

GEOMETRIC STRUCTURES ON 3-MANIFOLDS

① Show that $\mathbb{P}^3\mathbb{R} \# \mathbb{P}^3\mathbb{R}$ is a quotient of the model geometry $S^2 \times \mathbb{R}$.

(This is the only nontrivial connected sum with a geometric structure.)

② For $1 \leq b < a$ coprime, describe a Seifert-fibered structure on

the lens space $L_{b/a} := S^3 / \langle \begin{pmatrix} \cos 2\pi/a & -\sin 2\pi/a & 0 & 0 \\ \sin 2\pi/a & \cos 2\pi/a & 0 & 0 \\ 0 & 0 & \cos 2\pi b/a & -\sin 2\pi b/a \\ 0 & 0 & \sin 2\pi b/a & \cos 2\pi b/a \end{pmatrix} \rangle$.

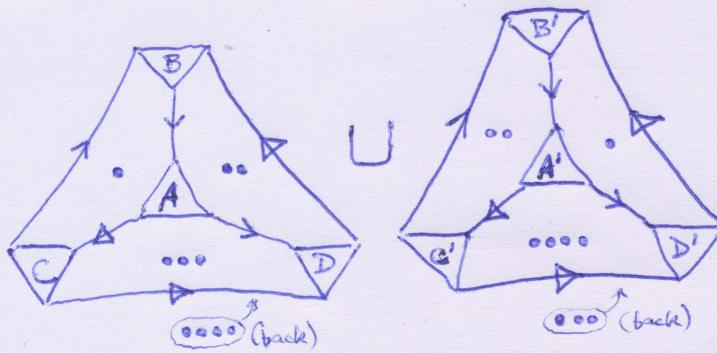
③ Show that $H^3, \mathbb{R}^3, S^3, H^2 \times \mathbb{R}, \widetilde{SL}_2, \text{Nil}, \text{Sol}$ (i.e. all 8 model geometries except $S^2 \times \mathbb{R}$) are Lie groups endowed with left-invariant metrics.

④ Let $\Gamma \subset SO_3 = S^3/\pm$ be the icosahedron group, and $C \subset SO_3$ finite.

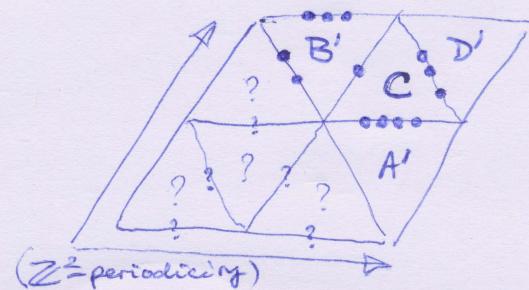
When is the $\Gamma \times C$ -action on SO_3 , given by $(\gamma, c) \cdot u = cu\gamma^{-1}$, free?

(Answer in terms of orders of elements in C). Can this happen for C non cyclic?

⑤ Consider the gluing of 2 regular ideal hyperbolic tetrahedra:



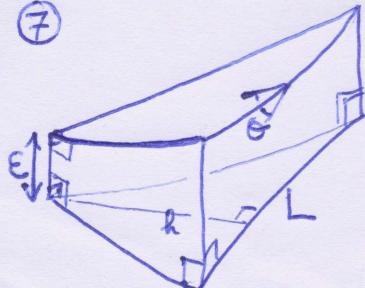
Check & complete the cusp pattern:



Show that this defines a hyperbolic metric on the punctured-torus bundle with monodromy $-(\frac{1}{1}, \frac{1}{1})$ (the "figure-8 sister manifold").

⑥ Estimate the rotational component of a Dehn filling of large slope $(p:q)$, in terms of $\tilde{q}^1 \bmod p$.
[the core geodesic of]

⑦



For a thin prism in H^3 , $\lim_{\epsilon \rightarrow 0} \frac{L \sinh(h)}{2} \epsilon$

and $\Theta = \frac{\pi}{2} - \epsilon \sinh(h) + o(\epsilon)$. (Can you show it?)

Deduce the Schlafli formula for polyhedra:

$$d\text{Vol} = -\frac{1}{2} \sum l_i d\theta_i \quad (\text{ } l_i = \text{edge lengths} \text{ } \theta_i = \text{dihedral angles})$$