?

  Not straightforward due to the “singular-drift problem” in the CLM caused by the near-zero eigenvalues the Dirac operator, but maybe possible.

- Generalizing the IR cutoffs to

$$\frac{1}{N}\mathrm{tr}\,\{(\tilde{A}_0)^2\}^p = \kappa^p \ , \qquad \frac{1}{N}\mathrm{tr}\,\{(A_i)^2\}^p = 1$$

  Previous studies suggest that we obtain universal results for $p \sim 1.5$, but the model becomes pathological for larger $p$.

# Discussion

- We further search for solutions and examine (3+1)-dim. space-time structure, matter contents and gauge interactions that the solutions give.

- We expect that there exists a solution that realizes the Standard model or beyond the Standard model and that it is indeed selected in the sense that our numerical simulation is connected to such a solution.

- Or we can calculate 1-loop effective actions around classical solutions we have found. We expect the effective action for the solution giving SM at low energy to be minimum.