

# Number Theory Days in Lille



## Rapport sur les contributions

ID de Contribution: 40

Type: **Non spécifié**

# The mean value of cubic L-functions over function fields

*lundi 8 juillet 2019 14:00 (1 heure)*

We present results about the first moment of L-functions associated to cubic characters over  $\mathbb{F}_q(T)$  when  $q$  is congruent to 1 modulo 3. The case of number fields was considered in previous work, but never for the full family of cubic twists over a field containing the third roots of unity. We will explain how to obtain an asymptotic formula with a main term, which relies on using results from the theory of metaplectic Eisenstein series about cancellation in averages of cubic Gauss sums over function fields. We will also discuss the case  $q$  congruent to 2 modulo 3.

## Summary

**Orateur:** LALÍN, Matilde (Université de Montréal)

**Classification de Session:** Analytic-additive number theory

ID de Contribution: 41

Type: Non spécifié

## Quadratic and Differential Forms over fields of characteristic 2

*mercredi 10 juillet 2019 11:00 (1 heure)*

In this talk  $F$  denotes a field of characteristic 2,  $W_q(F)$  the Witt of nonsingular quadratic forms over  $F$ ,  $W(F)$  the Witt ring of regular symmetric bilinear forms over  $F$ . For any integer  $m \geq 0$ , we denote by  $I_q^{m+1}(F)$  the group  $I^m F \otimes W_q(F)$ , where  $I^m F$  is the  $m$ -th power of the fundamental ideal  $I$  of  $W(F)$ , and  $\otimes$  is the module action of  $W(F)$  on  $W_q(F)$ .

Given a field extension  $K$  of  $F$ , we have two homomorphisms  $i_K : W_q(F) \rightarrow W_q(K)$  and  $j_K : W(F) \rightarrow W(K)$  induced by the inclusion  $F \subset K$ . A natural question that arises is to compute the kernels of these homomorphisms. This is an outstanding problem in the theory of quadratic forms.

A way to study the above problems is based on differential forms where we use a celebrated result of K. Kato which gives connections between quadratic forms and differential forms, and, by this way, obtain results in terms of graded Witt groups.

Following this approach we will provide examples who illustrate the use of tools.

### Summary

**Orateur:** ARAVIRE, Roberto (Universidad Arturo Prat)

**Classification de Session:** Quadratic forms

ID de Contribution: 42

Type: **Non spécifié**

## Pretorsion theories in general categories

*mercredi 10 juillet 2019 14:00 (1 heure)*

We will present a notion of (pre)torsion theory in general categories and two interesting examples of such pretorsion theories. Torsion theories in arbitrary categories have been studied by Grandis, Janelidze, Márki and several others. Our main examples will be in the category of preordered sets and the category of finite algebras with one operation, unary, and no axioms (i.e., the category of all mappings  $f: X \rightarrow X$ , where  $X$  is a finite set). Our results appear in joint papers with Carmelo Antonio Finocchiaro, Marino Gran and Leila Heidari Zadeh.

### Summary

**Orateur:** FACCHINI, Alberto (Università di Padova)

**Classification de Session:** Noncommutative algebra

ID de Contribution: 43

Type: **Non spécifié**

## Galois theory for cowreaths

*mercredi 10 juillet 2019 15:30 (1 heure)*

The Galois theory for monoidal cowreaths is developed. Cleft cowreaths are introduced in this context and its relation with the normal basis property investigated. The connection of this class of cowreath with some wreath algebra structures is obtained. Finally, several applications to quasi-Hopf algebras will be discussed. This is a joint work with D. Bulacu.

### Summary

**Orateur:** TORRECILLAS, Blas (Universidad de Almería)

**Classification de Session:** Noncommutative algebra

ID de Contribution: 44

Type: **Non spécifié**

## Counting points on algebraic varieties over the rational numbers

*jeudi 11 juillet 2019 15:00 (1 heure)*

For both conceptual and practical reasons it is useful to have estimates on the number of points of algebraic varieties over  $\mathbb{Q}$ , usually phrased in terms of asymptotics as the height of points increases. I will present a new such estimate, improving previous results by Bombieri, Pila, Heath-Brown, Browning, Salberger, Walsh and others. Time permitting, I will present an application to bounding the 2-torsion part of class groups of number fields.

This is joint work with Wouter Castryck, Raf Cluckers and Kien Huu Nguyen.

### Summary

**Orateur:** DITTMANN, Philip (KU Leuven)

**Classification de Session:** Arithmetic geometry and Galois theory

ID de Contribution: 45

Type: **Non spécifié**

# Applications of the orbit method to the analysis of automorphic forms

*mardi 9 juillet 2019 09:45 (1 heure)*

I will discuss joint work with Akshay Venkatesh in which we use microlocalized test vectors (inspired by the orbit method) and Ratner theory to study mean values of L-functions on Gross–Prasad pairs. I will also indicate some further applications of these methods, such as to the quantum variance problem for Hecke–Maass eigenforms.

## Summary

**Orateur:** NELSON, Paul (ETH Zurich)

**Classification de Session:** Galois representations and modular forms

ID de Contribution: 46

Type: **Non spécifié**

## Image of two-dimensional pseudorepresentations

*jeudi 11 juillet 2019 09:45 (1 heure)*

There is a general philosophy that the image of a Galois representation should be as large as possible, subject to its symmetries. This can be seen in Serre's open image theorem for non-CM elliptic curves, Ribet and Momose's work on Galois representations attached to modular forms, and recent work of the speaker and Conti-Iovita-Tilouine on Galois representations attached to  $p$ -adic families of modular forms. Recently, Bellaïche developed a way to measure the image of an arbitrary pseudorepresentation taking values in a local ring  $A$ . Under the assumptions that  $A$  is a domain and the residual representation is not too degenerate, we explain how the symmetries of such a pseudorepresentation are reflected in its image. This is joint work with Andrea Conti and Anna Medvedovsky.

### Summary

**Orateur:** LANG, Jaclyn (Université Paris 13)

**Classification de Session:** Galois representations and modular forms



ID de Contribution: 47

Type: **Non spécifié**

## On the distribution of the maximum of partial sums of exponential sums

*lundi 8 juillet 2019 11:00 (1 heure)*

In analogy with multiplicative character sums, we investigate the distribution of the maximum of partial sums of various families of exponential sums. We obtain precise estimates on the distribution function in a large uniform range, in the case where the Fourier transforms of these exponential sums are real valued, and satisfy some “natural” hypotheses. Important examples include Birch sums and Kloostermann sums. The proof uses a blend of analytic and probabilistic techniques together with deep tools from algebraic geometry. As an application, we exhibit large values of partial sums of these exponential sums, which we believe are best possible. This is a joint work with Pascal Autissier and Dante Bonolis.

### Summary

**Orateur:** LAMZOURI, Youness (Université de Lorraine / York University)

**Classification de Session:** Analytic-additive number theory

ID de Contribution: 48

Type: **Non spécifié**

## Quadratic forms and diophantine sets

*mercredi 10 juillet 2019 09:45 (1 heure)*

The interplay between valuations and certain geometrically rational varieties, in particular quadrics, has turned out to be very fruitful for proving that certain subsets of fields are existentially definable or diophantine. In particular, this has been used by J. Koenigsmann to prove that  $\mathbb{Q}\setminus\mathbb{Z}$  is diophantine in  $\mathbb{Q}$ . His proof combines several ingredients from classical number theory, involving in particular the Hasse-Minkowski local-global principle for quadratic forms. In my talk I want to highlight some ingredients of proofs for showing that certain subsets of fields are diophantine and some interesting questions for quadratic forms arising from this context.

### Summary

**Orateur:** BECHER, Karim Johannes (Universiteit Antwerpen)

**Classification de Session:** Quadratic forms

ID de Contribution: 49

Type: **Non spécifié**

## Density of specialization sets and Hasse principle in families of twisted Galois covers

*jeudi 11 juillet 2019 11:00 (1 heure)*

We discuss results on the structure of the set of all specializations of a Galois cover  $f: X \rightarrow \mathbb{P}^1$  with group  $G$  over a number field  $k$ . Hilbert's irreducibility theorem yields that this set contains infinitely many  $G$ -extensions of  $k$ . A natural question is then how large this specialization set is compared to the set of all  $G$ -extensions. We present evidence for the following conjecture: If  $f$  is not of a very special form, then the specialization set is "small" in a density sense (when counted by discriminant). For  $k = \mathbb{Q}$ , we make concrete progress on this conjecture, essentially reducing it to the abc-conjecture. This connects to a related "sparsity" result in a recent joint work with Dèbes, Legrand and Neftin about the set of rational pullbacks of Galois covers, and also extends results by Granville about the special case of hyperelliptic curves.

As an application of our result, we show that in certain families of "twisted Galois covers", there are "many" curves failing the Hasse principle.

This is joint work with F. Legrand.

### Summary

**Orateur:** KÖNIG, Joachim (KAIST)

**Classification de Session:** Arithmetic geometry and Galois theory

ID de Contribution: 50

Type: **Non spécifié**

## A Minkowski-type result for linearly independent subsets of ideal lattices

*lundi 8 juillet 2019 09:45 (1 heure)*

We estimate, in a number field, the maximal number of linearly independent elements with prescribed bounds on their valuations. As a by-product, we obtain new bounds for the successive minima of ideal lattices. Our arguments combine group theory, ramification theory, and the geometry of numbers. This is joint work with Mikołaj Frączyk and Péter Maga.

### Summary

**Orateur:** HARCOS, Gergely (Alfréd Rényi Institute of Mathematics)

**Classification de Session:** Analytic-additive number theory

ID de Contribution: 51

Type: **Non spécifié**

## Bootstrapping partition regularity of linear systems

*lundi 8 juillet 2019 15:30 (1 heure)*

Suppose that  $A$  is a  $k \times d$  matrix of integers such that for any  $r$  there is some  $N$  such that any  $r$ -colouring of  $\{1, \dots, N\}$  contains a monochromatic solution to  $A$ , meaning there is a colour class  $C$  and  $x \in C^d$  such that  $Ax = 0$ . Not all matrices  $A$  have this property (consider, for example, when all the entries of  $A$  are positive), but when they do they are called partition regular. In this talk we consider what bounds can be given on  $N$  in terms of  $r$  (and  $A$ ) when  $A$  is partition regular.

### Summary

**Orateur:** SANDERS, Tom (University of Oxford)

**Classification de Session:** Analytic-additive number theory

ID de Contribution: 52

Type: **Non spécifié**

## Overconvergent cohomology and automorphic p-adic L-functions

*mardi 9 juillet 2019 15:30 (1 heure)*

p-adic L-functions attached to automorphic representations and p-adic families of them, provide powerful tools to attack important problems such as Birch-Swinnerton-Dyer and Bloch-Kato conjectures. However, they are hard to construct and in fact beyond the case  $GL(2)$  the theory is poorly understood.

In this talk I will describe an approach based on the study of the overconvergent cohomology of locally symmetric spaces. This approach was introduced by G. Stevens in the nineties and the most general constructions available for  $GL(2)$  were based on it. Then I will describe an ongoing joint work with M. Dimitrov and C. Williams in which we construct p-adic L-functions for certain cuspidal automorphic representations of  $GL(2n)$  by the use of convergent cohomology. This construction extends previous results of Gehrman/Dimitrov-Januszewski-Raghuram to the non-ordinary setting and allows variation in p-adic families.

### Summary

**Orateur:** BARRERA, Daniel (Universidad de Santiago de Chile)

**Classification de Session:** Galois representations and modular forms

ID de Contribution: 53

Type: **Non spécifié**

## The Mumford–Tate conjecture implies the algebraic Sato–Tate conjecture

*jeudi 11 juillet 2019 13:30 (1 heure)*

The famous Mumford–Tate conjecture asserts that, for every prime number  $\ell$ , Hodge cycles are  $\mathbb{Q}_\ell$  linear combinations of Tate cycles, through Artin’s caparisons theorems between Betti and étale cohomology. The algebraic Sato–Tate conjecture, introduced by Serre and developed by Banaszak and Kedlaya, is a powerful tool in order to prove new instances of the generalized Sato–Tate conjecture. This previous conjecture is related with the equidistribution of Frobenius traces. Our main goal is to prove that the Mumford-Tate conjecture for an abelian variety  $A$  implies the algebraic Sato-Tate conjecture for  $A$ . The relevance of this result lies mainly in the fact that the list of known cases of the Mumford-Tate conjecture was up to now a lot longer than the list of known cases of the algebraic Sato-Tate conjecture.

This is a joint work with Johan Commelin.

### Summary

**Orateur:** CANTORAL-FARFAN, Victoria (ICTP)

**Classification de Session:** Arithmetic geometry and Galois theory

ID de Contribution: 54

Type: **Non spécifié**

## Hopf Ore extensions and the antipode

*mercredi 10 juillet 2019 16:45 (1 heure)*

In this talk we deal with Hopf Ore extensions, the role they play in the classification of low dimensional Hopf algebras, and the property of a Hopf algebra to be almost involutive (meaning that the square of the antipode has a square root that is an automorphism of Hopf algebras).

### Summary

**Orateur:** FERRER, Walter (Universidad de la República)

**Classification de Session:** Noncommutative algebra



ID de Contribution: 55

Type: **Non spécifié**

## **Birch and Swinnerton-Dyer Formula for modular forms of arbitrary weight in the cases of analytic ranks 0 and 1**

*mardi 9 juillet 2019 11:00 (1 heure)*

In this talk, I will report on recent results on the computation of the  $p$ -part of the leading term of the  $L$ -function of a modular form of arbitrary weight at the central point in the cases when the order of vanishing is at most one. Unlike the classical case of weight 2 modular forms, qualitatively different arguments are needed in the higher-weight case. After explaining the difference, I will indicate how one can use level-raising and (non-ordinary)  $p$ -adic deformations together with some of the arguments in weight 2 to obtain results in the case of general weights.

This is joint work with Chris Skinner and Xin Wan.

### **Summary**

**Orateur:** JETCHEV, Dimitar (EPFL)

**Classification de Session:** Galois representations and modular forms

ID de Contribution: 56

Type: **Non spécifié**

## On a conjecture of Gross and explicit formula for Gross-Stark units

*mardi 9 juillet 2019 14:00 (1 heure)*

In this talk I will report on my joint work in progress with Samit Dasgupta on the tower of fields conjecture first formulated by Gross. This proves a conjecture of Dasgupta on explicit p-adic analytic formulae for Gross-Stark units. These units, when considered for all primes of a totally real number field  $F$ , generate the maximal abelian CM extension of  $F$  and therefore our work can be considered as giving a p-adic analytic solution to Hilbert's 12th problem.

### Summary

**Orateur:** KAKDE, Mahesh (King's College London)

**Classification de Session:** Galois representations and modular forms