Gibbs measures of nonlinear Schrödinger equations as limits of many-body quantum states in dimension $d \leq 3$.

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Gibbs measures of nonlinear Schrödinger equations are a fundamental object used to study low-regularity solutions with random initial data. In the dispersive PDE community, this point of view was pioneered by Bourgain in the 1990s. We prove that the Gibbs measures of nonlinear Schrödinger equations arise as high-temperature limits of appropriately modified thermal states in many-body quantum mechanics. We consider bounded defocusing interaction potentials and work either on the d-dimensional torus or on $\mathbb{R}^d$ with a confining potential. The analogous problem for $d = 1$ and in higher dimensions with smooth non translation-invariant interaction potentials was previously studied by Lewin, Nam, and Rougerie by means of a variational formulation. In our work, we apply a perturbative expansion in the interaction, motivated by ideas from field theory. The terms of the expansion are analyzed using a diagrammatic representation and their sum is controlled using Borel resummation techniques. When $d = 2, 3$, we apply a Wick ordering renormalisation procedure. Moreover, in the one-dimensional setting, our methods allow us to obtain a microscopic derivation of time-dependent correlation functions for the nonlocal cubic nonlinear Schrödinger equation.

This is joint work with Jürg Fröhlich, Antti Knowles, and Benjamin Schlein.