Ten adventures in mathematical statistical physics

François Dunlop

Laboratoire de Physique Théorique et Modélisation (CNRS - UMR 8089),
Université de Cergy-Pontoise, https://dunlop.u-cergy.fr

April 2019, Villa Finaly, Florence

UNIVERSITÉ
de Cergy-Pontoise

cnrs

Multicomponent Field Theories and Classical Rotators

François Dunlop

Inst. des Hautes Études Scientifiques, F-91 Bures-sur-Yvette, France

Charles M. Newman*

Dept. of Mathematics, Indiana University, Bloomington, Indiana 47401, USA

Received April 8, 1975

Abstract. It is shown that a D-component Euclidean quantum field, $\boldsymbol{\varphi}=(\varphi^1, \ldots, \varphi^D)$, with $\lambda|\boldsymbol{\varphi}|^4+\beta|\boldsymbol{\varphi}|^2$ interaction, can be obtained as a limit of (ferromagnetic) classical rotator models; this extends a result of Simon and Griffiths from the case $D=1$. For these Euclidean field models, it is then shown that a Lee-Yang theorem applies for $D=2$ or 3 and that Griffiths' second inequality is valid for $D=2$; a complete proof is included of a Lee-Yang theorem for plane rotator and classical Heisenberg models. As an application of Griffiths' second inequality for $D=2$, an interesting relation between the "parallel" and "transverse" two-point correlations is obtained.

Physics

Geometric Expansion of the Boundary Free Energy of a Dilute Gas

Pierre Collet and François Dunlop

Centre de Physique Théorique de l'Ecole Polytechnique, Plateau de Palaiseau, F-91128 Palaiseau Cedex, France

Abstract. We consider a dilute classical gas in a volume $\varepsilon^{-1}\Lambda$ which tends to $\mathbb{R}^d$ by dilation as $\varepsilon \to 0$. We prove that the pressure $p(\varepsilon^{-1}\Lambda)$ is C^q in ε at $\varepsilon = 0$ (thermodynamic limit), for any $q \in \mathbb{N}$, provided the boundary $\partial\Lambda$ is C^q and provided the Ursell functions $u_n(x_1, \ldots, x_n)$ admit moments of degree q and have "nice" derivatives.

Commun. Math. Phys. 121, 401–419 (1989)

Communications in Mathematical Physics

On the Microscopic Validity of the Wulff Construction and of the Generalized Young Equation

Joël De Coninck,* François Dunlop and Vincent Rivasseau
Centre de Physique Théorique (CNRS–LP 14), Ecole Polytechnique, F-91128 Palaiseau Cedex, France

Abstract. For a large class of 1 + 1 dimensional interfaces of the Solid-On-Solid type we prove on a microscopic basis the validity of the Wulff construction and of the generalized Young equation which gives the contact angle of a sessile drop on a wall. Our proof relies on a new method to treat random walks with a finite number of global constraints.

1. Introduction

Multilayer Wetting in Partially Symmetric q-State Models

François Dunlop,[1,2] Lahoussine Laanait,[1,3] Alain Messager,[1] Salvador Miracle-Sole,[1] and Jean Ruiz[1]

Received August 4, 1989; revision received January 23, 1990

When several phases coexist, the interface between two phases can be wetted by several films of the other phases. This is called *multilayer wetting* and can be characterized by the behavior of the *spreading coefficients*, which relate the surface tensions between the different phases. In this paper we consider a class of models which can exhibit a sequence of phase transitions. With some new correlation inequalities, we prove the positivity of a family of spreading coefficients. These inequalities, together with a thermodynamic argument, lead to the conclusion of multilayer wetting. These results generalize earlier results where single-layer interfacial wetting was obtained for the Potts model.

Pinning of an Interface by a Weak Potential

François Dunlop,[1] Jacques Magnen,[1] Vincent Rivasseau,[1] and Philippe Roche[1]

Received January 3, 1991

We prove that in a two-dimensional Gaussian SOS model with a small attractive potential the height of the interface remains bounded no matter how small the potential is; this is in sharp contrast with the free situation in which the interface height diverges logarithmically in the thermodynamic limit.

KEY WORDS: Interfaces; Solid-On-Solid model; cluster expansions.

Product Measures and Dynamics for Solid-on-Solid Interfaces

Pierre Collet,[1] François Dunlop,[2] Damien P. Foster,[3] and Thierry Gobron[4]

Received August 23, 1996; final April 22, 1997

A two-species asymmetric exclusion process is considered with general transition rates subject only to the constraint of charge conservation. Conditions for the existence of a stationary product measure are found in both the cases of odd–even parallel dynamics and continuous-time dynamics. The results are then applied to a one-dimensional restricted solid-on-solid model, considered as a model of driven interfacial growth, showing a nontrivial dependence of the stationary measure on the external driving field. The dependence of the growth velocity on the slope of the interface is given and interface shapes in finite volume with opposite boundary conditions are investigated numerically.

KEY WORDS: R-SOS model; stochastic lattice gas; two-species exclusion process; odd–even dynamics.

Dynamics of the Contact Line: Contact Angle Hysteresis

P. Collet,[1] J. De Coninck,[2] F. Dunlop,[3] and A. Regnard[3]
[1]*Centre de Physique Théorique, Ecole Polytechnique, 91128 Palaiseau, France*
de Recherche en Modélisation Moléculaire, Université de Mons-Hainaut, 20 Place du Parc, 7000 Mons,
oratoire de Physique Théorique et Modélisation, Université de Cergy-Pontoise, 95302 Cergy-Pontoise, Fr
(Received 1 May 1997)

An effective dynamic for the edge of a three-dimensional sessile drop is derived from the capillary waves model of two-dimensional interfaces. The corresponding Monte Carlo evolution is implemented to study wetting of a disordered substrate. Hysteresis associated with a stick-slip mechanism is measured as a function of disorder and system size. For a given system size, no measurable hysteresis is found at weak disorder. For a given disorder, even small, a measurable contact angle hysteresis is found above a certain system size. [S0031-9007(97)04480-3]

PACS numbers: 68.45.Kg, 05.20.–y, 68.10.Cr

y of contact line motion in the dynamics of ting is a long standing problem, going back Adamson [1]. Many experiments have been They were devoted mainly to the measurement angle as a function of the characteristics of the It is known that if the substrate is sufficiently

results and get useful insights. Other alterna consist of molecular dynamics approaches [9] then be extremely difficult to analyze this phenomenon.

To study the contact line on the substrate, w sidered a wedge of the spreading phase as d

Dynamic One-Dimensional Interface Interacting th a Wall

M. Dunlop,[1] P. A. Ferrari,[2] and L. R. G. Fontes[2]

Received March 7, 2001; acceped November 2, 2001

We study a symmetric randomly moving line interacting by exclusion with a wall. We show that the expectation of the position of the line at the origin when it starts attached to the wall satisfies the following bounds:

$$c_1 t^{1/4} \leqslant \mathbb{E}\xi_t(0) \leqslant c_2 t^{1/4} \log t$$

The result is obtained by comparison with a "free" process, a random line that has the same behavior but does not see the wall. The free process is isomorphic to the symmetric nearest neighbor one-dimensional simple exclusion process. The height at the origin in the interface model corresponds to the integrated flux of particles through the origin in the simple exclusion process. We compute explicitly the asymptotic variance of the flux and show that the probability that this flux exceeds $Kt^{1/4} \log t$ is bounded above by const. t^{2-K}. We have also performed numerical simulations, which indicate $\mathbb{E}\xi_t(0)^2 \sim t^{1/2} \log t$ as $t \to \infty$.

Layering and Wetting Transitions for an SOS Interface

Kenneth S. Alexander · François Dunlop · Salvador Miracle-Solé

Received: 31 March 2010 / Accepted: 12 January 2011 / Published online: 22 January 2011

Abstract We study the solid-on-solid interface model above a horizontal wall in three dimensional space, with an attractive interaction when the interface is in contact with the wall, at low temperatures. There is no bulk external field. The system presents a sequence of layering transitions, whose levels increase with the temperature, before reaching the wetting transition.

Keywords SOS model · Wetting · Layering transitions · Interface · Entropic repulsion

Wetting Transitions for a Random Line in Long-Range Potential

P. Collet[1] · F. Dunlop[2] · T. Huillet[2]

Received: 20 November 2014 / Accepted: 8 June 2015 / Published online: 20 June 2015

Abstract We consider a restricted Solid-on-Solid interface in $\mathbb{Z}_+$, subject to a potential $V(n)$ behaving at infinity like $-\mathrm{w}/n^2$. Whenever there is a wetting transition as $b_0 \equiv \exp V(0)$ is varied, we prove the following results for the density of returns $m(b_0)$ to the origin: if $\mathrm{w} < -3/8$, then $m(b_0)$ has a jump at b_0^c; if $-3/8 < \mathrm{w} < 1/8$, then $m(b_0) \sim \left(b_0^c - b_0\right)^{\theta/(1-\theta)}$ where $\theta = 1 - \frac{\sqrt{1-8\mathrm{w}}}{2}$; if $\mathrm{w} > 1/8$, there is no wetting transition.

Constructive Quantum Field Theory

The 1973 "Ettore Majorana" International School of Mathematical Physics

Edited by J. Ehlers

Springer