One day conference on Calculus of Variations 2018. Abstracts

1. Filippo Santambrogio: *Least gradient functions and optimal transport.*

The least gradient problem (minimizing the BV norm with given boundary data), motivated by both image processing applications and connections with minimal surfaces, is known to be equivalent, in the plane, to the Beckmann minimal-flow problem with source and target measures located on the boundary of the domain. Sobolev regularity of functions of least gradient is equivalent in this setting to L^p-bounds on the solution of the Beckmann problem (i.e. on the transport density) and can be attacked with techniques which are now standard in optimal transport. From the transport point of view, the novelty of the estimates that I will present, coming from a joint paper with S. Dweik, lies in the fact they are obtained fro transport between measures which are concentrated on the boundary. From the BV point of view, a new result is the $W^{1,p}$-regularity of the least gradient function whenever the boundary datum is $W^{1,p}$ as a 1D function: moreover, the optimal transport framework is strong enough to deal with arbitrary strictly convex norms instead of the Euclidean one with almost no effort.

2. Vincent Millot: *Applications s-harmoniques, régularité et singularités.*

Dans cet exposé, je présenterai des résultats de régularité partielle pour les applications s-harmoniques à valeurs dans une sphère. JâĂŹexpliquerai également un résultat de classification des singularités dans le cas $s = 1/2$, pour des applications minimisantes dâĂŹun domaine plan à valeurs dans le cercle.

3. Flaviana Iurlano: *Concentration analysis of brittle damage.*

This talk is concerned with an asymptotic analysis of a variational model of brittle damage, when the damaged zone concentrates into a set of zero Lebesgue measure, and, at the same time, the stiffness of the damaged material becomes arbitrarily small. In a particular non-trivial regime, concentration leads to a limit energy with linear growth as typically encountered in plasticity. I will show that, while the singular part of the limit energy can be easily described, the identification of the bulk part of the limit energy requires a subtler analysis of the concentration properties of the displacements. I will present a candidate bulk density that arises from a possible scenario. This is an ongoing work with J.-F. Babadjian and F. Rindler.

4. Antonin Monteil: *Ginzburg-Landau relaxation for harmonic maps valued into manifolds.*

We will look at the classical problem of minimizing the Dirichlet energy of a map $u : \Omega \subset \mathbb{R}^2 \to N$ valued into a compact Riemannian manifold N and subjected to a Dirichlet boundary condition $u = \gamma$ on $\partial\Omega$. It is well known that if γ has a non-trivial homotopy class in N, then there are no maps in the critical Sobolev space

$H^1(\Omega, N)$ such that $u = \gamma$ on $\partial\Omega$. To overcome this obstruction, a way is to rather consider a relaxed version of the Dirichlet energy leading to singular harmonic maps with a finite number of topological singularities in Ω. This was done in the 90's in a pioneering work by Bethuel-Brezis-Helein in the case $N = \mathbb{S}^1$, related to the Ginzburg-Landau theory. In general, we will see that minimizing the energy leads at main order to a non-trivial combinatorial problem which consists in finding the energetically best topological decomposition of the boundary map γ into minimizing geodesics in N. Moreover, we will introduce a renormalized energy whose minimizers correspond to the optimal positions of the singularities in Ω.

5. Élie Bretin: *Phase field approximation of the Steiner problem: a numerical investigation.*

We analyze in this talk the abitity of different phase field models to approximate solutions of the Steiner problem. In particular, we will first focus on the recent phase field model introduced by Bonnivard, Lemenant and Santanbrogio that couples a Cahn Hilliard type functional with a penalyzed term forcing the compacness of the desired set. We then propose and justify the convergence of some slightly modified versions, which improve the regularity of its solution and use a better uniform contribution of the penalized term. In particular, we show that this phase field model are able to consider a large number of points in dimension 2 and 3. Finally, we also propose in comparison some numerical experiments using the approach of Chambolle, Ferrari and Merlet.

6. Antonin Chambolle: *A « Total variation »with curvature penalization.*

In this joint work with T. Pock (TU Graz) we propose a convex variant of the total variation which penalizes the curvature of the level lines, and is based on a Gauss map (lifting) of curves to represent curvature dependent energies as convex functionals. Applications to "image inpainting" are presented.