Title: Ginzburg-Landau relaxation for harmonic maps valued into manifolds

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Abstract: We will look at the classical problem of minimizing the Dirichlet energy of a map $u : \Omega \subset \mathbb{R}^2 \to N$ valued into a compact Riemannian manifold N and subjected to a Dirichlet boundary condition $u = \gamma$ on $\partial\Omega$. It is well known that if γ has a non-trivial homotopy class in N, then there are no maps in the critical Sobolev space $H^1(\Omega, N)$ such that $u = \gamma$ on $\partial\Omega$. To overcome this obstruction, a way is to rather consider a relaxed version of the Dirichlet energy leading to singular harmonic maps with a finite number of topological singularities in Ω. This was done in the 90's in a pioneering work by Bethuel-Brezis-Helein in the case $N = \mathbb{S}^1$, related to the Ginzburg-Landau theory. In general, we will see that minimizing the energy leads at main order to a non-trivial combinatorial problem which consists in finding the energetically best topological decomposition of the boundary map γ into minimizing geodesics in N. Moreover, we will introduce a renormalized energy whose minimizers correspond to the optimal positions of the singularities in Ω.