Models for $\mathrm{Out}(F_n)$

1. X = graph. Show $\pi_0 HE(X) \cong \mathrm{Out}(\pi_1 X)$
2. X = handlebody.
   - Show $\pi_0 \mathrm{Homeo}(X) \twoheadrightarrow \mathrm{Out}(\pi_1 X)$
   - A Dehn twist on ∂X extends to a homeomorphism of X. Show this is in the kernel of the map above

*3. X = punctured surface

A homeomorphism of X preserves the intersection form on $H_1(X)$, so the eigenvalues of the map on $H_1(X)$ are 1 or come in pairs (λ, λ^{-1})

Use this to find an automorphism of F_3 that cannot be realized on any surface.

Stallings folds

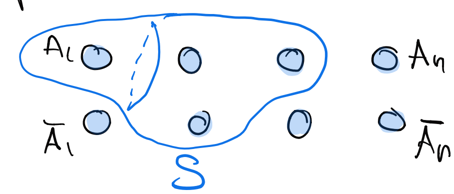
1. If $f: X \to R_n$ is a graph morphism that is locally injective, prove that f is a homeomorphism (Do not assume X is a rose)
2. - Use Stallings folds to factor the automorphism

   $$a \mapsto cba, \quad b \mapsto a^{-1}c^{-1}ba, \quad c \mapsto ca$$

   of $F_3 = F\langle a,b,c\rangle$ into a product of ρ_{xy}'s and λ_{xy}'s.
   -

1. Suppose $A_1, \dots, A_n$ are spheres cutting M_n into a $2n$-punctured 3-ball, and $\{a_1, \dots, a_n\}$ the dual basis of $\pi_1 M_n$.
Let S be a sphere in $M_n - \cup A_i$ separating A_1 and $\bar{A}_1$

eg:



Let f be a diffeomorphism exchanging A_1 and S and fixing all other A_i. What is the automorphism of $\pi_1 M_n = \langle a_1, \dots, a_n \rangle$ induced by f?

2. Use Whitehead's algorithm to decide whether

$$\begin{cases} a \mapsto ab \\ b \mapsto b a^{-1} b c^{-1} \\ c \mapsto cab \end{cases} \quad \text{and} \quad \begin{cases} a \mapsto ab \\ b \mapsto bcab \\ c \mapsto b^{-1} a^{-1} c \end{cases}$$

are automorphisms.

- Choose a random set of words and check whether it's a basis.

Lecture comprehension questions

1. Why does a Dehn twist on a 2-sphere in $M_n = \# S^1 \times S^2$ have order 2 in $\pi_0 \mathrm{Homeo}(M_n)$?
2. Describe an inner automorphism in terms of folding.
3. Prove the cut vertex lemma