Some problems on Boundaries

July 11, 2019

These are some problems related to the talk on the "Boundaries of hyperbolic and relatively hyperbolic groups", given at IHES in July 2019 by Genevieve Walsh.

1. Thin rectangles: show that, in a δ hyperbolic metric space, $\forall x, y, z, w$:

$$d(x,w) + d(y,z) \leq max(d(x,y) + d(w,z), d(x,z) + d(y,w)) + 2\delta$$

2. Show that the relation on sequences given by $(a_i) \sim (b_i)$ if $\lim(a_i, b_j)_x = \infty$ is transitive if X is δ-hyperbolic and not necessarily transitive if X is not hyperbolic (hint: try $\mathbb{R}^2$).

3. Show that if X is δ hyperbolic, then

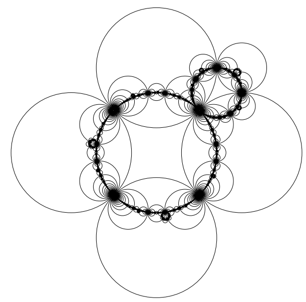
$$(x,y)_p \leq d(p,[x,y]) \leq (x,y)_p + \delta$$

4. Show that the boundary of a trivalent infinite tree is a Cantor set. (hint: try putting the basepoint in the middle of an edge)

5. Show that if Γ is hyperbolic, and $\partial\Gamma$ is a Cantor set then Γ is virtually free. (Hint: use theory of ends of groups also look in Scott and Wall about graphs of groups.)

6. Using the construction of gluing surfaces together (as in the three surfaces example) make a hyperbolic group whose boundary is not planar. (Hint: use Claytor's embedding theorem)

7. What is a possible relatively hyperbolic group pair such that $\partial(G, \mathcal{P})$ is the following limit set? The circles are not part of the limit set, they are a hint.



8. Use Curt McMullen's lim program, available at `http://www.math.harvard.edu/~ctm/programs/index.html`, to make some fun pictures of limit sets of Kleinian groups. This is what was used to make the above picture.