GROWTH AND ISOPERIMETRIC INEQUALITIES IN GROUPS, "ASPECTS OF GEOMETRIC GROUP THEORY", IHES, JULY 2019

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Exercise 1. $G = \mathbb{Z}^d$. $s_1^{\pm 1}, s_2^{pm1}, \ldots s_d^{\pm 1}$ are the standard generators of $\mathbb{Z}^d$. Evaluate the growth function $v_{G,S}(n)$.

Exercise 2 (Unitriangular mathrices)**.** Let G_m be a group of unitriangular (upper triangular with 1 on the main diagonal) interger valued mathrices $(m+1) \times (m+1)$.

(1) Show that G is nilpotent.
(2) Find d such that the growth funciton of G_3 satisfies
$$C_1 n^d \leqslant v_{G,S}(n) \leqslant C_2 n^d.$$
(3) Find $d = d_m$ such that the growth funciton of G_m satisfies
$$C_1 n^d \leqslant v_{G,S}(n) \leqslant C_2 n^d.$$
(4) Show that G_m admits a dilation.

Exercise 3 (Wreath products)**.** Let $G = \mathbb{Z} \wr \mathbb{Z}$. Find a sequence of sets Ω_n, such that $|\partial \Omega_n|/|\Omega_n| \leqslant 1/n$ and such that $|\Omega_n| \leqslant C n^n$ for some $C > 0$ and all n.

Exercise 4. Let Γ_d be a regular tree of degree $d \geqslant 3$.

(1) Show that there exists $C_d > 0$ such that for any finite subset $V \subset \Gamma_d$ it holds
$$\frac{|\partial V|}{|V|} \geqslant C_d$$
(2) Find $\inf C_d$ of possible constants C_d as above.

Exercise 5 (Lower bounds for self-similar groups)**.**

(1) Let $k \geqslant 2$. Let $v(n)$ be an increasing function, such that $v(n) \to \infty$ as n tends to ∞. Assume that there exist $C_1, C_2 > 0$ such that for all n
$$v(n) \geqslant C_1 v^k(C_2 n).$$
Prove that there exist $\alpha > 0$ such that $v(n) \geqslant \exp(n^\alpha)$
(2) Let G, H be finitely generated groups. Assume that both G and G^k admit finite index subgroups isomorphic to H. Prove that there exist $\alpha > 0$ such that $v_{G,S}(n) \geqslant \exp(n^\alpha)$,

Exercise 6 (*, First Grigorchuk group))**.** Show that for any element g in the first Grigorchuk groups there exists n_g such that $g^{2^{n_g}} = e$. Evaluate n_g for $g = (ad)$; for $g = ac$; for $g = ab$.