Iwasawa 2019

mercredi 19 juin 2019 - vendredi 28 juin 2019

Université de Bordeaux, building A33

Programme Scientifique

Mini courses

1. Frauke BLEHER (University of Iowa). Chern classes and Iwasawa theory.

Abstract: This series of talks is about the study of higher codimension behavior of Iwasawa modules. Classical main conjectures can be interpreted as saying that the first Chern class of an Iwasawa module is given by a *p*-adic *L*-function. First Chern classes describe the codimension one behavior of modules. A conjecture by Greenberg is that the first Chern classes of various natural Iwasawa modules vanish. This leads to the idea that to obtain a better insight into the structure of an Iwasawa module, one needs to study its higher codimension behavior, given by higher Chern classes. After introducing Chern classes and relating firstChern classes to classical main conjectures, I will briefly discuss Greenberg's conjecture. Then I will concentrate on the case of CM fields and show how two appropriately chosen Katz *p*-adic *L*-functions describe the second Chern classes of modules given by quotients of top exterior powers of Iwasawa modules. The goal is to give an insight into the techniques used to prove this result. This is based on joint work with T. Chinburg, R. Greenberg, M. Kakde, R. Sharifi and M. Taylor, building on prior joint work of these authors and G. Pappas.

2. Ellen EISCHEN (University of Oregon). p-adic L-functions.

Abstract: This course will provide an introduction to *p*-adic *L*-functions, one of the key objects in Iwasawa theory. To start, I will introduce *p*-adic measures and their connections with elements of Iwasawa algebras, and I will explain how to realize certain *p*-adic *L*-functions as p-adic measures (or equivalently, as elements of an Iwasawa algebra). I will also discuss connections with *p*-adic families of modular forms, an important tool for constructing *p*-adic *L*-functions. To help participants prepare for the research conference the following week, I will emphasize ingredients shared by many constructions.

As per the organizers' instructions, this course will be addressed to PhD students, so will include an introduction suitable for beginners. Useful references for getting started include Washington?s book Cyclotomic Fields and Serre's article "Formes modulaires et fonctions zeta *p*-adiques".

3. Victor ROTGER (Universitat Politècnica de Catalunya). On the arithmetic of elliptic curves via triple products of modular forms.

Abstract: The aim of this course is to explain the insights on the arithmetic of of elliptic curves that one can obtain by means of the *p*-adic *L*-functions and Euler systems associated a triple of modular forms.

Thanks to the works of Gross, Zagier and Kolyvagin in the eighties, the classical theory of Heegner points allows us to understand Mordell-Weil groups of rank 0 or 1 of elliptic curves over the field of rational numbers or imaginary quadratic fields.

It turns out that the theory of triple-products of modular forms sheds some light in ranks 0, 1 and 2, and for a larger collection of ground fields. During this series of lectures, I will describe the picture emerging from my works with Henri Darmon and Alan Lauder, which although highly conjectural for the most of it, there is compelling numerical evidence. If time permits, I will explain unconditional results on this conjecture by Rivero, variations of this conjecture due to Gatti, Guitart and Masdeu, and mod *p* tame refinements due to Darmon, Harris and Venkatesh. .

4. Jan VONK (Oxford). Overconvergent modular forms and their explicit arithmetic.
Abstract: The theory of overconvergent modular forms provides a framework for understanding classical congruences between modular forms, through the notion of *p*-adic families of modular forms. This course will start with the basics of the theory, and discuss some arithmetic applications. The focus will be on explicit computations, and we will explore the concrete nature of these objects throughout.

Main conference

1. Daniel BARRERA SALAZAR (Universidad de Santiago de Chile). Triple product p-adic L-functions and Selmer groups over totally real number fields.
Abstract: During the nineties Kato obtained deep results on the Birch and Swinnerton-Dyer conjecture in rank 0 for twists of elliptic curves over **Q** by Dirichlet characters. More recently, Bertolini-Darmon-Rotger and Darmon-Rotger developed analogous methods to treat twists by certain Artin representations of dimension 2 and 4. The aim of this talk is to explain the main ideas of joint ongoing work with Molina and Rotger which aims to generalize the methods used by Kato, Bertolini-Darmon-Rotger and Darmon-Rotger to totally real number fields, by exploiting the techniques of Andreatta and Iovita.

2. Ted CHINBURG (University of Pennsylvania). Group homology and exterior quotients in Iwasawa theory.
Abstract: Higher codimension Iwasawa theory concerns the support in codimension greater than one of Iwasawa modules. A useful technique when relating this support to *p*-adic *L*-functions is to consider the quotient of the top exterior power of an Iwasawa module *M* of rank *r* by the sum of the *r*-th exterior powers of submodules arising from various Panciskin conditions. A natural question is then to give a Galois theoretic interpretation of such exterior quotients.

In this talk I will discuss such an interpretation for *r >= 2* involving group homology. The particular homology group involved is *H_{r-2}(A,T)* when *A* and *T* are the first and second graded quotients in the derived series of a pro-*p* Galois group. One consequence is that the Galois theoretic information provided by second Chern classes in the case of Iwasawa theory over CM fields seems to governed by the first two graded quotients of the derived series, rather than being about higher graded quotients. This is joint work with F. Bleher, R. Greenberg, M. Kakde, R. Sharifi and M. J. Taylor.

3. Mladen DIMITROV (Université de Lille). Geometry of the eigencurve and Iwasawa theory.

4. Adrian IOVITA (Concordia University and Università degli studi di Padova). Katz type *p-adic L-functions when p is not split in the CM field and applications.**
Abstract: *With F. Andreatta we constructed p-adic* L*-functions attached to a triple* (F, K, p) *where* F *is a classical, elliptic modular eigenform,* K *a quadratic imaginary field and* p *a prime integer, all satisfying certain assumptions of which the most important is that p is not split in* K*. Such* p *adic* L*-functions have been constructed by N. Katz (during the 70') if* F *is an Eisenstein series and by Bertolini-Darmon-Prasana (2013) when* F *is a cuspform, when the prime* p *is split in* K.* I will also present some arithmetic applications of these constructions.

5. Joaquin RODRIGUES JACINTO (Aix-Marseille Université). Norm-compatible cohomology classes in Siegel varieties.

Abstract: We will explain how to construct towers of interesting classes in the cohomology of Siegel sixfolds. We will study their complex regulator and we will give an application to Iwasawa theory. This is joint work with Antonio Cauchi and Francesco Lemma.

6. Yukako KEZUKA (Universität Regensburg). On the conjecture of Birch and Swinnerton-Dyer for certain elliptic curves with complex multiplication.
Abstract: This talk will describe recent joint work in progress with J. Coates, Y. Li and Y. Tian. Let *K* be the imaginary quadratic field **Q***(sqrt{-q})*, where *q* is any prime congruent to 7 modulo 16. Let *A* be the Gross curve defined over the Hilbert class field *H* of *K*, with complex multiplication by the ring of integers of *K*. In their most recent work, Coates and Li found a large family of quadratic twists *E* of *A* whose complex *L*-series *L(E/H,s)* does not vanish at *s=1*. We will discuss the *p*-part of the Birch and Swinnerton-Dyer conjecture for these curves for every prime *p* which splits in *K* (in particular, this includes *p=2*).

7. Guido KINGS (Universität Regensburg). Equivariant motivic Eisenstein classes and a generalization of the Damerell/Shimura/Katz theorem (joint with J. Sprang).
Abstract:The equivariant polylogarithm allows to construct in a very general setting cohomology classes of arithmetic groups with values in motivic cohomology. Using the regulator to algebraic de Rham cohomology gives interesting algebraic Eisenstein classes. We use this theory to generalize the results of Damerell, Shimura and Katz on the algebraicity of special values of *L*-functions for Hecke characters for CM fields *K* to the case of finite extensions *L/K* over CM fields *K*.

8. Antonio LEI (Université Laval). Pseudo-null modules and codimension two cycles for supersingular elliptic curves.
Abstract: Let *E/* **Q** be an elliptic curve with supersingular reduction at an odd prime *p* and *a_p(E)=0*. Let *K* be an imaginary quadratic field where *p* splits and write *K_\infty* for the compositum of all *\mathbb{Z}_p*-extensions of *K.* Generalizing Kobayashi's plus and minus Selmer groups over cyclotomic extensions of **Q**, Kim defined *\pm/\pm*-Selmer groups for *E* over *K_\infty*. We present numerical examples where the intersection of a pair of these Selmer groups is pseudo-null. This allows us to give explicit examples which affirm the pseudo-nullity conjecture of Coates and Sujatha. We will also explain how to relate these Selmer groups to Loeffler's *2*-variable *p*-adic *L*-functions via codimension two cycles. If time permits, we will discuss how our technique can be extended to the setting of tensor products of Hida families. This is joint work with Bharath Palvannan.

9. Zheng LIU (McGill University). *p-adic families of Klingen Eisenstein series and theta series.**
Abstract: *p-adic interpolations of Eisenstein series and theta series give explicit examples of* p*-adic families of automorphic forms. Their congruences with other automorphic forms help show lower bounds of certain Selmer groups. I will first explain the construction of a* p*-adic Klingen Eisenstein family for symplectic groups, and then discuss its connection with a* p*-adic family of theta lifts.

10. David LOEFFLER (University of Warwick). *P-adic L-functions and Euler systems for GSp(4).**
Abstract: *I will explain how the higher Hida theory recently introduced by Pilloni can be used to construct p-adic L-functions interpolating the criticalvalues of the degree 4 (spin) L-functions of automorphic forms on* GSp(4)*, and the degree 8* L*-functions of cusp forms on* GSp(4) x GL(2)*.This is joint work with Vincent Pilloni, Chris Skinner and Sarah Zerbes. I will conclude by describing work in progress to relate the GSp(4)* p*-adic* L*-function to the images of Euler system classes under the* p-*adic syntomic regulator map.

11. Jan NEKOVÁŘ (Sorbonne Université). The plectic polylogarithm.
Abstract: We are going to describe the Hodge realisation of the plectic polylogarithm and its relation to special values of *L*-functions. This is a joint work with A.J. Scholl.

12. Jishnu RAY (University of British Columbia). Selmer groups of elliptic curves and Iwasawa algebras.
Abstract:The Selmer group of an elliptic curve over a number field encodes several arithmetic data of the curve providing a *p*-adic approach to the Birch and Swinnerton-Dyer, connecting it with the *p*-adic *L*-function via the Iwasawa main conjecture. Under suitable extensions of the number field, the dual Selmer becomes a module over the Iwasawa algebra of a certain compact *p*-adic Lie group over *Z_p* (the ring of *p*-adic integers), which is nothing but a completed group algebra. The structure theorem of *GL(2)* Iwasawa theory by Coates, Schneider and Sujatha (C-S-S) then connects the dual Selmer with the “reflexive ideals” in the Iwasawa algebra. We will give an explicit ring-theoretic presentation, by generators and relations, of such Iwasawa algebras and sketch its implications to the structure theorem of C-S-S. Furthermore, such an explicit presentation of Iwasawa algebras can be obtained for a much wider class of *p*-adic Lie groups viz. pro-*p* uniform groups and the pro-*p* Iwahori of *GL(n,Z_p).* Alongside Iwasawa theoretic results, we will state results counting the dimension of first cohomology group of the pro-*p* Iwahori subgroup of any reductive group over *Z_p* and thus prove the Inverse Galois problem for *p*-adic Lie extensions. We finally conclude by connecting *GL(2)* Iwasawa theory of (C-S-S) with *PGL(2)* Iwasawa theory, thus moving down the Iwasawa theoretic tower, unlike (C-S-S) where their arguments circles on moving up the Iwasawa theoretic tower.

13. Giovanni ROSSO (Concordia University and Cambridge). Families of Drinfeld modular forms.
Abstract: Seminal work of Hida tells us that for eigenforms that are ordinary at p we can always find other eigenforms, of different weights, that are congruent to our given form. Even better, it also says that we can find *q*-expansions whose coefficients are analytic functions of the weight variable *k*, that when evaluated at positive integers give the q-expansion of classical ordinary eigenforms.This talk will explain how similar results can be obtained for Drinfeld modular forms. We shall explain how to construct families for Drinfeld modular forms, both ordinary and of positive slope, and how to decide if an overconvergent form of small slope is classical. Joint work with Marc-Hubert Nicole.

14. Ryotaro SAKAMOTO (University of Tokyo). An application of the theory of higher rank Euler, Kolyvagin, and Stark systems.
Abstract: Recently, we established the theory of higher rank Euler, Kolyvagin, and Stark systems when a coefficient ring is Gorenstein. In this talk, I will discuss two applications of this theory.First, I will discuss equivariant BSD conjecture. Second, I will outline the construction of a higher rank Euler system for *\mathbb{G}_{m}* over a totally real field and explain that all higher Fitting ideals of a certain *p*-ramified Iwasawa module are described by analytic invariants canonically associated with Stickelberger elements.The first part is joint work with David Burns and Takamichi Sano.

15. Romyar SHARIFI (UCLA). Eisenstein cocycles in motivic cohomology.
Abstract: I will describe joint work with Akshay Venkatesh on the construction of a 1-cocycle on *GL_2(Z)* valued in a quotient of a limit of second motivic cohomology groups of open subschemes of the square of **G**_m over **Q**. I’ll show how the cohomology class of this cocycle is annihilated by an Eisenstein ideal, and I’ll explain how the cocycle specializes to homomorphisms from first homology groups of modular curves to second *K*-groups of rings of cyclotomic integers. I also hope to mention a related construction over imaginary quadratic fields.

16. Florian SPRUNG (Arizona State University). Shedding light on Selmer groups for elliptic curves at supersingular primes in *Z_p^2-extensions via chromatic Selmer groups.***
Abstract: *We present some results and techniques concerning Selmer groups in Z_p^2-extensions for elliptic curves at supersingular primes, focusing on the case* a_p *not equal to* 0. *In this case, a convenient pair of objects to consider is the 'chromatic Selmer groups' (also called 'signed Selmer groups' when* a_p=0*).

17. Eric URBAN (Columbia University and CNRS). Towards an Euler system for the standard L-function attached to Siegel modular forms.

18. Shunsuke YAMANA (Kyoto University). On central derivatives of (twisted) triple product p-adic L-functions..
Abstract: We will construct twisted triple product *p*-adic *L*-functions and discuss its trivial or non-trivial zeros at the center of the functional equation. In the split and *+1* sign case we will determine the trivial zeros of cyclotomic *p*-adic *L*-functions associated to three ordinary elliptic curves and identify the double or triple derivatives of the *p*-adic *L*-function with the product of the algebraic part of central *L*-values and suitable *L*-invariants. If time permits, we will formulate the *p*-adic Gross-Zagier formula in the -1 sign case. This is a joint work with Ming-Lun Hsieh.