$\mathscr{R}$encontre ANR $\mathscr{F}$atou – du 14 au 16 novembre 2018

Mercredi 14 novmebre 2018

10h: Accueil des participants - café

10h45 - 12h15: Pierre Berger (CNRS - Université Paris 13): **Abondance des applications d'allure Hénon fortement régulières I**

Pause déjeuner

14h-15h30: Sylvain Crovisier (CNRS - Université Paris-Sud): **$\mathscr{C}^1$-density of stable ergodicity**

Pause café

16h-17h30: Duc Viet Vu (University of Cologne): **Theory of tangent currents and applications to complex dynamics I**

Jeudi 15 novembre 2018

9h30-10h30: Zhuchao Ji (Sorbonne Université): **Topological Collet-Eckmann condition in polynomial skew products**

10h45-12h15: Duc Viet Vu (University of Cologne): **Theory of tangent currents and applications to complex dynamics II**

Pause déjeuner

14h-15h30: Pierre Berger (CNRS - Université Paris 13): **Abondance des applications d'allure Hénon fortement régulières I**

Vendredi 16 novembre 2018

9h30-10h30: Yves Cornulier (CNRS - Université Lyon 1): **Distortion in the Cremona group**

10h45-11h45: Yûsuke Okuyama (Kyoto Institute of Technology): **An improvement of McMullen's finiteness theorem**

Résumés

Pierre Berger: Abondance des applications d'allure Hénon fortement régulières I & II

Ce mini-cours s'intéresse aux applications d'allure Hénon de la forme $f_a : (x, y) \mapsto (x^2 + a, 0) + B(a, x, y)$, avec $B \in C^2([-2, 2]^3, \mathbb{R}^2)$. Nous introduirons la notion d'applications fortement régulières dans ce cadre. Ces applications ont la propriété de laissé invariant un attracteur mélangeant muni d'une mesure SRB. Nous montrerons leur abondance: pour B assez petit, l'application f_a est fortement régulière pour un ensemble de mesure de Lebesgue positive de paramètre a. Cela implique en particulier les théorèmes énoncés par Benedicks-Carleson, Mora-Viana, Wang-Young et Takahasi.

Il s'agit d'une solution du programme de J.C. Yoccoz énoncé durant son premier cours au collège de France en 1996. Ce mini-cours se focalisera principalement sur la définition des applications fortement régulières et sur la sélection des paramètres. Après une courte introduction sur les mesures SRB, et l'idée naïve de la sélection des paramètres, nous introduirons un foncteur entre un pseudo-monoïde combinatoire (formé de mots sur un alphabet dénombrable) et un pseudo-monoïde de pièces (formé d'itérations de la dynamique restreintes à des boites ayant certaines propriétés d'expansion). Chacun de ces deux pseudo-monoïdes est muni de deux lois de composition que le foncteur laisse équivariant. Ici, une pièce est un analogue en dimension réelle 2 de la notion de pièces de puzzle en dimension complexe 1. L'étude combinatoire de ce pseudo-monoïd permet de déduire les propriétés géométriques et analytiques de ces pièces. Le foncteur permet aussi de définir de façon combinatoire certaines variétés stables ou instable de Pesin qui sont proches d'être verticales ou horizontales, et d'obtenir certaines bornes sur la distorsion de la dynamique restreinte ou encore d'approximer la dépendance de celles-ci en fonction de la combinatoire ou du paramètre. Il permet aussi de majorer la mesure de l'espace l'espace des phases n'appartenant pas à des pièces d'ordres données. Ce formalisme permet surtout d'énoncer clairement sur quel intervalle de paramètres ces pièces persistent, la mesure du complémentaire de ces intervalles dans l'espace des paramètres, le tout sans hypothèse de récurrence. De là, le théorème d'abondance se déduit facilement, sans hypothèse de récurrence.

Yves Cornulier: Distortion in the Cremona group

Given x in a group G, we say that x has distortion at least f if there exists a finite subset S of G such that, for every n, the ball S^n contains an element of the form x^m for some $m \geq f(n)$. It is said to have distortion at most f if for every S, there exists C such that any element x^m in S^n satisfies $m \leq f(Cn)$. The element x is said to be undistorted if it has linear distortion.

In linear groups $GL_n(K)$, the distorted elements are precisely the virtually unipotent ones; when they have infinite order (which occurs only in characteristic zero), they have exponential distortion.

We study distortion in Cremona groups, notably in dimension 2. When the dynamical degree is greater than 1 (i.e., degrees of powers grow exponentially), it is obvious that the element is undistorted. In dimension 2, we describe the other cases: for elements of bounded degree growth (typically, elements of PGL(3)) and infinite order, the distortion is doubly exponential or exponential according to whether they are virtually unipotent. When the degree growth is linear or quadratic (Jonquières/Halphen twists), the elements are undistorted. The Jonquières case is due to Blanc-Deserti, using that these element have a linear basepoint growth. The Halphen case (which has bounded basepoint growth) is treated by using the action on the Picard-Manin space (an infinite-dimensional Lobachevski-Poincaré space): while for this action, Halphen twists have distorted orbit growth, we remove an invariant family of horoballs and endow the remaining space with the intrinsic length metric; for the resulting metric we establish that the Halphen twists have a linear growth.

Sylvain Crovisier: $\mathscr{C}^1$-density of stable ergodicity

We prove a $\mathscr{C}^1$ version of a conjecture by Pugh and Shub: among partially hyperbolic volume-preserving $\mathscr{C}^2$ diffeomorphisms, the stably ergodic ones are $\mathscr{C}^1$-dense. (Joint work with A. Avila and A. Wilkinson).

This is a consequence of a perturbation result for horseshoes which allow to produce "blenders" from horseshoes with large topological entropy.

Zhuchao Ji: Topological Collet-Eckmann condition in polynomial skew products

The dynamics of non-uniformly hyperbolic polynomials of $\mathbb{C}$ restrict on the Fatou set are fully understood: for Topological Collet-Eckmann polynomials the Fatou set equal to the union of finite number of attracting basins, and form a full measure subset in $\mathbb{C}$. In this talk we consider polynomial skew products of $\mathbb{C}^2$ instead. Let f be a polynomial skew product which leaves invariant a vertical line L, and assume f restricts on L satisfies Topological Collet-Eckmann condition, we show that the one-dimensional attracting basins of L bulge to two-dimensional Fatou components, and the union of these Fatou components form a full measure subset in the immediate basin in L (in the case L is attracting or parabolic), or form a full measure subset in a neighborhood of L (in the case L is elliptic). As an easy consequence there are no wandering domains in the immediate basin or in the neighborhood of L.

Yûsuke Okuyama: An improvement of McMullen's finiteness theorem

Let Rat_d be the set of all rational functions of (a given) degree $d > 1$ (of one variable). The dynamical moduli M_d of rational functions of degree d is the set of all Mobius conjugacy classes of them. The multiplier morphism s_n from M_d to A^{d^n+1} associates each Mobius conjugacy class $[f]$ of a rational function f of degree d to the ordered collection of all (the values of) $d^n + 1$ symmetric polynomials for the set of the multipliers of all periodic points of f having the given period n. We first recall the Minor-Silverman isomorphism theorem between M_2 and A^2 and McMullen's "finiteness" theorem of the above multiplier morphisms s_n from M_d to A^{d^n+1} for $n >> 1$, except for the flexible Lattès locus L_d in M_d, and then present our recent improvement of McMullen's finiteness theorem, which asserts that the "formally exact version" of the above multiplier morphisms is (already) finite for $n >> 1$, necessarily except for L_d.

This talk is based on our joint work with Thomas Gauthier and Gabriel Vigny.

Duc Viet Vu: **Theory of tangent currents and applications to complex dynamics**

A fundamental question in the pluripotential theory is to establish a theory of intersection of closed positive currents on complex manifolds. Such a theory has been extensively developed in the case of (1,1)-currents and proved to be a powerful tool in complex geometry. On the other hand, the case of higher bi-degree remains very challenging. The aim of this minicourse is to present the theory of tangent currents coined by Dinh-Sibony which is a recent breakthrough in this research direction. We also see how this theory is applied to obtain an (optimal) upper bound for the number of isolated periodic points of meromorphic self-maps which had been inaccessible before.