

11th Meeting of the GDR DYNQUA
Bordeaux
13-15 February 2019

CONFERENCE SCHEDULE

Wednesday february 13	Thursday february 14	Friday february 15
<p>12h-13h30 : Welcome and lunch 13h30-14h20 : C. Shirley 14h30-15h20 : L. Bonnet 15h20-15h50 :Coffee break 15h50-16h40 : V.D. Dinh 16h50-17h40 : M. Khalile 17H50-18H40 : F. Hérau</p>	<p>9h-10h30 : F. Klopp (course 1) 10h30-11h : Coffee break 11h-11h50 : R. Ducatez 12h-12h50 : M. Simon 13h-14h30 : Lunch break 14h30-15h20 : B. Nectoux 15h30-16h20 : G. Klein 16h20-16h50 : Coffee break 16h50-17h40 : N. Cunéo</p> <p>19h30 : Conference dinner</p>	<p>9h-10h30 F. Klopp (course 2) 10h30-11h : Coffee break 11h-11h50 : A. Duca 12h-12h50 : G. Lemarié End of the conference</p>

WIFI LOGIN/PASSWORD

Choose the 'REAUMUR' wireless network
 In your internet browser, permit pop-up and cookies
 Pop-up maintain the connexion open
 Start your internet browser et try to acces a web site in HTTP not in HTTPS
 For example, open the page <http://www.free.fr>
 DON'T USE <http://www.google.com>
 Follow the instructions
 Choose 'Conferences/Invites'
 Identify you with login and password
Login: DYNQUA-n-1
Password: xY6P86n

CHRISTOPHER SHIRLEY
Wednesday february 13 – 13h30-14h20

Transport properties of stationary Schrödinger operators at small disorder

Abstract : in this presentation, I will first go back over the connections between the spectrum of Schrödinger operators and their transport properties as well as the Anderson conjecture. Secondly, we will see how we can obtain results on ballistic transport until times that depend on the strength of the disorder and the type of stationarity using approximate spectral theory.

LAURENT BONNET
Wednesday february 13 – 14h30-15h20

Semiclassical initial value theory of rotationnally inelastic scattering
some remarks on the phase index in the interaction picture

Abstract : the talk will be about the treatment of quantum interferences in the semiclassical initial value theory of rotationally inelastic scattering in the interaction picture. Like many semiclassical methods, the previous approach involves a phase index related to sign changes of a Jacobian whose square root is involved in the calculations. It is shown that replacing the original phase index by a new one extends the range of applicability of the theory. The resulting predictions are in close agreement with exact quantum scattering results for a model of atom-rigid diatom collision involving strong interferences. The developments are presented within the framework of the planar rotor model, but are readily applicable to threedimensional collisions.

REFERENCE

Laurent Bonnet, J. Chem. Phys. 148, 194104 (2018).

VAN DUONG DINH
Wednesday february 13 – 15h50-16h40

On Strichartz estimates for the fractional Schrodinger equation on asymptotically Euclidean manifolds

Abstract : the fractional nonlinear Schrodinger equation has attracted a lot of interest recently both in physics and mathematics. In this talk, I will present known results of Strichartz estimates for the fractional Schrodinger equation on manifolds. I will focus mainly on global-in-time Strichartz estimates for the fractional Schrodinger equation on asymptotically Euclidean manifolds. The result says that under the non-trapping condition one has global-in-time Strichartz estimates with no loss of derivatives as in the flat case.

MAGDA KHALILE
Wednesday february 13 – 16h50-17h40

Asymptotics of Robin eigenvalues in domains with corners

Abstract : we study the eigenvalues of the Laplacian with a strong attractive Robin boundary condition in curvilinear polygons. There exists a number $N \in \mathbb{N}$ depending on the domain such that the asymptotics of the N first eigenvalues is essentially determined by the corner openings. While only a rough estimate is available for the next eigenvalues, we prove under some geometric assumptions the existence of an effective self-adjoint operator, acting on the boundary of the domain with boundary conditions at the corners, which leads the asymptotic behavior of any eigenvalue beyond the critical number N .

FRÉDÉRIC HÉRAU
Wednesday february 13 – 17h50-18h40

A Korn-Wirtinger inequality

Abstract : in kinetic theory or in other fields, some control of the gradient by the symmetric gradient of the macroscopic velocity of a system of particle may be necessary, since only the second quantity appears naturally in the equations. This type of inequality is known on bounded domains with or without axisymmetries. In this talk, we present a version on the whole space equipped with a probability measure, and give an example where this type of tool may be useful. This is a joint work with K. Carrapatoso, S. Mischler, J. Dolbeault, C. Mouhot, and C. Schmeiser.

FRÉDÉRIC KLOPP
Thursday february 14 – 9h00-10h30

Localization (course 1)

Abstract : the lectures will be devoted to localization for systems of interacting quantum particles in a random field. After a short review of available results for a single particle, we will turn to the case of systems with a positive density of particles.

RAPHAEL DUCATEZ
Thursday february 14 – 11h00-11h50

Analysis of the one dimensional inhomogeneous Jellium model with the Birkhoff-Hopf Theorem.

Abstract : the Jellium model is a system of charged particles in a charged background all interacting with Coulomb interaction. In the one dimension case, we study this system at non zero temperature in a inhomogeneous background. We use the Hilbert distance on cones and the Birkhoff-Hopf Theorem to prove decay of correlation, analyticity of the free energy and a central limit theorem.

MARIELLE SIMON
Thursday february 14 – 12h00-12h50

Diffusion normale et anormale de l'énergie dans les chaînes d'oscillateurs

Résumé : récemment, des comportements anormaux de l'énergie ont été observés numériquement dans les modèles de chaînes d'oscillateurs en dimension 1. L'obtention rigoureuse de tels comportements à partir de systèmes déterministes de particules newtoniennes est actuellement hors de portée. Une façon de montrer rigoureusement les propriétés macroscopiques diffusives (ou non) de ces chaînes soumises à une dynamique Hamiltonienne est d'y ajouter un bruit stochastique qui conserve l'énergie. L'objectif de l'exposé sera de donner les résultats que l'on peut mathématiquement prouver dans ce contexte pour les fluctuations macroscopiques de l'énergie autour de l'équilibre. En particulier, en changeant le bruit stochastique choisi, on obtiendra les deux régimes possibles : diffusion normale et anormale (de type Laplacien fractionnaire), ainsi qu'une transition entre ces deux régimes. En collaboration avec C. Bernardin, P. Gonçalves, M. Jara and M. Sasada.

BORIS NECTOUX
Thursday february 14 – 14h30-15h20

Eigenvalues of the Witten Laplacian with Dirichlet boundary conditions
the case with critical points on the boundary

Abstract : metastability in molecular dynamics is one the most important challenges and is a very active field of research. Its understanding is crucial in order to justify the use of accelerated dynamics algorithms (e.g. those introduced by A.F. Voter *et al.* in Los Alamos) which are based on so-called asymptotic *Eyring-Kramers* formulas for the exit event from a metastable state $\Omega \subset \mathbb{R}^d$ when the temperature $h > 0$ of the system goes to 0. In many applications, the boundary of the domain Ω contains saddle points of the potential energy $f : \mathbb{R}^d \rightarrow \mathbb{R}$ of the system. Motivated by this issue, I will present recent results [1] which consist in giving asymptotic Eyring-Kramers type formulas when $h \rightarrow 0$ of the smallest eigenvalues of the Dirichlet realization of the Witten Laplacian

$$\Delta_{f,h} = -h^2\Delta + |\nabla f|^2 - h\Delta f$$

on a bounded domain $\Omega \subset \mathbb{R}^d$ when the function f has saddle points on the boundary of Ω .

[1] D. Le Peutrec and B. Nectoux. *Eigenvalues of the Witten Laplacian with Dirichlet boundary conditions: the case with critical points on the boundary*, 2019. In preparation.

GUILLAUME KLEIN
Thursday february 14 – 15h30-16h20

Spectral asymptotics for the vectorial damped wave equation

Abstract : in this talk we will be interested in the distribution of the eigenfrequencies of a damped wave equation on a compact Riemannian manifold. In the case of a scalar equation J. Sjöstrand has shown that "the majority" of the eigenfrequencies were located in a strip parallel to the real axis. The width and the position of this strip are determined by the average of the damping term along the geodesics of the manifold. In the case of a vectorial equation the damping term is no longer a real valued function but a matrix valued one and I will present the analog of the result of Sjöstrand in this setting. The width and position of the strip are then determined by the Lyapunov exponents of some cocycle defined from the damping term.

NÉO CUNÉO
Thursday february 14 – 16h50-17h40

Nonequilibrium steady states for chains of oscillators and rotors: an overview

Abstract : I will talk about chains of oscillators and rotors interacting with stochastic heat baths at different temperatures. I will introduce these very simple models in the framework of the (yet unsolved!) problem of heat conduction. Then, we will focus on a much more elementary question: the existence of an invariant measure (called non-equilibrium steady state), which has been proved only in some specific cases over the past 20 years. I will explain how distinct models lead to distinct difficulties, and sketch some of the ideas used to overcome them.

FRÉDÉRIC KLOPP
Friday february 15 – 9h00-10h30

Localization (course 2)

Abstract : the lectures will be devoted to localization for systems of interacting quantum particles in a random field. After a short review of available results for a single particle, we will turn to the case of systems with a positive density of particles.

Controllability of localized quantum states on infinite graphs through bilinear control fields

Abstract : we consider a particle constrained in a graph structure and excited by an external controlling field. Its dynamics is modeled by the bilinear Schrödinger equation $i\partial_t\psi = -\Delta\psi + u(t)B\psi$ in the Hilbert space $L^2(\mathcal{G}, \mathbb{C})$ where \mathcal{G} is the graph. The Laplacian $-\Delta$ is equipped with self-adjoint boundary conditions. The action of the field is represented by the bounded symmetric operator B and the control function $u \in L^2((0, T), \mathbb{R})$ with $T > 0$, which accounts its intensity.

The exact controllability of the bilinear Schrödinger equation on bounded intervals was widely studied in literature (see for instance [BL10, Mor14, MN15]). Nevertheless, the bilinear Schrödinger equation on graphs is in general a more delicate matter so it was only studied in [Duc18b, Duc18a] on compact networks. Up to our knowledge, the controllability on infinite graphs is still an open problem and the main reason can be found on the dispersive phenomena characterizing the equation (not considering the difficulties already appearing on compact graphs). A peculiarity of the Schrödinger equation is the loss of localization of the wave packets during the evolution, the dispersion. This effect can be measured by L^∞ -time decay (see [AAN15, AAN17]).

In this talk, we present the work [AD18] where we study the bilinear Schrödinger equation on infinite graphs. In particular, we show the existence of suitable subspaces of $L^2(\mathcal{G}, \mathbb{C})$ where the equation is well-posed. In such spaces, we define assumptions on the structure of the graph and on the control field such that the global exact controllability is guaranteed. The result leads to the so-called "energetic controllability".

Work in collaboration with Kais Ammari.

REFERENCES

- [AAN15] F. Ali Mehmeti, K. Ammari and S. Nicaise, Dispersive effects and high frequency behaviour for the Schrödinger equation in star-shaped networks, *Port. Math.*, **72** (2015), 309–355.
- [AAN17] F. Ali Mehmeti, K. Ammari and S. Nicaise, Dispersive effects for the Schrödinger equation on a tadpole graph, *Journal of Mathematical Analysis and Applications*, **448** (2017), 262–280
- [AD18] K. Ammari and A. Duca. Controllability of localized quantum states on infinite graphs through bilinear control fields. *submitted: <https://arxiv.org/abs/1811.04273>*, 2018.
- [BL10] K. Beauchard and C. Laurent. Local controllability of 1D linear and nonlinear Schrödinger equations with bilinear control. *J. Math. Pures Appl. (9)*, 94(5):520–554, 2010.
- [Duc18a] A. Duca. Advances on the controllability of bilinear quantum. *submitted: <https://hal.archives-ouvertes.fr/hal-01830297>*, 2018.
- [Duc18b] A. Duca. Bilinear quantum systems on compact graphs: well-posedness and global exact controllability. *submitted: <https://hal.archives-ouvertes.fr/hal-01830297>*, 2018.
- [MN15] M. Morancey and V. Nersisyan. Simultaneous global exact controllability of an arbitrary number of 1D bilinear Schrödinger equations. *J. Math. Pures Appl. (9)*, 103(1):228–254, 2015.
- [Mor14] M. Morancey. Simultaneous local exact controllability of 1D bilinear Schrödinger equations. *Ann. Inst. H. Poincaré Anal. Non Linéaire*, 31(3):501–529, 2014.

Glassy properties of Anderson localization: pinning, avalanches and chaos

Abstract : localization is a key mechanism for the emergence of non-ergodicity in disordered quantum systems. In this seminar, I will present some numerical results which suggest that quantum localization shares a close analogy with the physics of spin glasses, another paradigm of non-ergodic behavior in classical disordered systems. I will focus on the quantum transport through a disordered two-dimensional sample in the localized regime and show that it verifies several important glassy properties: pinning, avalanches and chaos. This problem is addressed by following the well-known analogy between quantum localization and the physics of directed polymers, one of the simplest model of statistical physics in which disorder plays a non-trivial role, as in spin glasses. I will first recall how to observe the dominant paths taken by the transport and demonstrate that these paths verify pinning and avalanches. Then, I will characterize the spin glass chaos property: Two infinitesimally perturbed replicas of the same sample have their correlation which vanishes at the thermodynamic limit following a characteristic single parameter scaling law.