Google matrix analysis of the World Trade Network (WTN)

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Google Matrix: fundamentals, applications and beyonds

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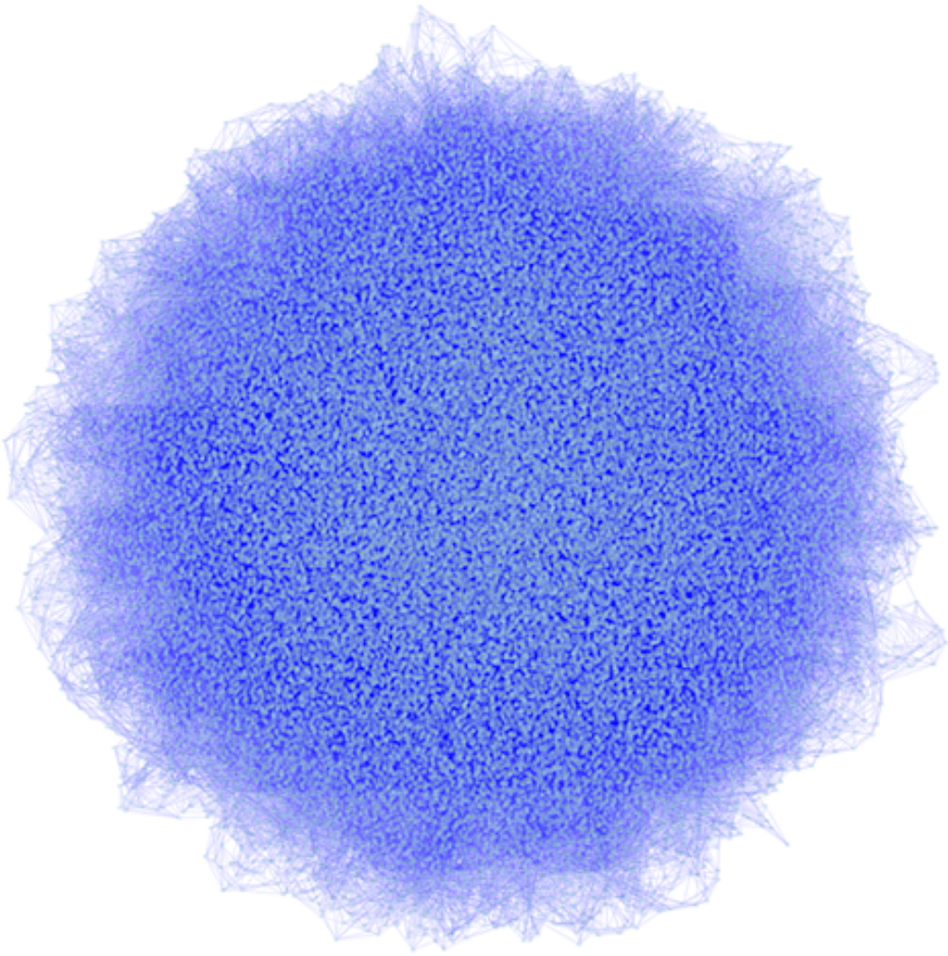
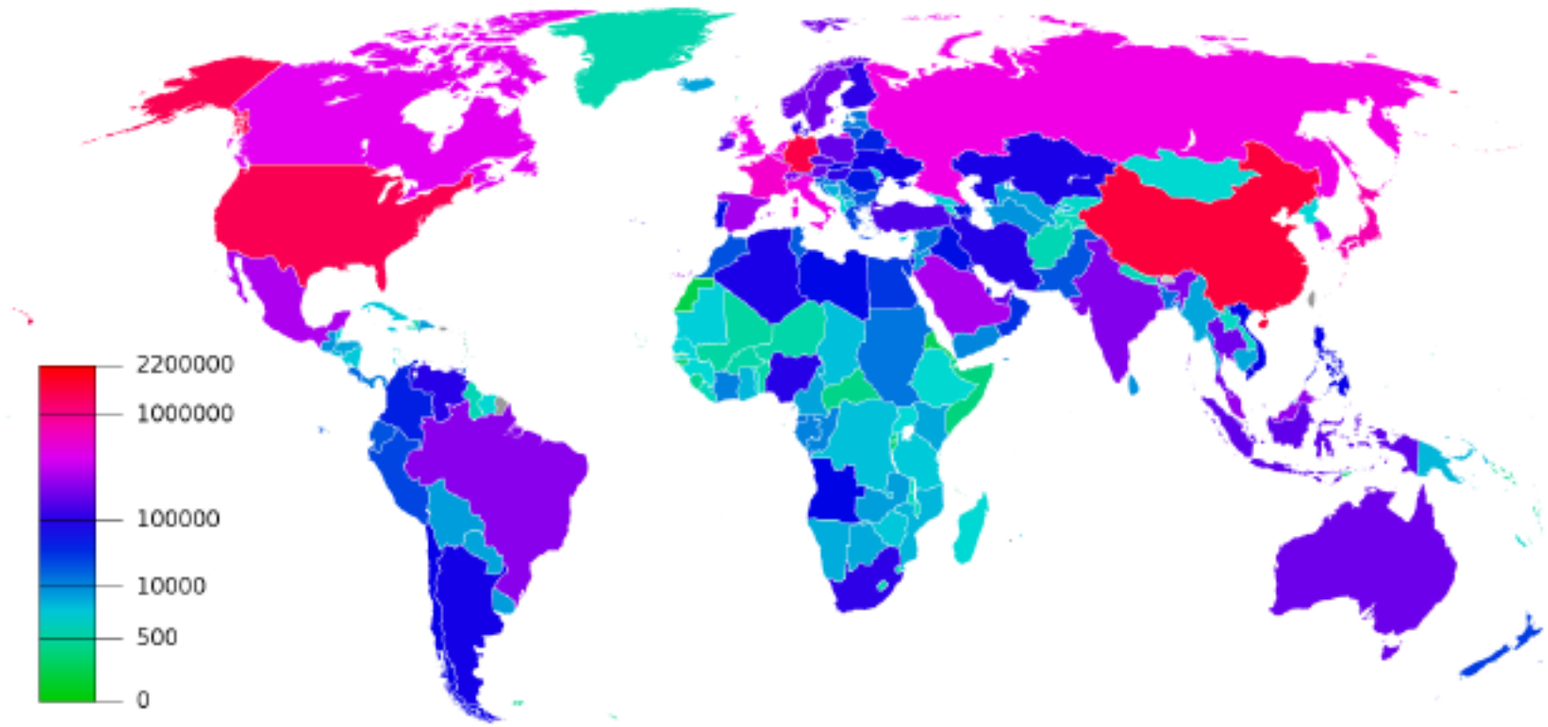
IHES, Paris, France

Motivations

Google approach to the World Trade Network



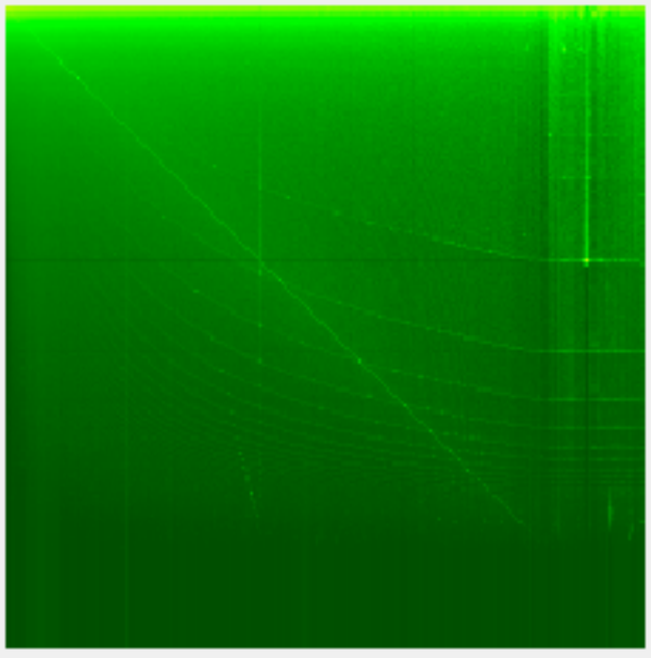


Analyze, model, simplify, visualize and highlight useful information of large networks using nontrivial and global properties

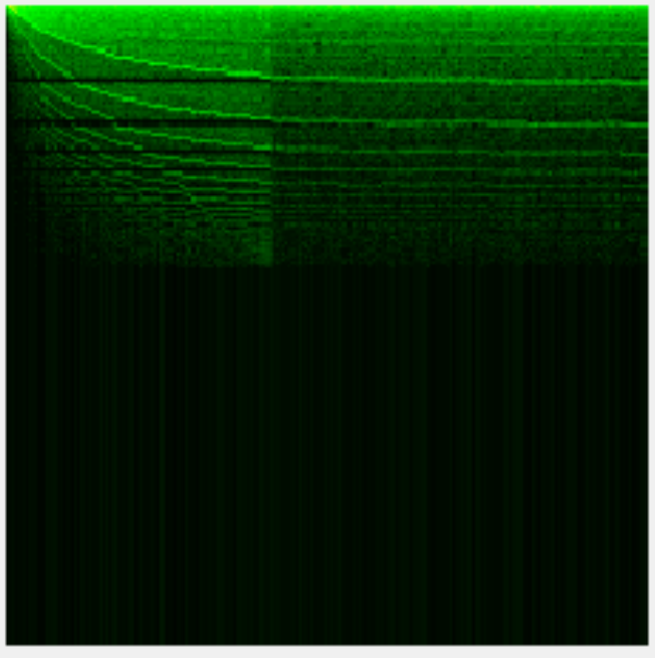
Google Matrices

- www: Wikipedia (diff. lang.), Universities (UK, France)
- Operating Systems: Kernel Linux; NOMAO (spot recommendation)
- Bitcoin network
- Ulam method for Dynamical Systems: Standard Map

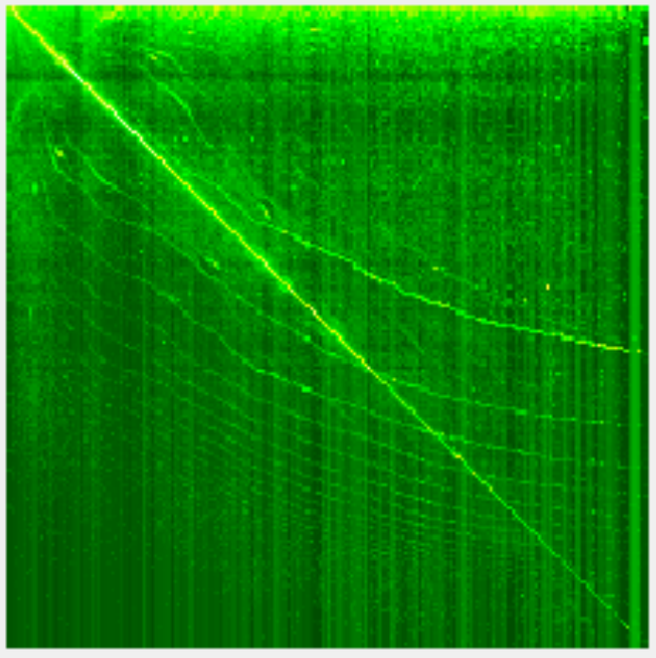
Wikipedia



Kernel Linux V2.6



Cambridge 2006



World Trade Network

Import-Export trade database:

United Nation Commodities Trade Network

HTTP://COMTRADE.UN.ORG/DB/

* Each year from 1962 to 2017
* All UN countries: ~ 230 (Nc=227 in 2008)
* Product classification (SITC Rev. 1): Np=61
* Trade volume is given in USD (N=13847 x 50 years)

Money Matrices

$$M_{c,c'} = \$\ (c' \to c) \qquad M^{p}_{c,c'} = \$\ (c' \to c)$$

Google Matrix of WTN

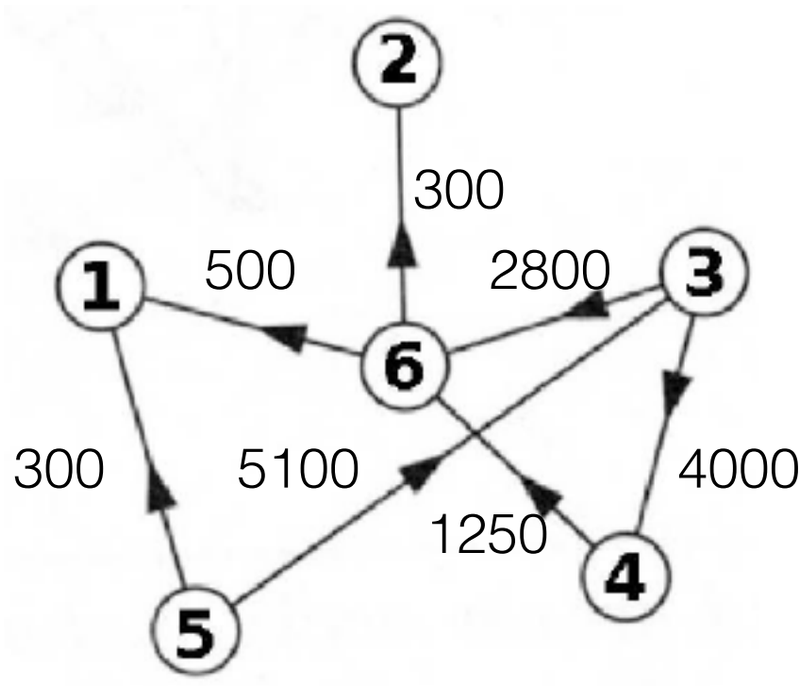
Brin and Page (1998)

Introduced by Frahm yesterday

centrality measure:
Spectral Indices

- directed networks
- easy to compute
- incoming links
- non-local properties

directed network



Money matrix

$$M = \begin{pmatrix} 0 & 0 & 0 & 0 & 300 & 500 \\ 0 & 0 & 0 & 0 & 0 & 300 \\ 0 & 0 & 0 & 0 & 5100 & 0 \\ 0 & 0 & 4000 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2800 & 1250 & 0 & 0 \end{pmatrix}$$

Google Matrix

Brin and Page (1998)

centrality measure:
Spectral Indices

- directed networks
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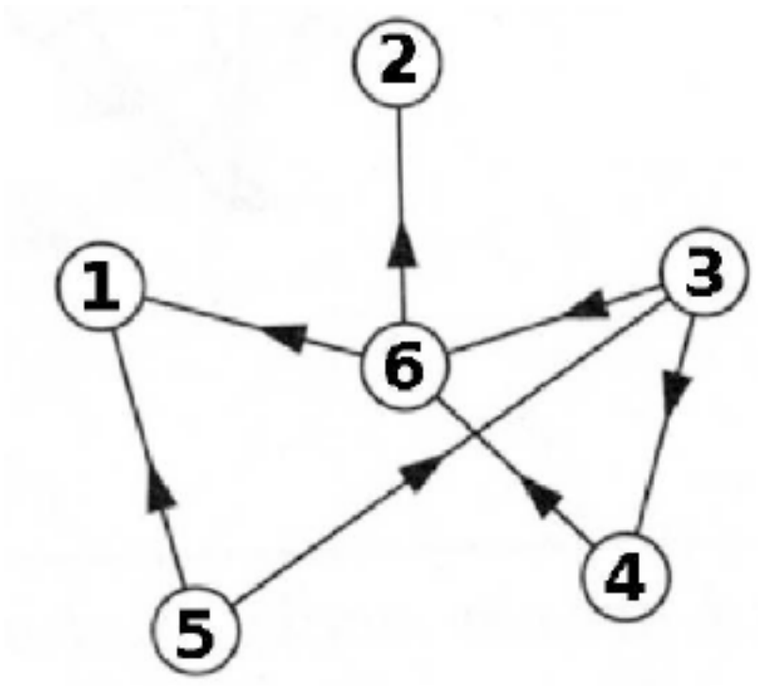
directed network



weighted adjacency matrix and dangling nodes

$$S = \begin{pmatrix} 1/6 & 1/6 & 0 & 0 & 0.055 & 0.625 \\ 1/6 & 1/6 & 0 & 0 & 0 & 0.375 \\ 1/6 & 1/6 & 0 & 0 & 0.945 & 0 \\ 1/6 & 1/6 & 0.5888 & 0 & 0 & 0 \\ 1/6 & 1/6 & 0 & 0 & 0 & 0 \\ 1/6 & 1/6 & 0.412 & 1 & 0 & 0 \end{pmatrix}$$

- $\sum_i S_{i,j} = 1$
- Perron-Frobenius (non-negative)
- $\lambda_1 = 1$ (degeneracy)

Google Matrix

Brin and Page (1998)

PageRank

$$\mathbf{G}P = P$$

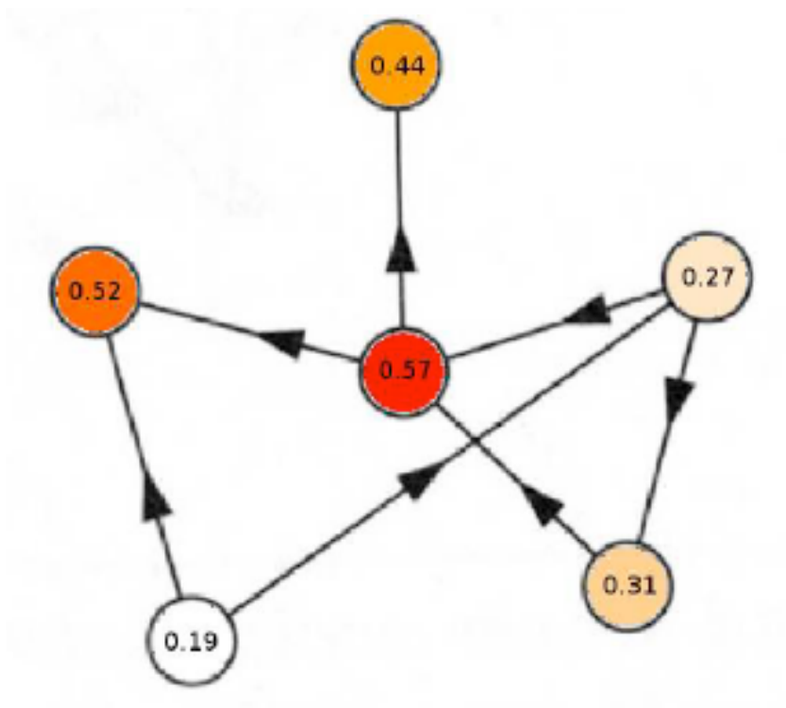
centrality measure:

Spectral Indices

- directed networks
- easy to compute
- incoming links
- non-local properties

$$\mathbf{G} = \alpha\mathbf{S} + (1-\alpha)\mathbf{E}/N \ (\alpha = 0.85)$$

directed network

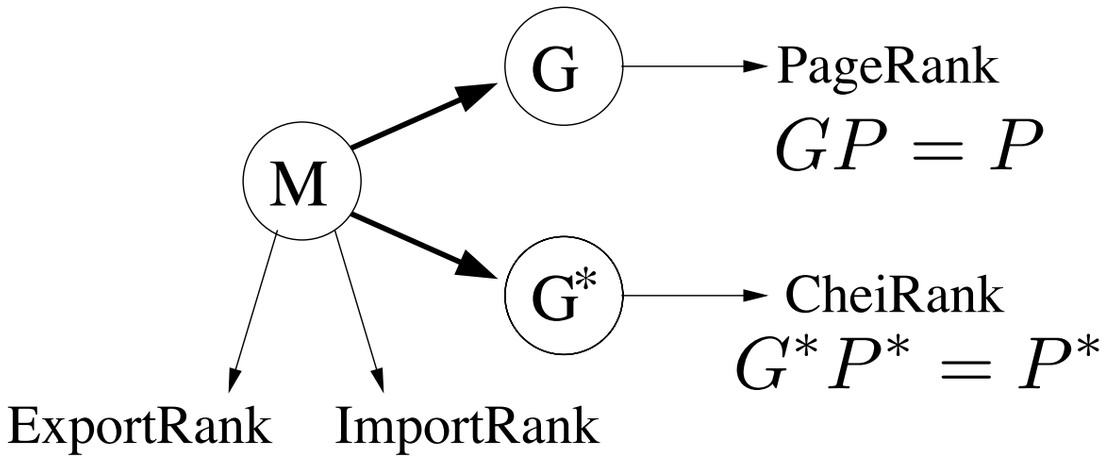


Google Matrix

$$G = \begin{pmatrix} 0.166 & 0.166 & 0.025 & 0.025 & 0.07175 & 0.556 \\ 0.166 & 0.166 & 0.025 & 0.025 & 0.025 & 0.344 \\ 0.166 & 0.166 & 0.025 & 0.025 & 0.828 & 0.025 \\ 0.166 & 0.166 & 0.5248 & 0.025 & 0.025 & 0.025 \\ 0.166 & 0.166 & 0.025 & 0.025 & 0.025 & 0.025 \\ 0.166 & 0.166 & 0.375 & 0.875 & 0.025 & 0.025 \end{pmatrix}$$

- $\alpha \to S, (1-\alpha) \to$ random node
- Perron-Frobenius (positive) $\lambda_1 = 1$
- $\Delta \geq (1-\alpha)$ (global convergence)

Google matrix of the WTN

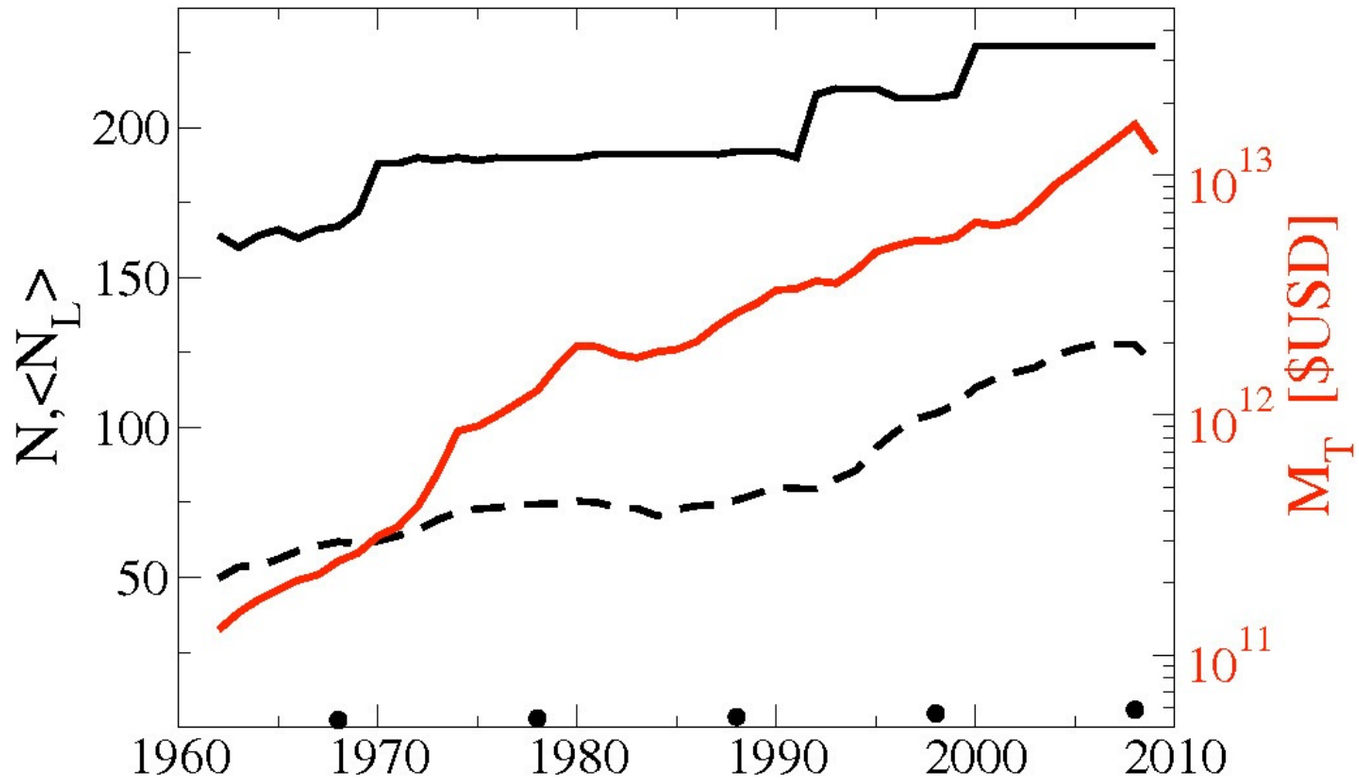


$(\tilde{K}, \tilde{K}^*)$

ImportRank, ExportRank

$(K, K*)$

PageRank, CheiRank

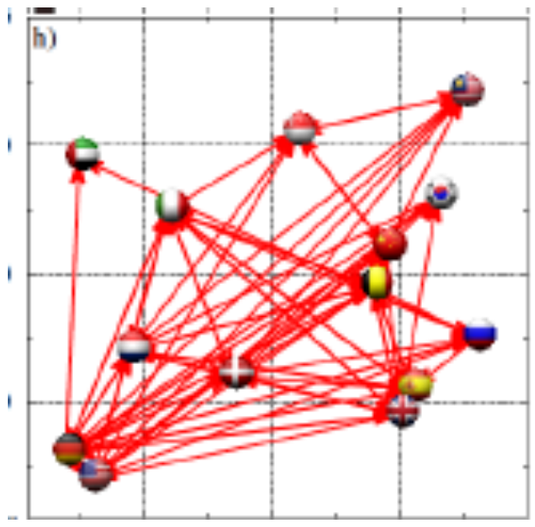


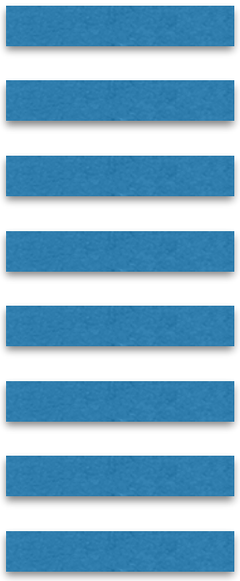
Democracy in countries but not in products

Google matrix of the WTN

1) WTN (all com. or 1 prod)
$N=N_c=227$

non-interacting products



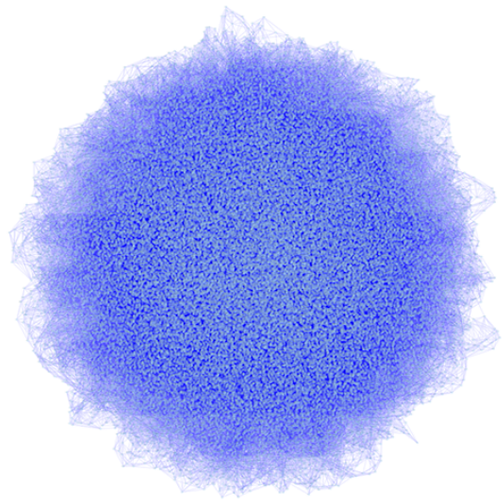


L. Ermann and D.L. Shepelyansky, APPA, Vol. 120, A-158 (2011),
http://www.quantware.ups-tlse.fr/QWLIB/tradecheirank

2) WTN (multiprod)
$N_p=61$; $N=13847$

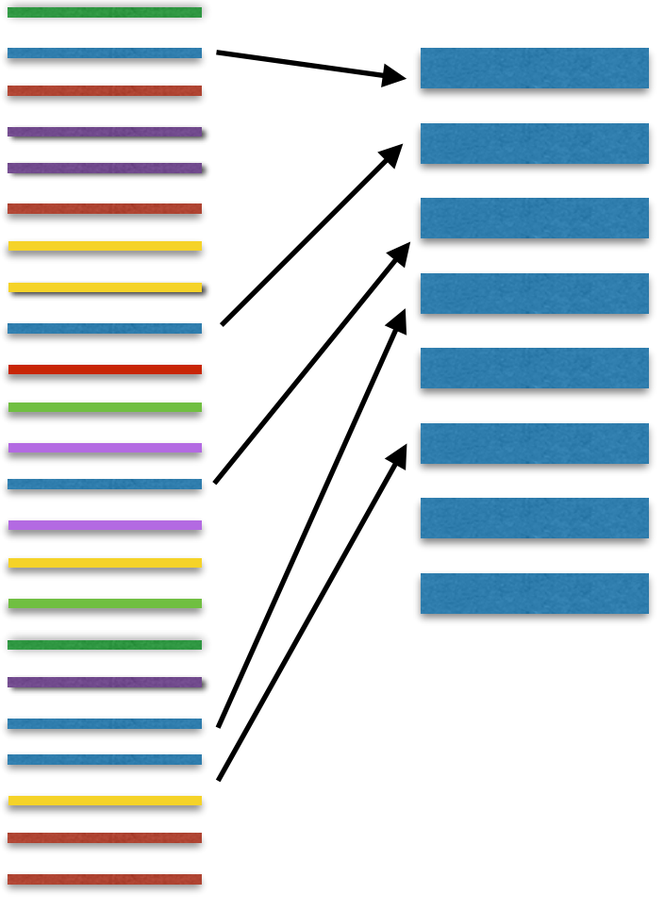
personalized vector ⟶ prop to Vp of each c

2nd iteration ⟶ reduced Pp for all c



$$v_i = \frac{V_c^p}{N_c \sum_{p'} V_c^{p'}}\,,\; v_i^* = \frac{V_c^{*p}}{N_c \sum_{p'} V_c^{*p'}}\,,$$

$$v'(i) = \frac{P_p}{N_c}\,,\; v'^*(i) = \frac{P_p^*}{N_c}\,.$$



L. Ermann and D.L. Shepelyansky, EPJB (2015).

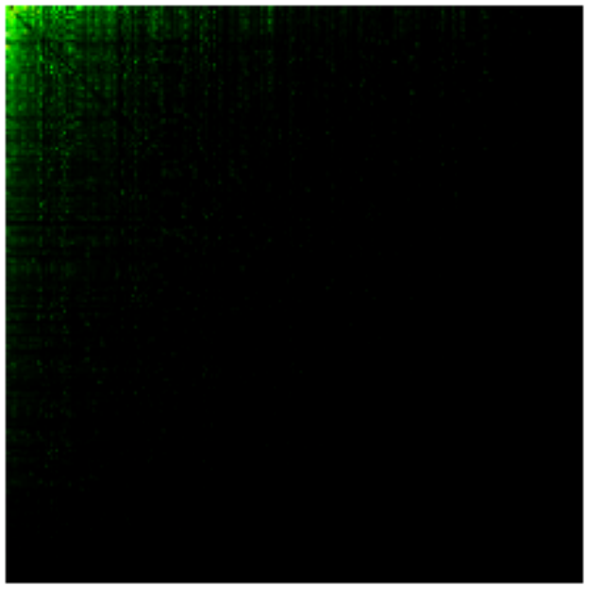
all commodities and given products (N=227)

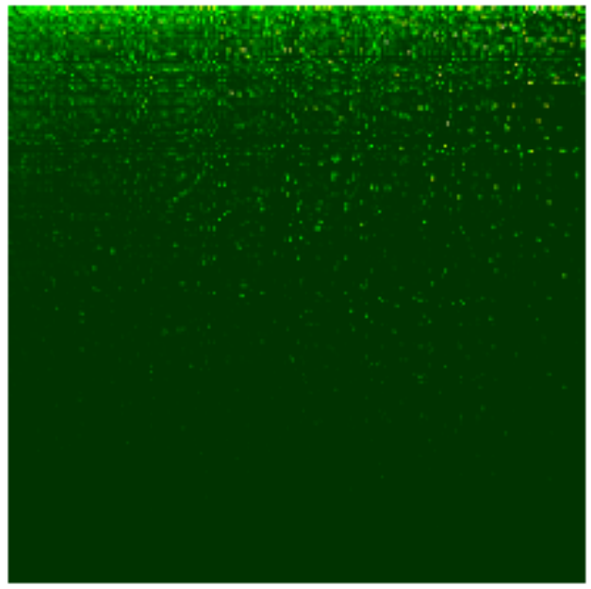
M

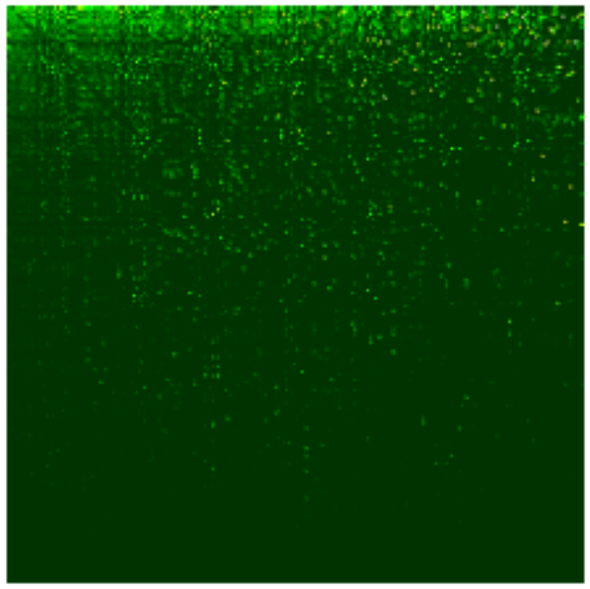
G

G*

all commodities

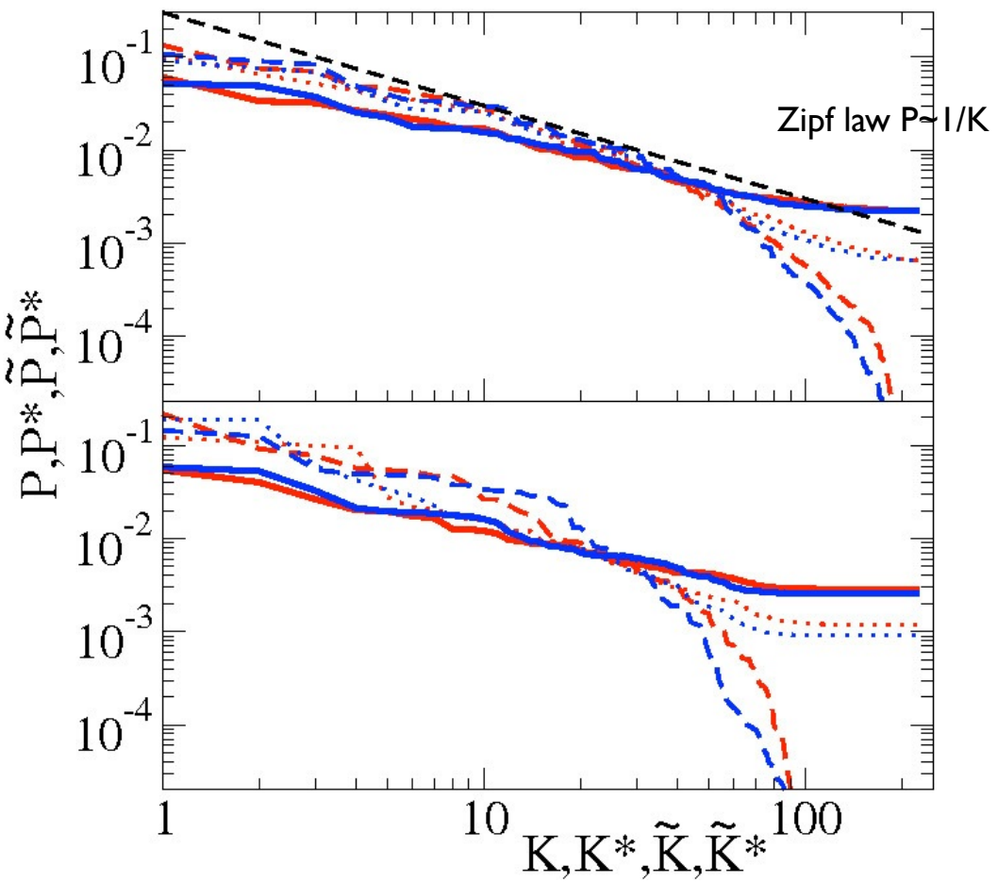


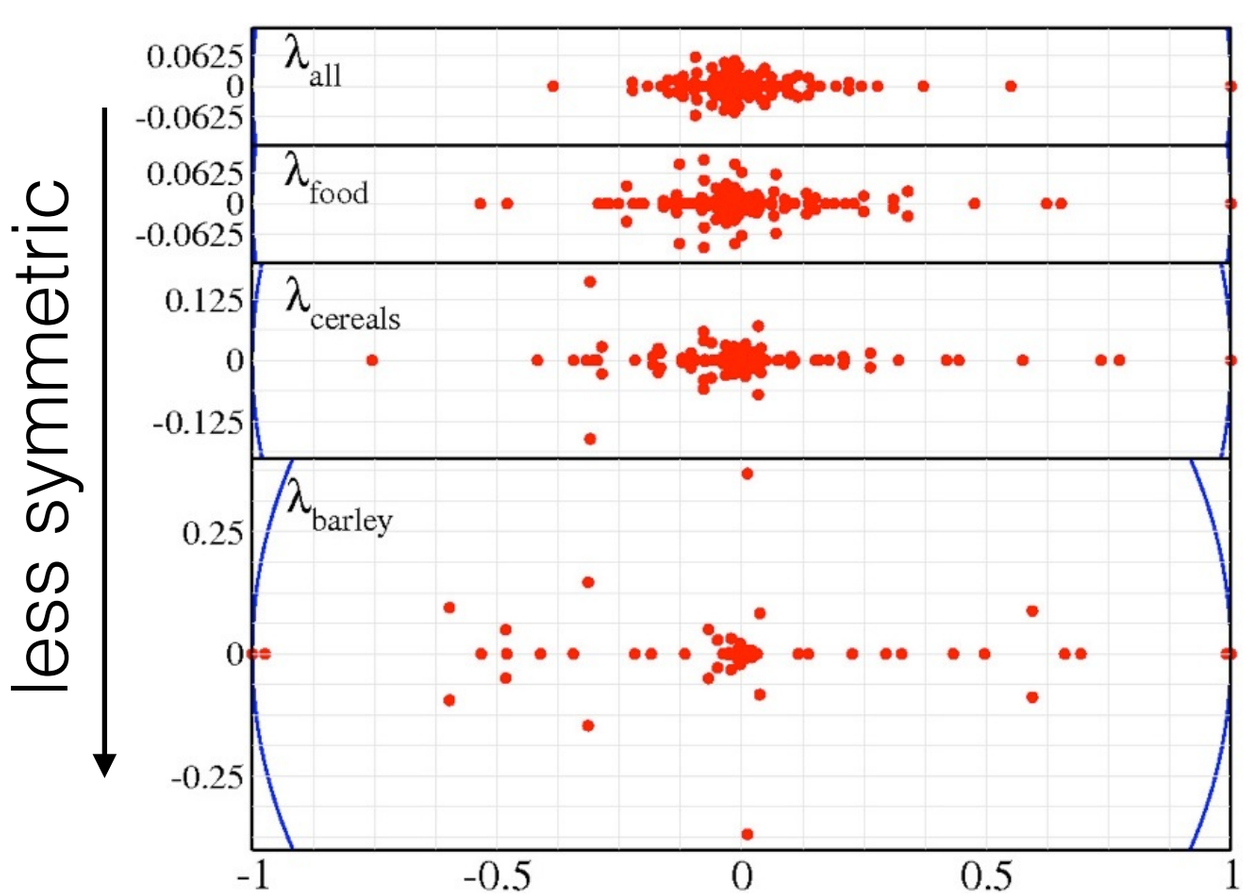




PageRank, CheiRank, ImportRank, ExportRank $\alpha = 0.5$

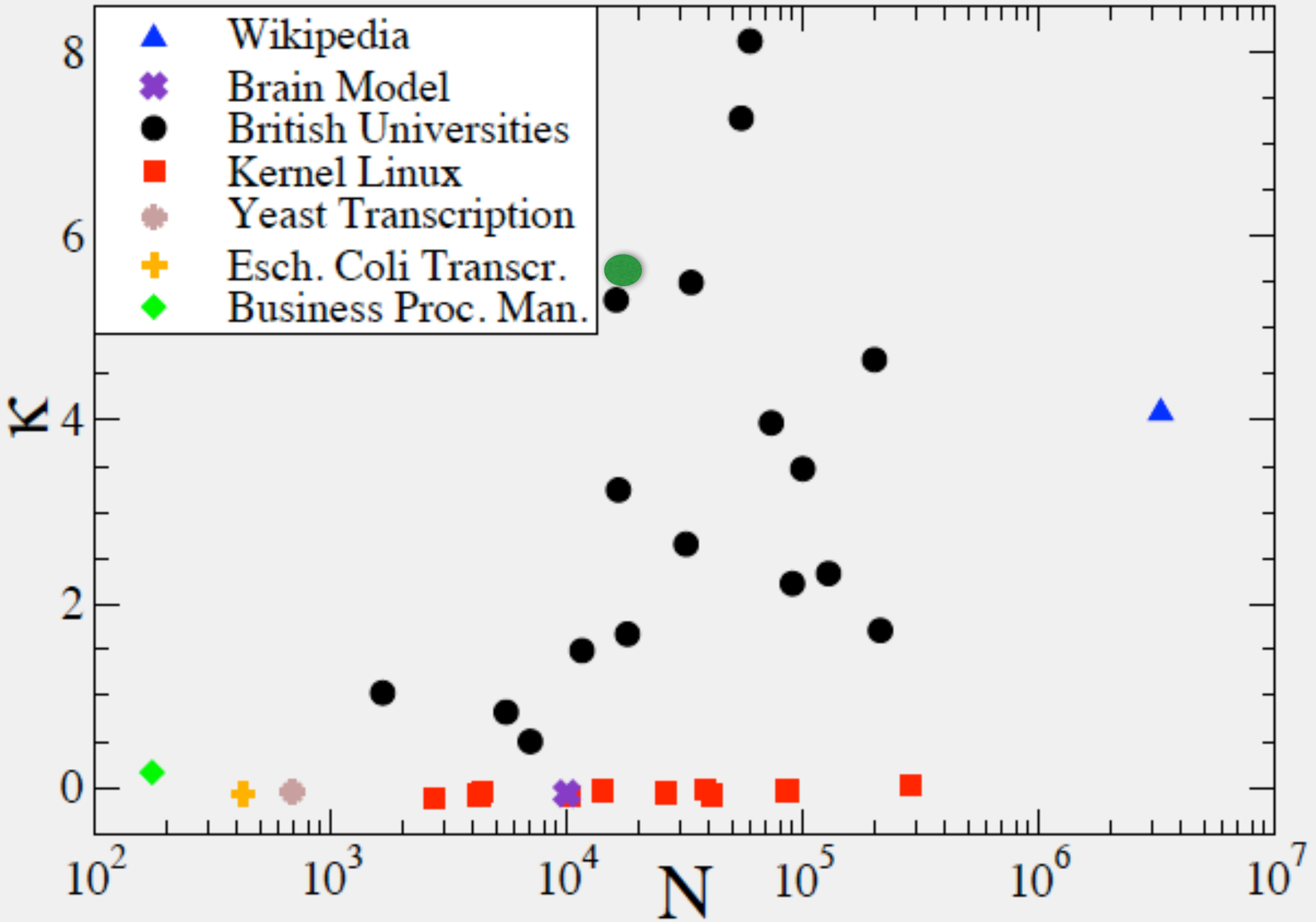
Spectra $\alpha = 1$



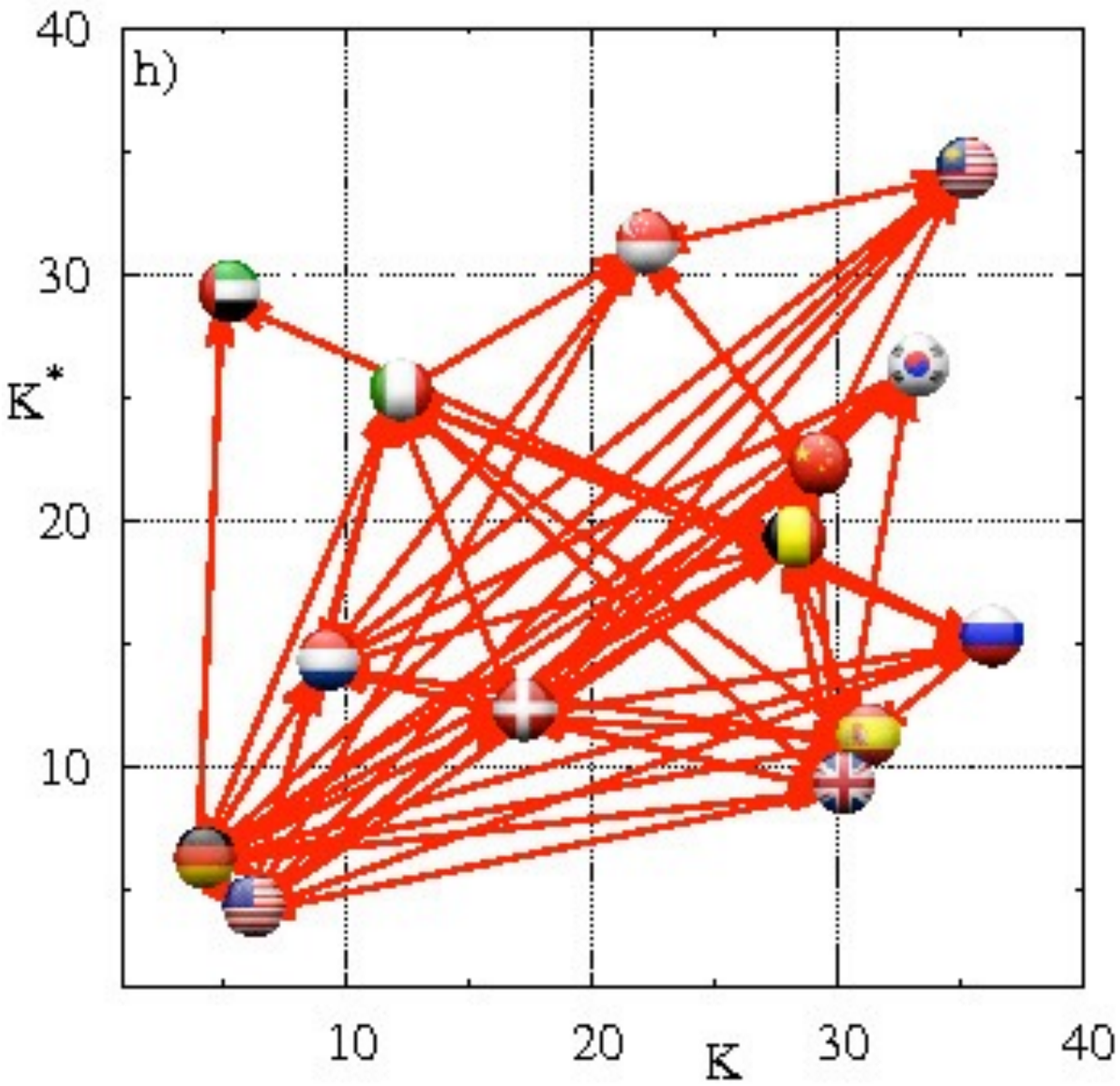


PageRank CheiRank correlator

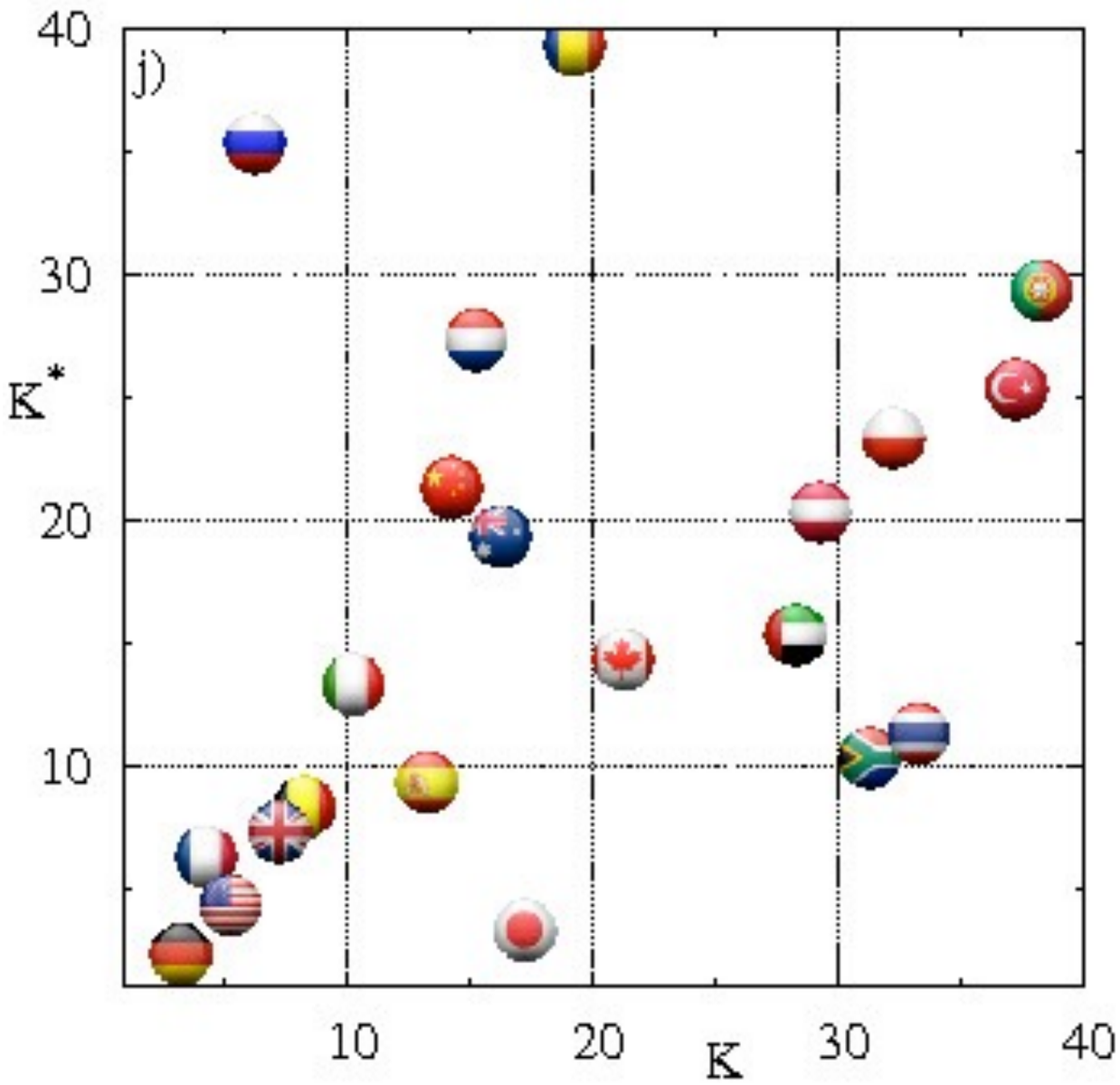
$$\kappa = N \sum_i P(i) P^*(i) - 1$$



2D rank examples: barley and cars

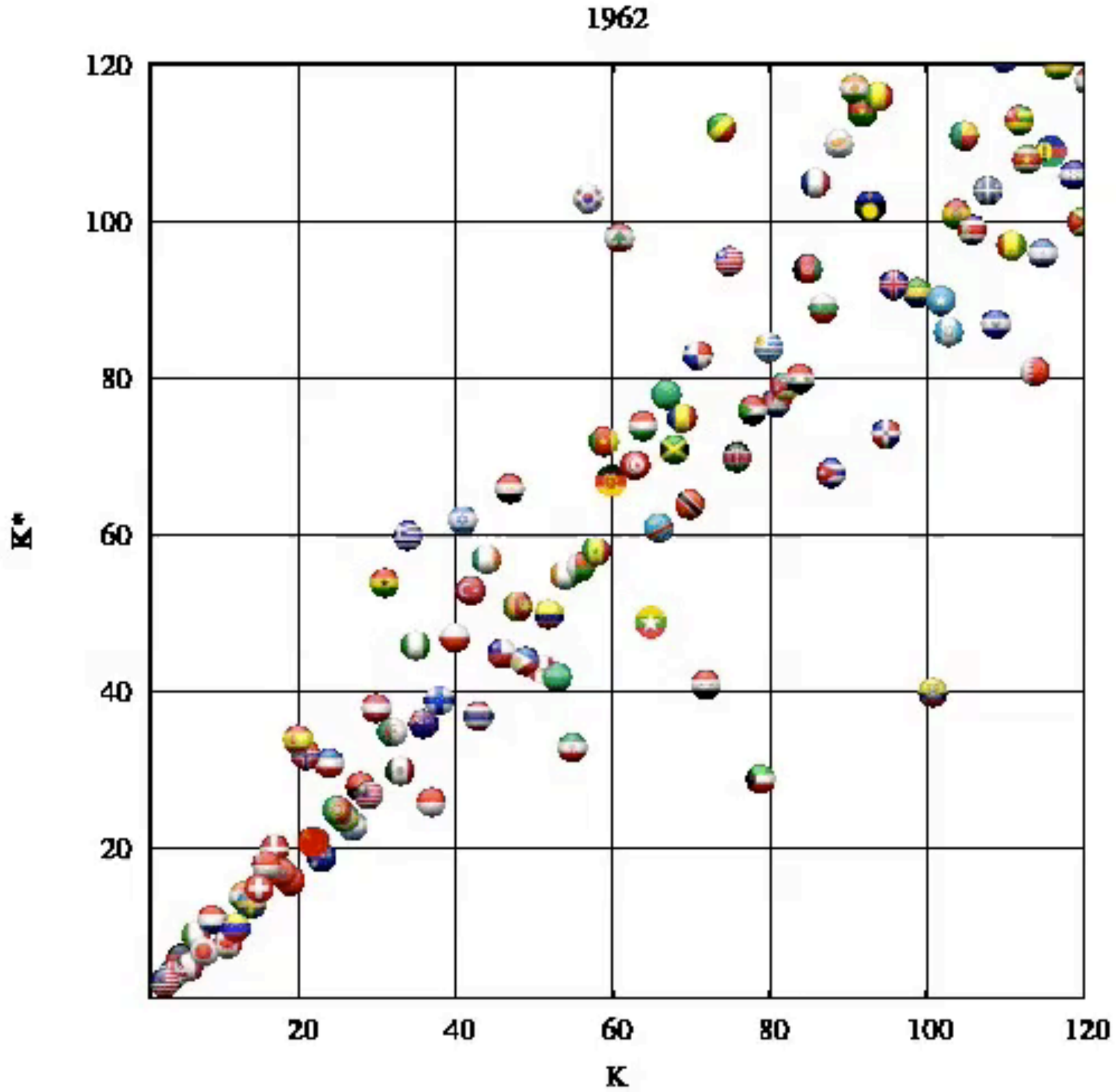


Ukraine (K* 1st to 6th) USA (K* 8th to 3rd)



France (K* 7th to 3rd) Thailand (K* 19th to 10th)

2d rank evolution (all comm.)



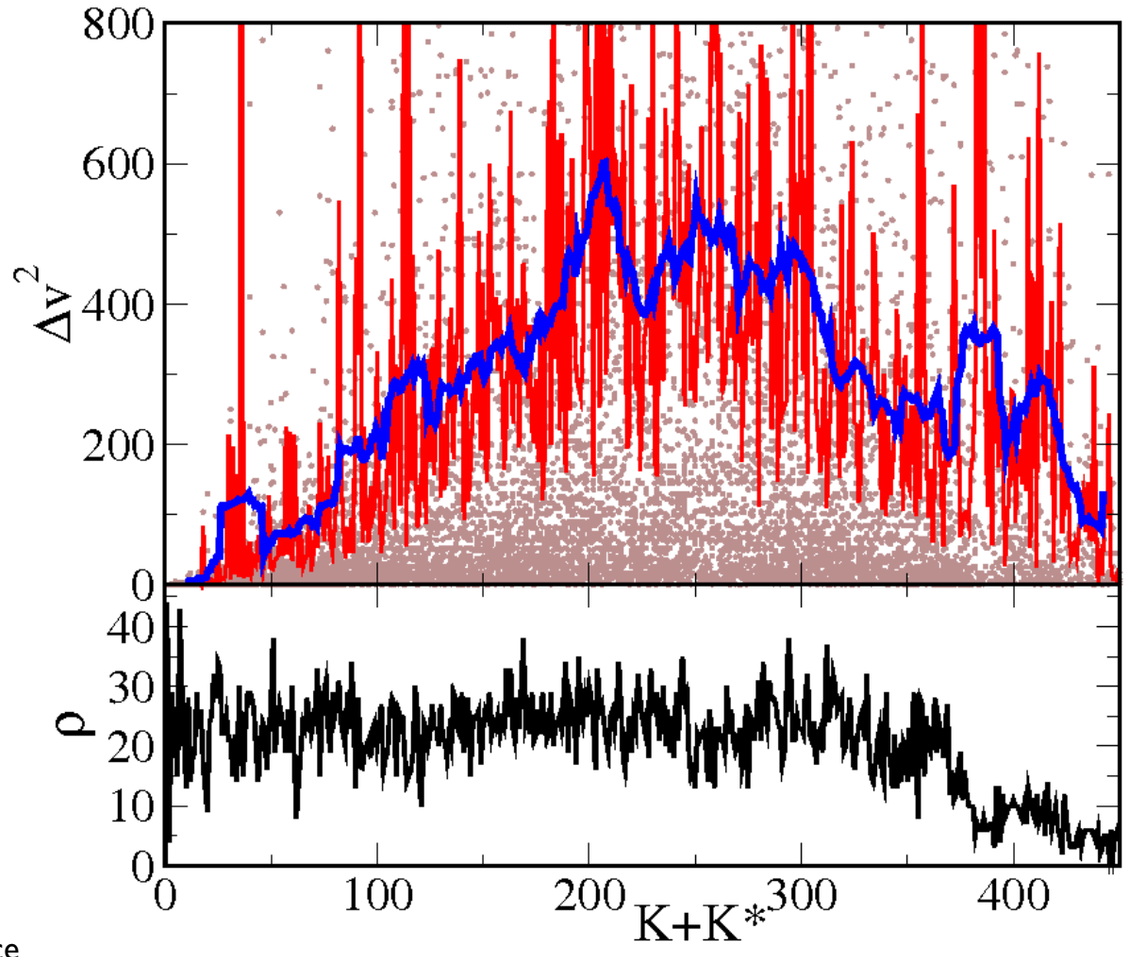
2d rank evolution

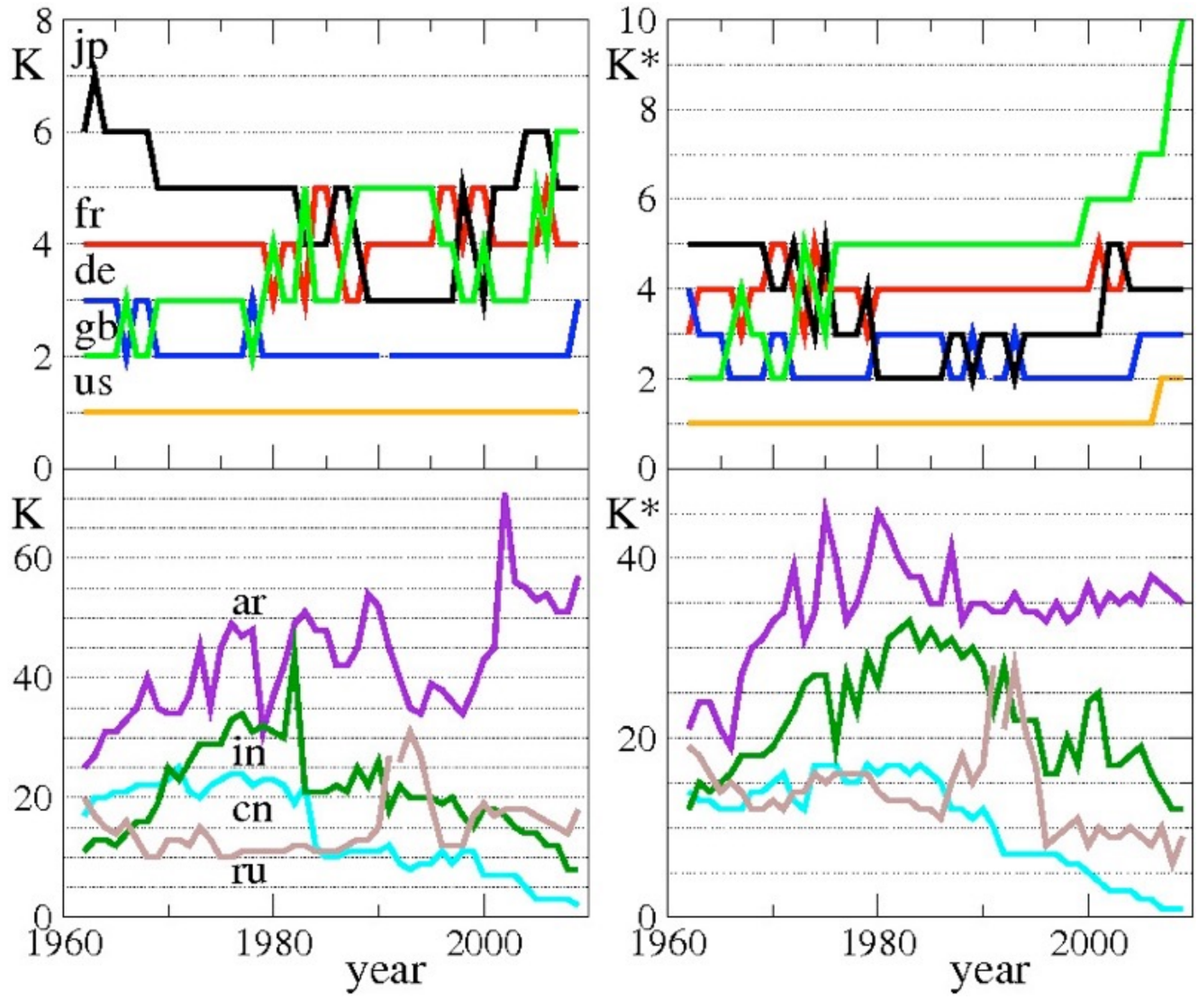
Velocity square vs. K+K*

$$\Delta v^2 = [K(t) - K(t-1)]^2 + [K^*(t) - K^*(t-1)]^2$$

average per $K + K^*$

average in $[K + K^* - 10, K + K^* + 10]$





Japan
France
Fed. Rep. of Germany and Germany
Great Britain (sublimation?)
USA

crisis detection

Argentina
India
China (deposition)
USSR and Russian Fed.

WTN model

- Gravity model of trade:

  $M_{i,j} = g m_i m_j / D_{i,j}$

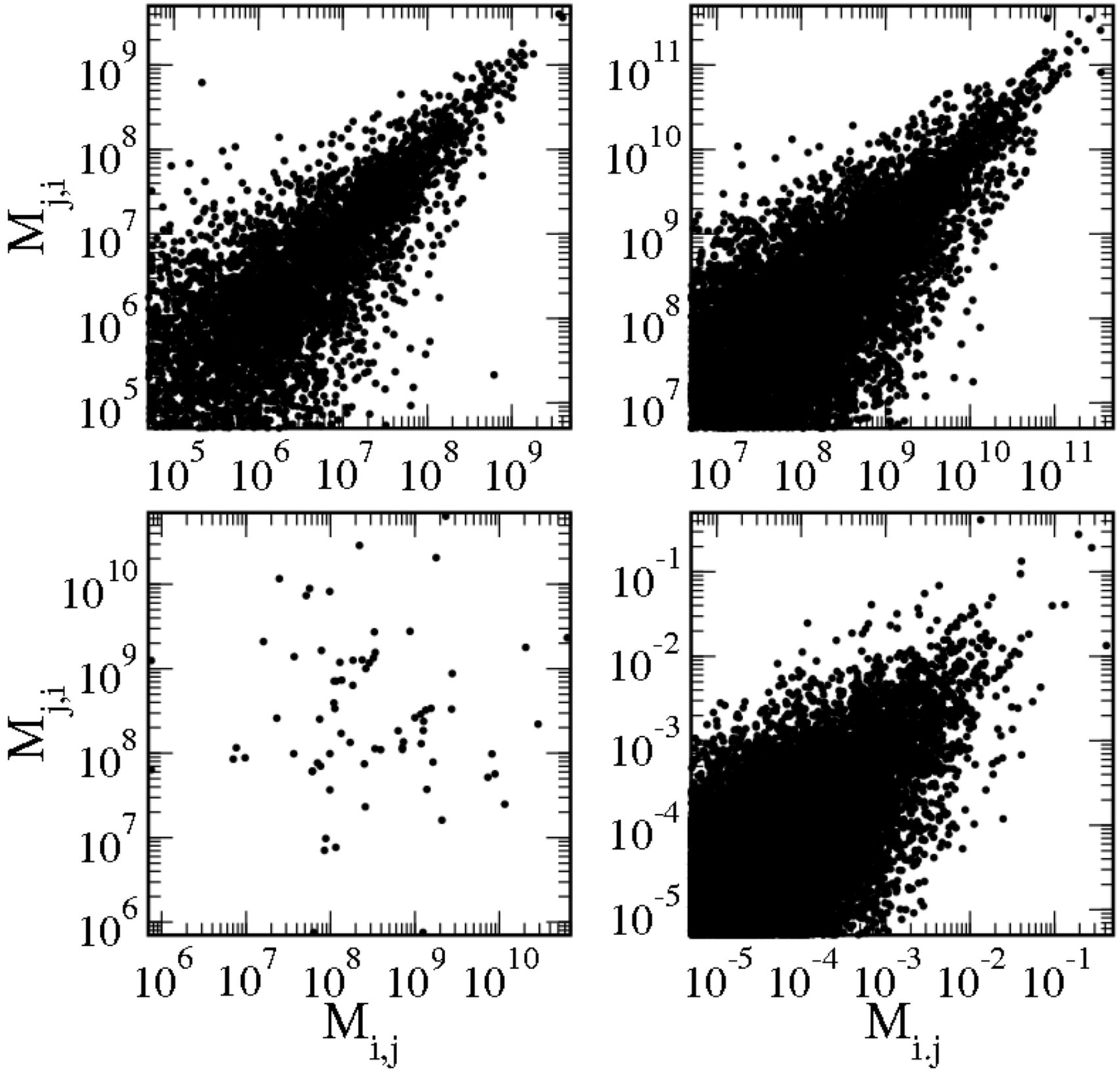
  (symmetric)

- Random model

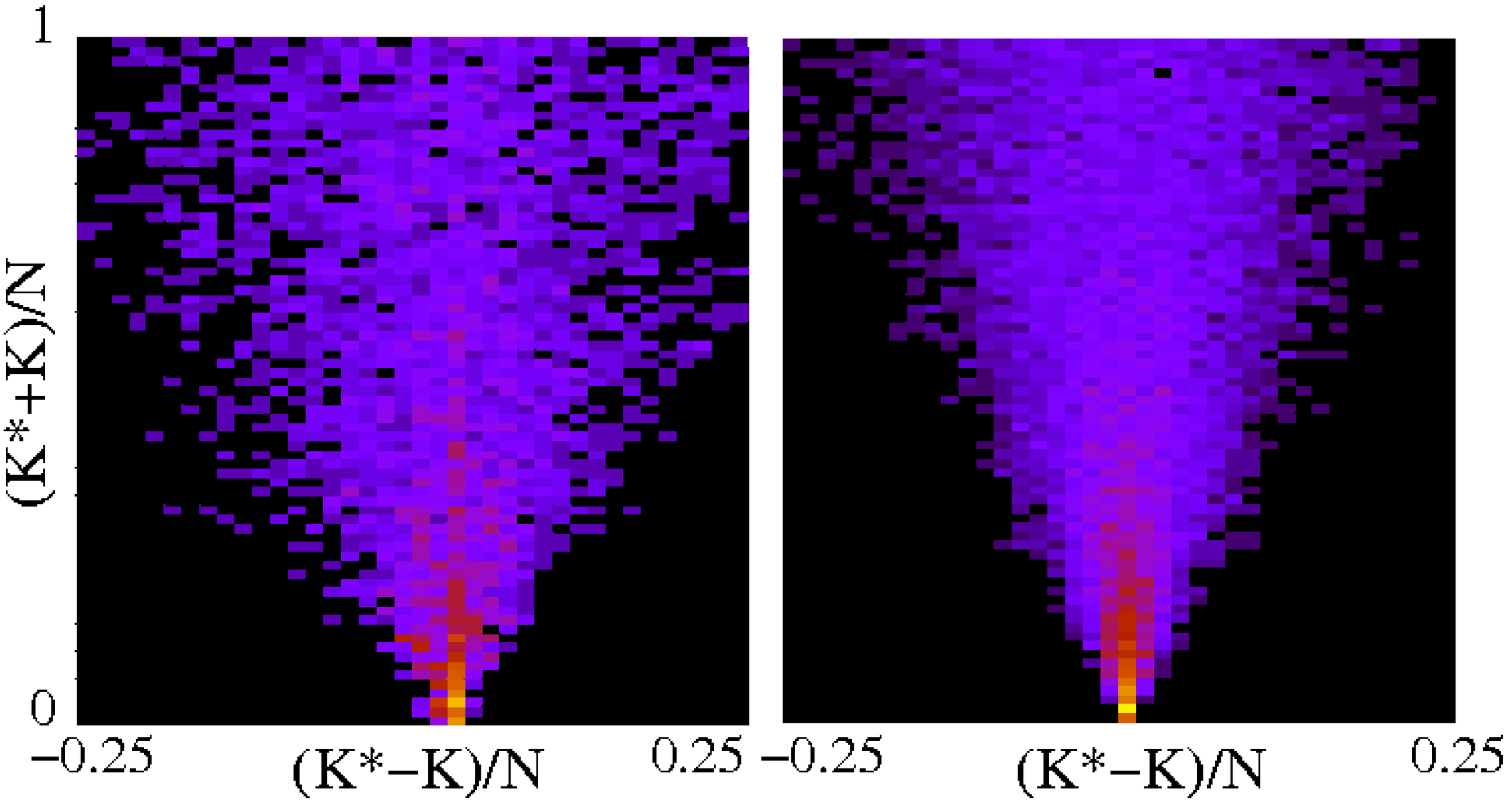
$M_{i,j} = \epsilon_i \epsilon_j / ij \qquad \epsilon_{i,j} \in [0,1)$

(preserves Zipf law)

t:: all commodities (1962, 2008);
b: crude petroleum (2008), random model



model statistics



Crisis

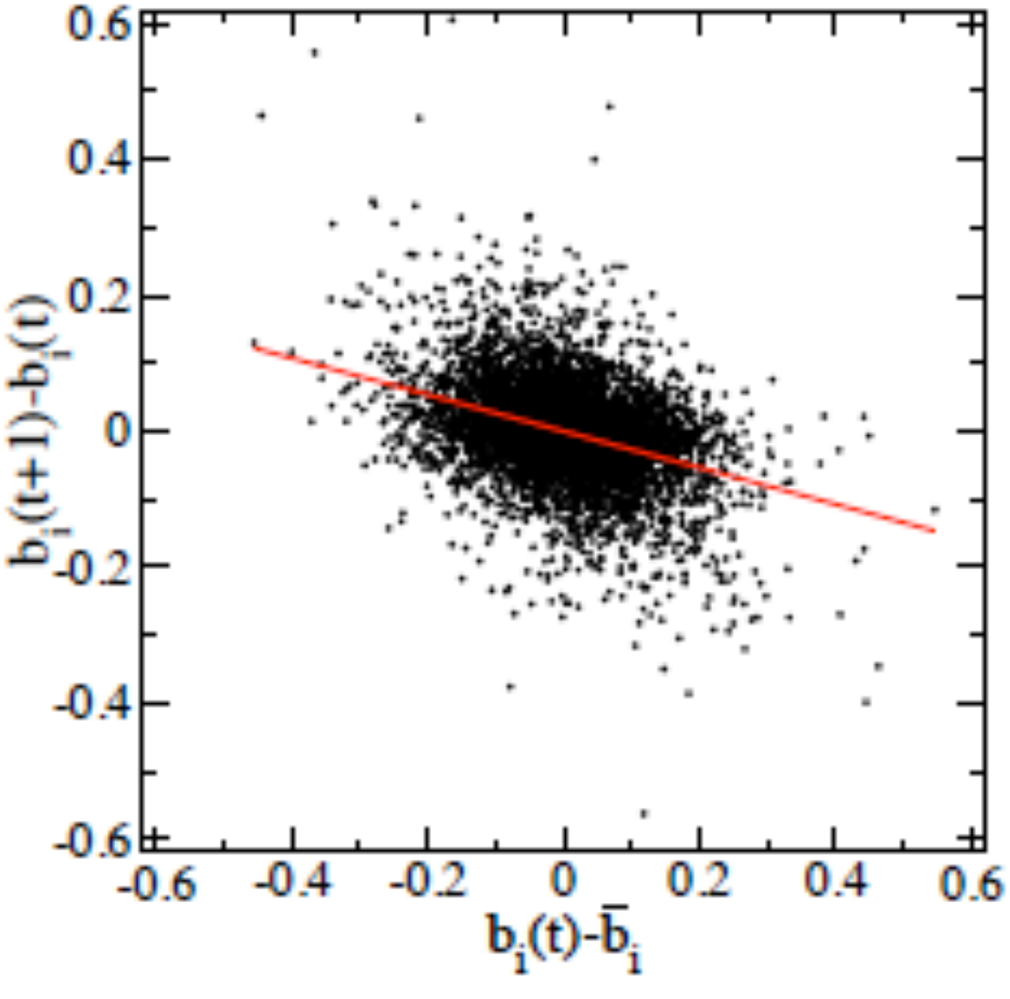
balance $b_i = \dfrac{P_i^* - P_i}{w_i}$

weight $w_i = P(i) + P^*(i)$

Rank	country	b_i (negative)	global rank
1	Greece	0.5131	1
2	Spain	0.2505	15
3	Romania	0.2322	19
4	Portugal	0.2222	23
5	Mexico	0.1743	37
6	Canada	0.1633	40
7	USA	0.1457	45
8	UK	0.1397	49
9	Poland	0.1326	51
10	France	0.1086	62

2008; w>0.05 (~20%)

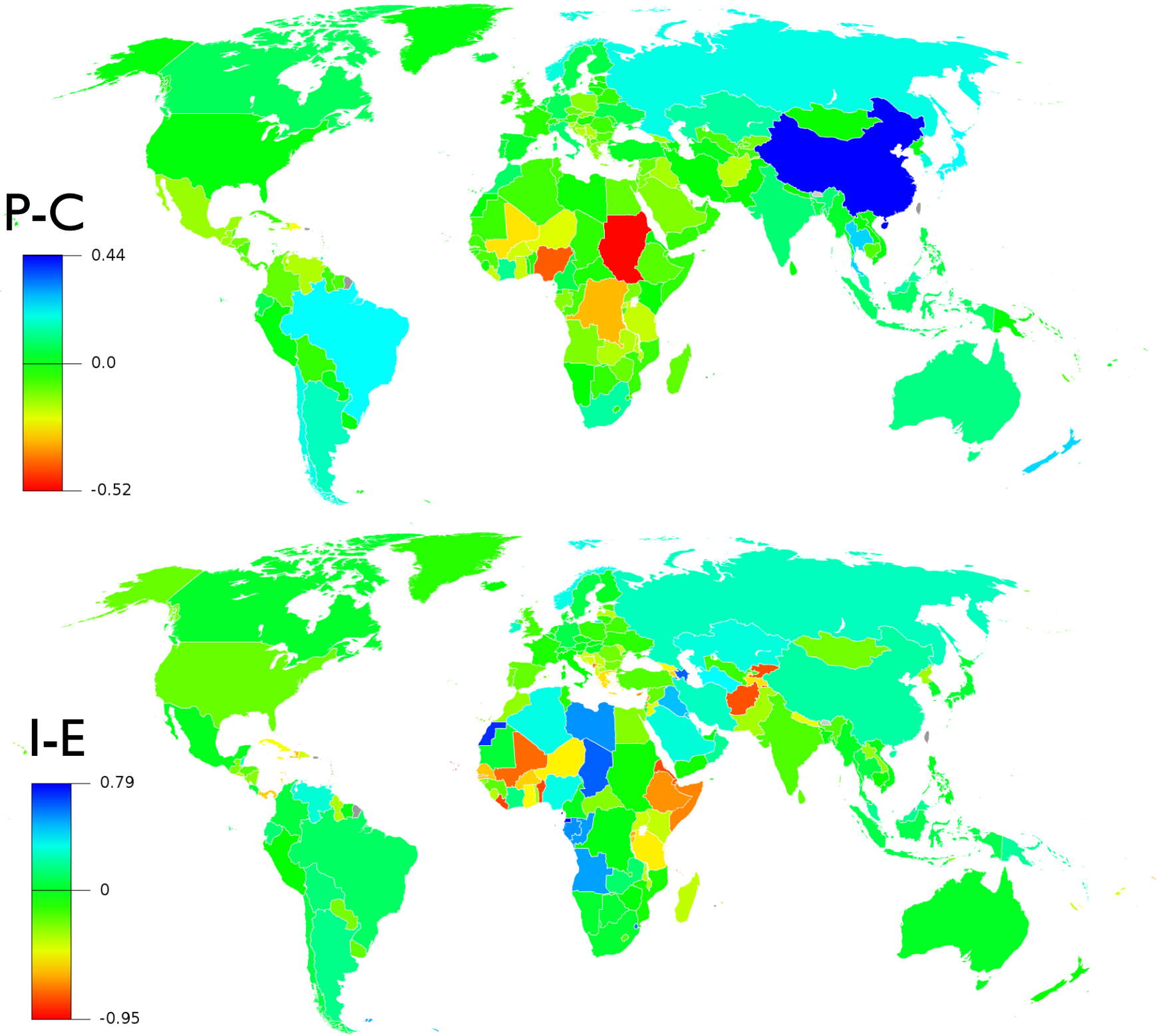
Rank	year	country	$b_i(t+1) - b_i(t)$
1	1973	Lebanon	-0.376
2	2001	Argentina	-0.269
3	1981	Mexico	-0.258
4	1983	Nigeria	-0.253
5	2002	Saudi Arabia	-0.250
6	1982	Venezuela	-0.247
7	1997	Indonesia	-0.244
8	1962	Venezuela	-0.236
9	1973	Nigeria	-0.230
10	1994	Mexico	-0.230
11	1997	Rep. of Korea	-0.219
12	1983	U. Arab Emir.	-0.213
13	2005	Iran	-0.210
14	1978	Iran	-0.210
15	1993	Turkey	-0.204
16	1975	India	-0.202
17	1998	Russian Fed.	-0.202
18	1976	Iraq	-0.200
19	1987	Argentina	-0.196
20	1989	Venezuela	-0.192



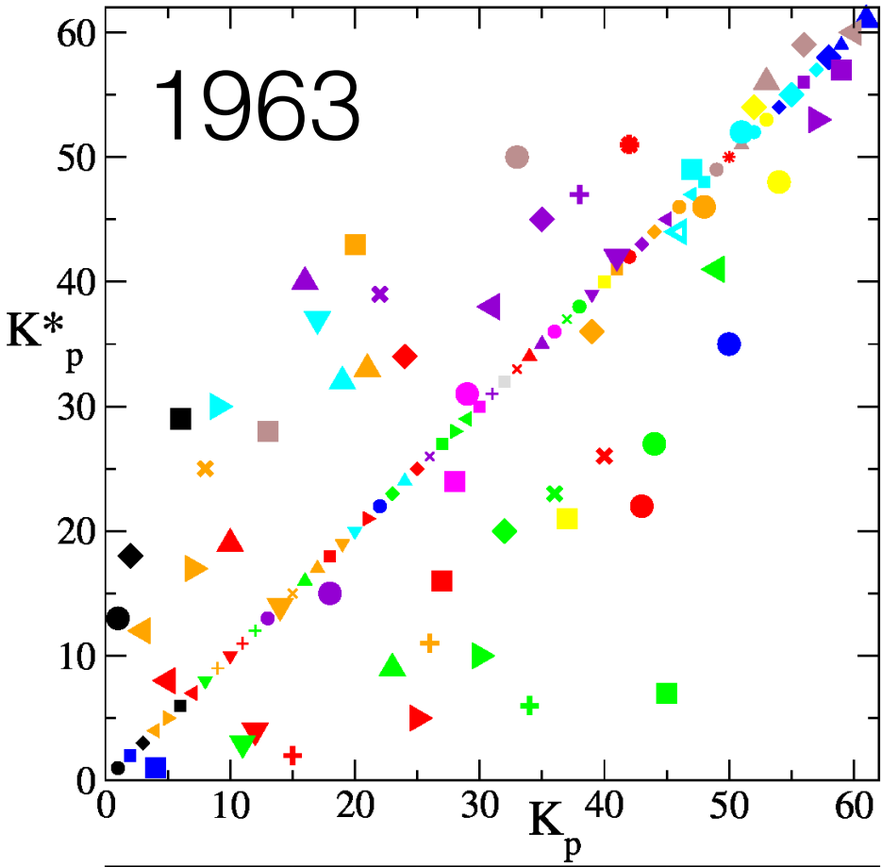
Multi-product WTN

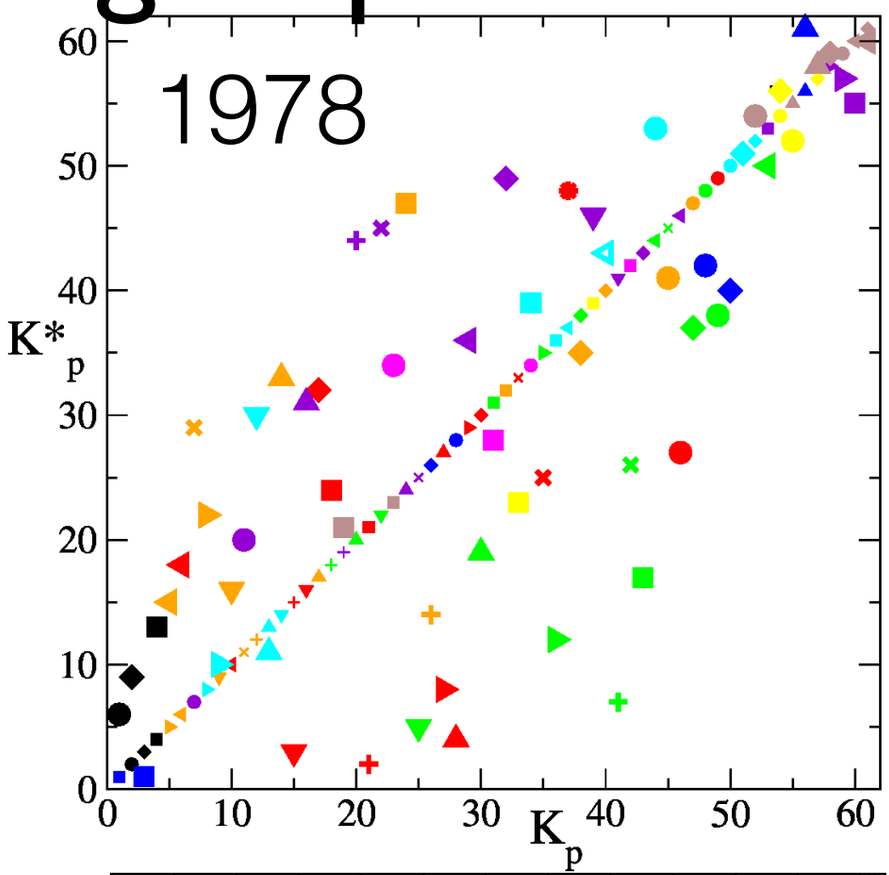
Country balance

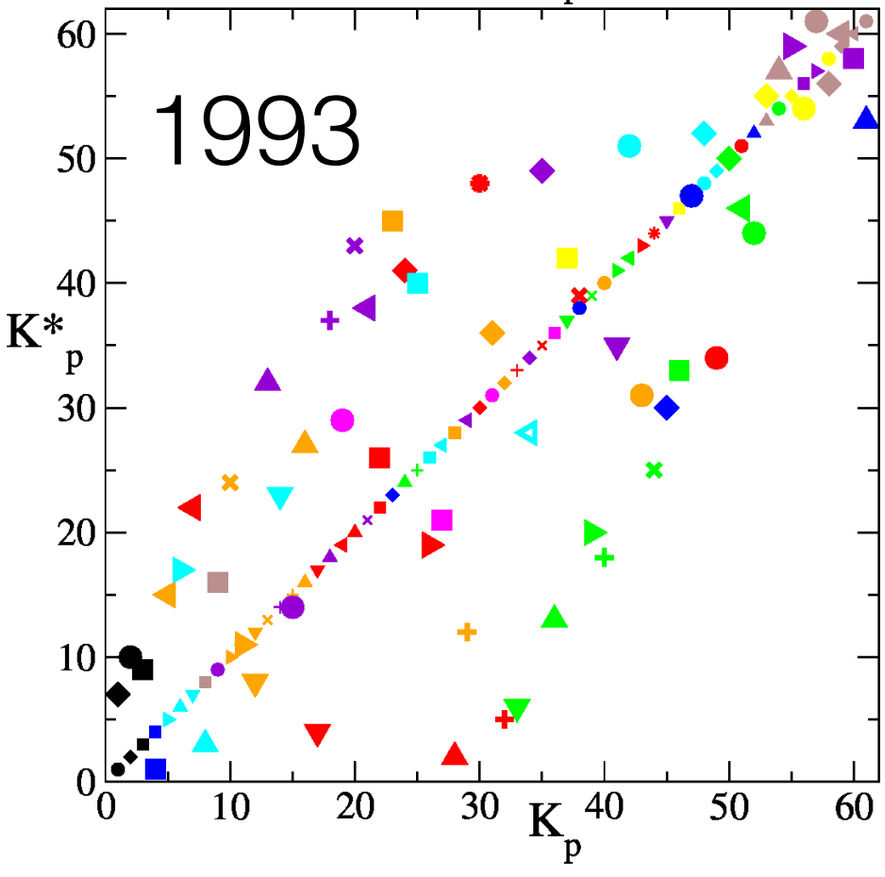
$$B_c = \sum_p (P^*_{cp} - P_{cp}) / \sum_p (P^*_{cp} + P_{cp}) = (P^*_c - P_c)/(P^*_c + P_c).$$

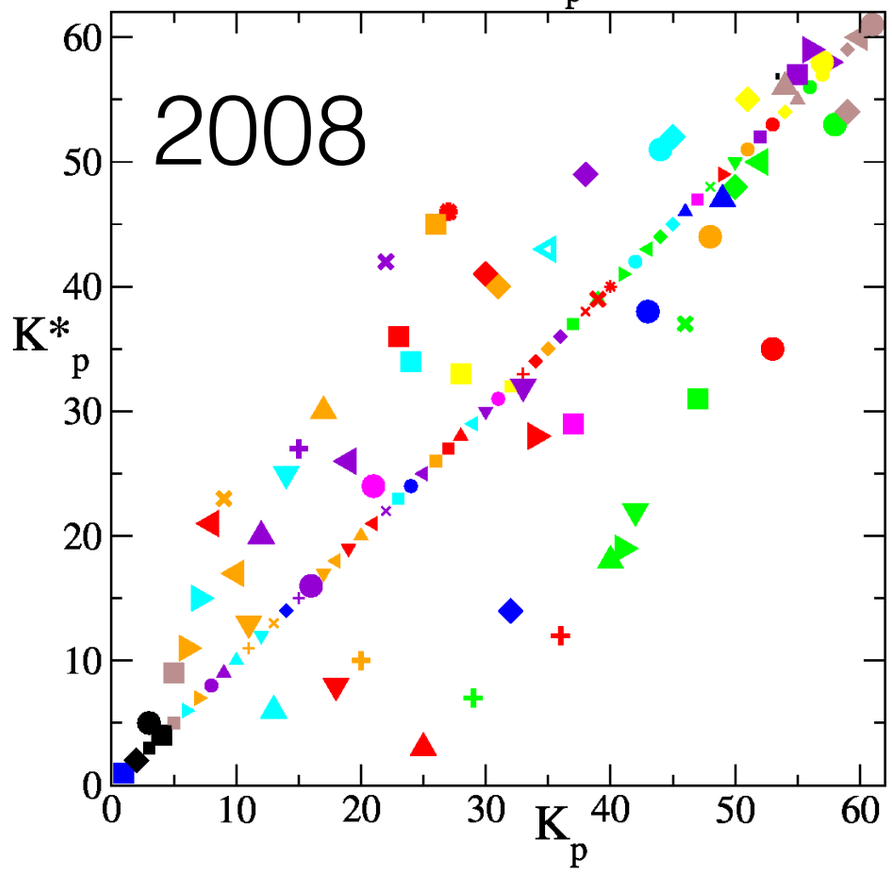


2d ranking of products





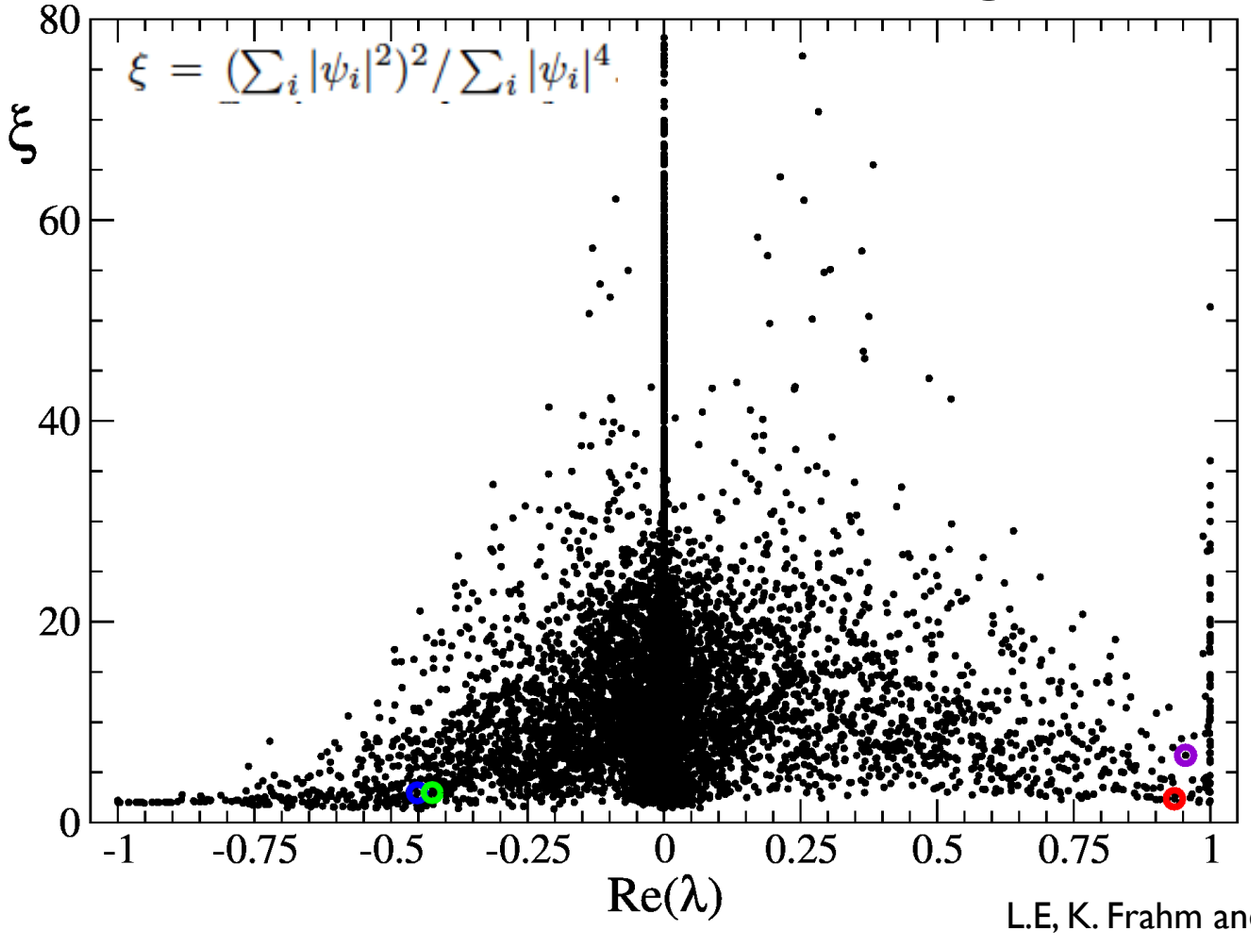


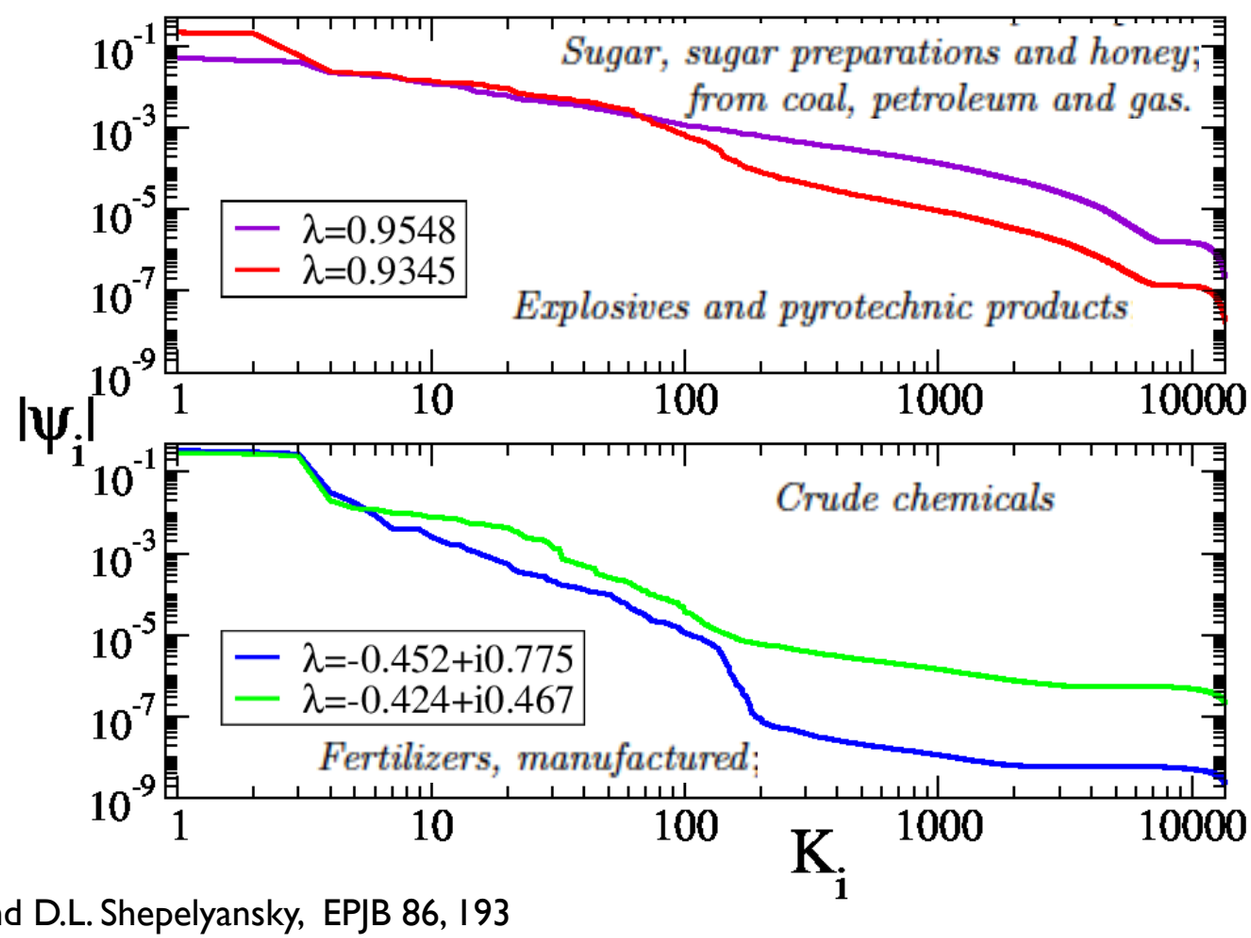


multi-prod WTN spectrum

K_i	$\|\psi_i\|$	country	$\|\psi_i\|$	country	$\|\psi_i\|$	country	$\|\psi_i\|$	country
		prod: 57		prod:06		prod:56		prod:52
1	0.052	USA	0.216	Mali	0.332	Brazil	0.288	Japan
2	0.044	Tajikistan	0.201	Guinea	0.304	Bolivia	0.279	Rep. of Korea
3	0.042	Kyrgyzstan	0.059	USA	0.274	Paraguay	0.245	China
4	0.022	France	0.023	Germany	0.031	Argentina	0.020	Australia
5	0.021	Mexico	0.021	Mexico	0.017	Uruguay	0.013	USA
6	0.018	Italy	0.021	Canada	0.009	Chile	0.012	U Arab Em
7	0.018	Canada	0.018	UK	0.004	Portugal	0.010	Canada
8	0.015	Germany	0.015	Israel	0.004	Angola	0.010	Singapore
9	0.013	U Arab Em	0.015	C d'Ivoire	0.004	Spain	0.009	Germany
10	0.012	Qatar	0.014	Japan	0.003	France	0.008	New Zealand

eigenstate communities

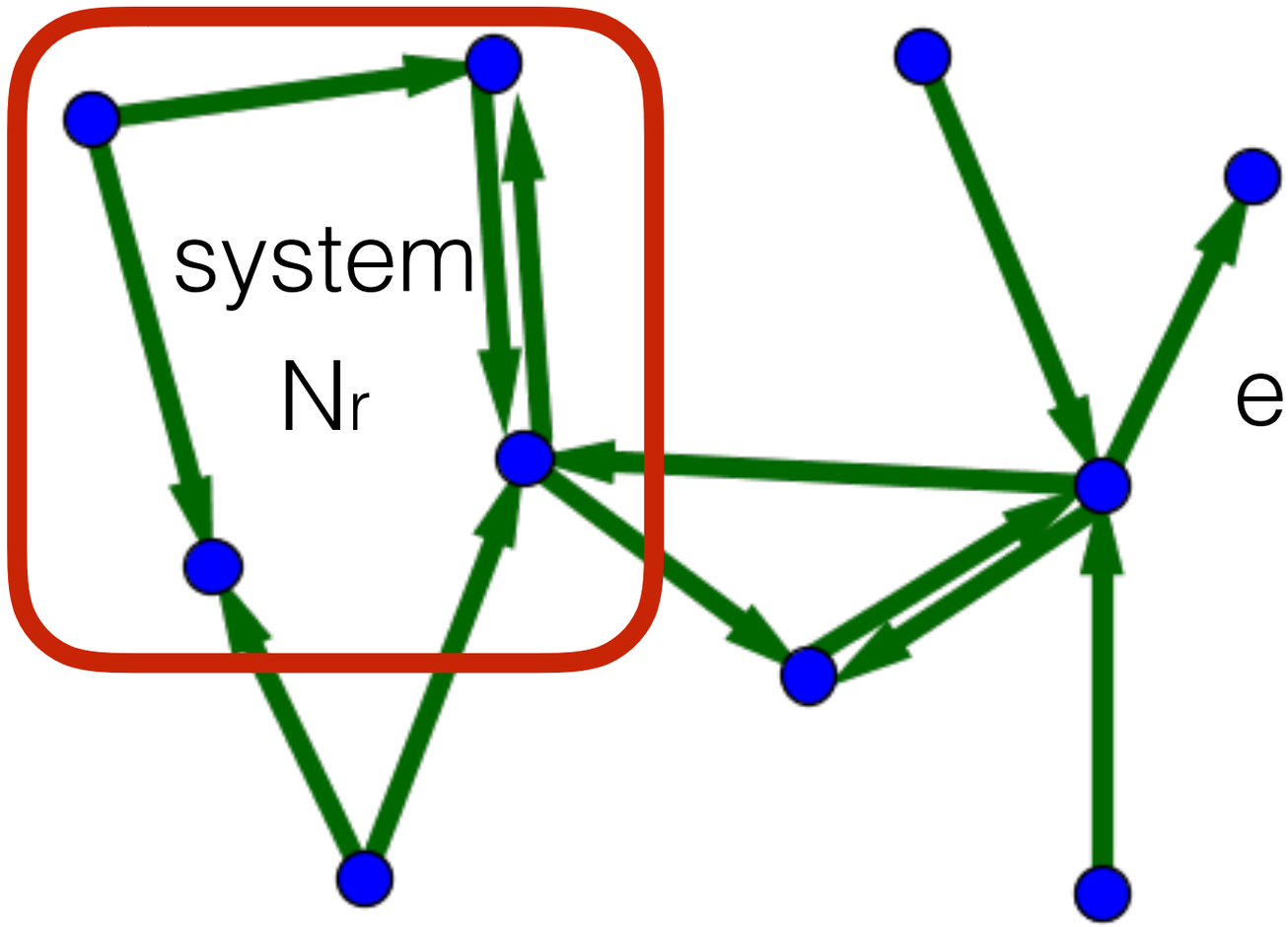




L.E, K. Frahm and D.L. Shepelyansky, EPJB 86, 193

Reduced G matrix of WTN

reduced Google matrix



$$G = \begin{pmatrix} G_{\rm rr} & G_{\rm rs} \\ G_{\rm sr} & G_{\rm ss} \end{pmatrix}$$

$$G_{\rm R} = G_{\rm rr} + G_{\rm pr} + G_{\rm qr}$$

$$G_{\rm R} P_r = P_r$$

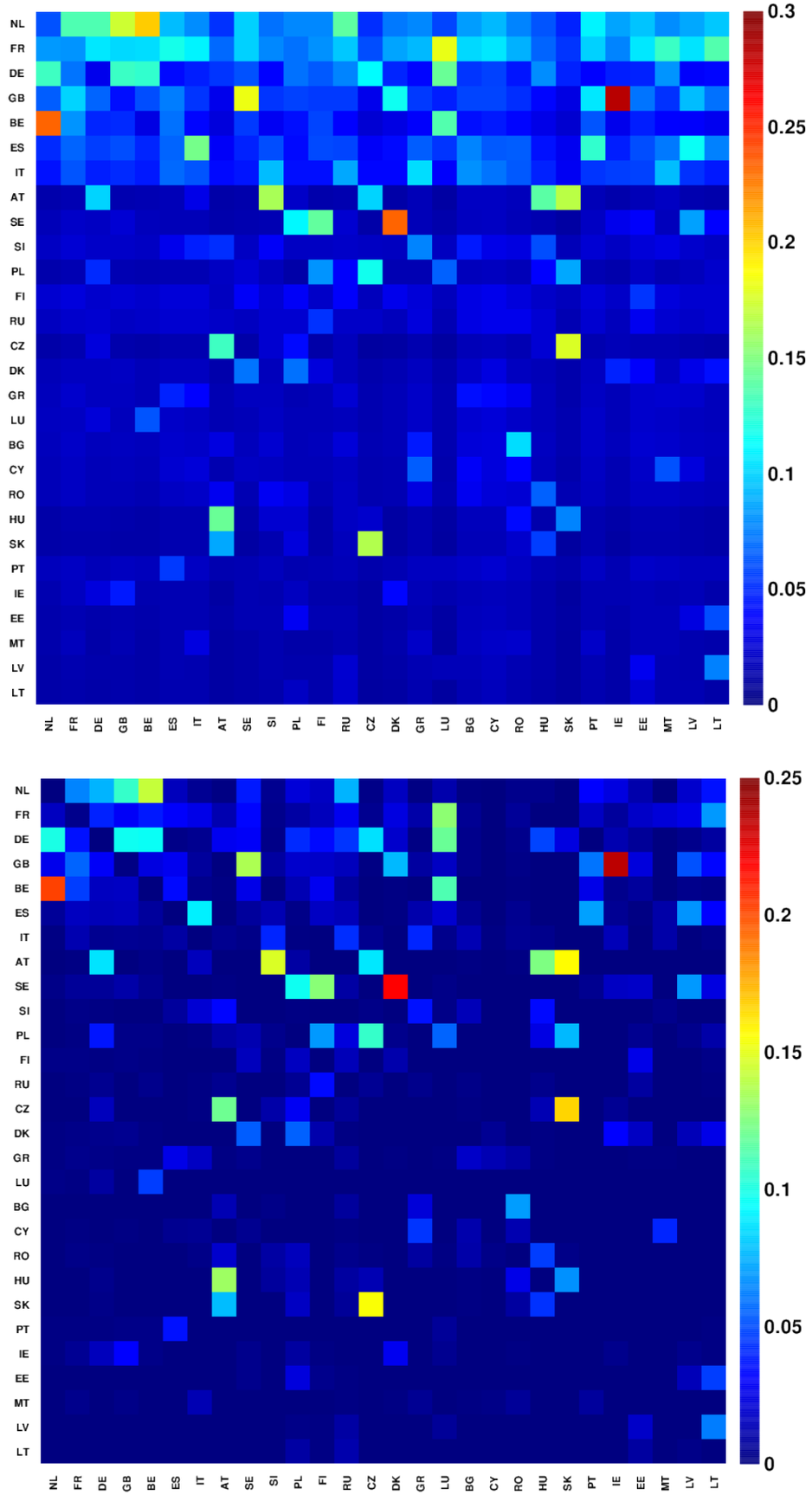
$$G_{\rm R} = G_{rr} + G_{rs}(1 - G_{ss})^{-1} G_{sr}$$

K. Frahm and D.L. Shepelyansky, arxiv:1602.02394 (yesterday Frahm's introduction)

Colab. C Coquide, J. Lages & D. Shepelyansky

Petroleum for 27 European countries + Russia

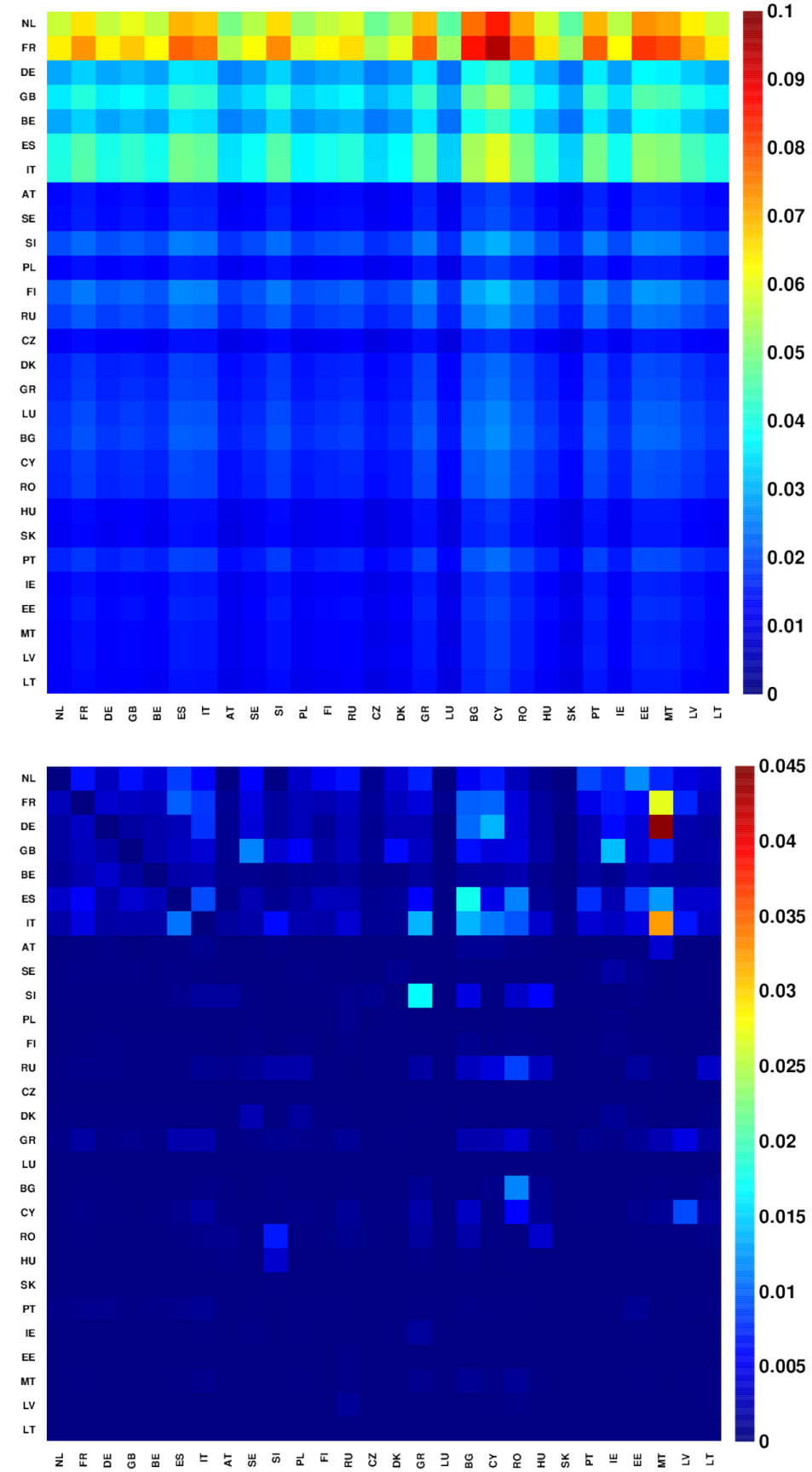
G_R



G_{pr}

W_{rr}=0.65

(W_{rr}~0.95 Wiki ex.)



G_{rr}

W_{rr}=0.31

(W_{rr}~0.011 Wiki ex,)

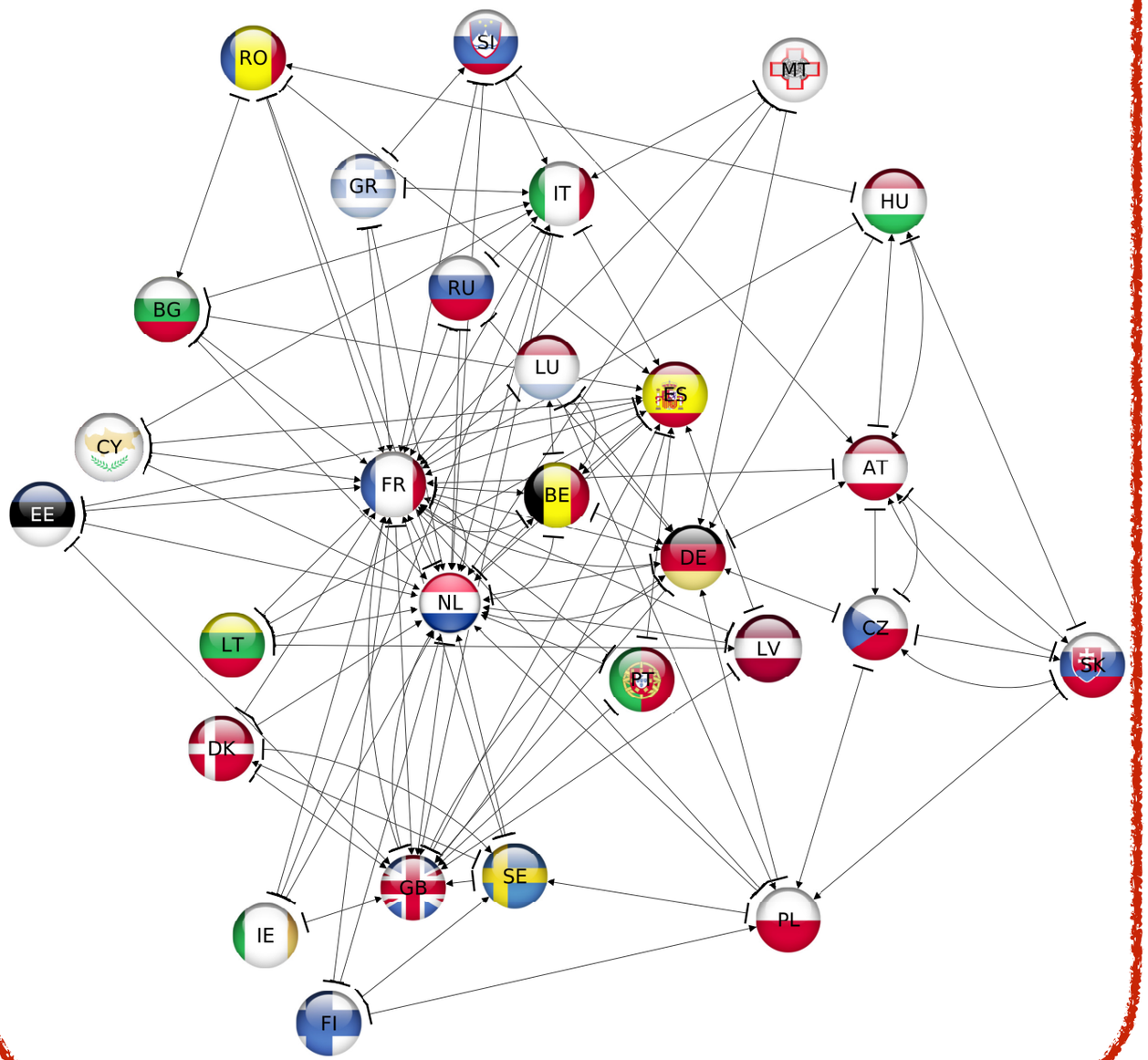
G_{qrnd}

W_{rr}=0.037

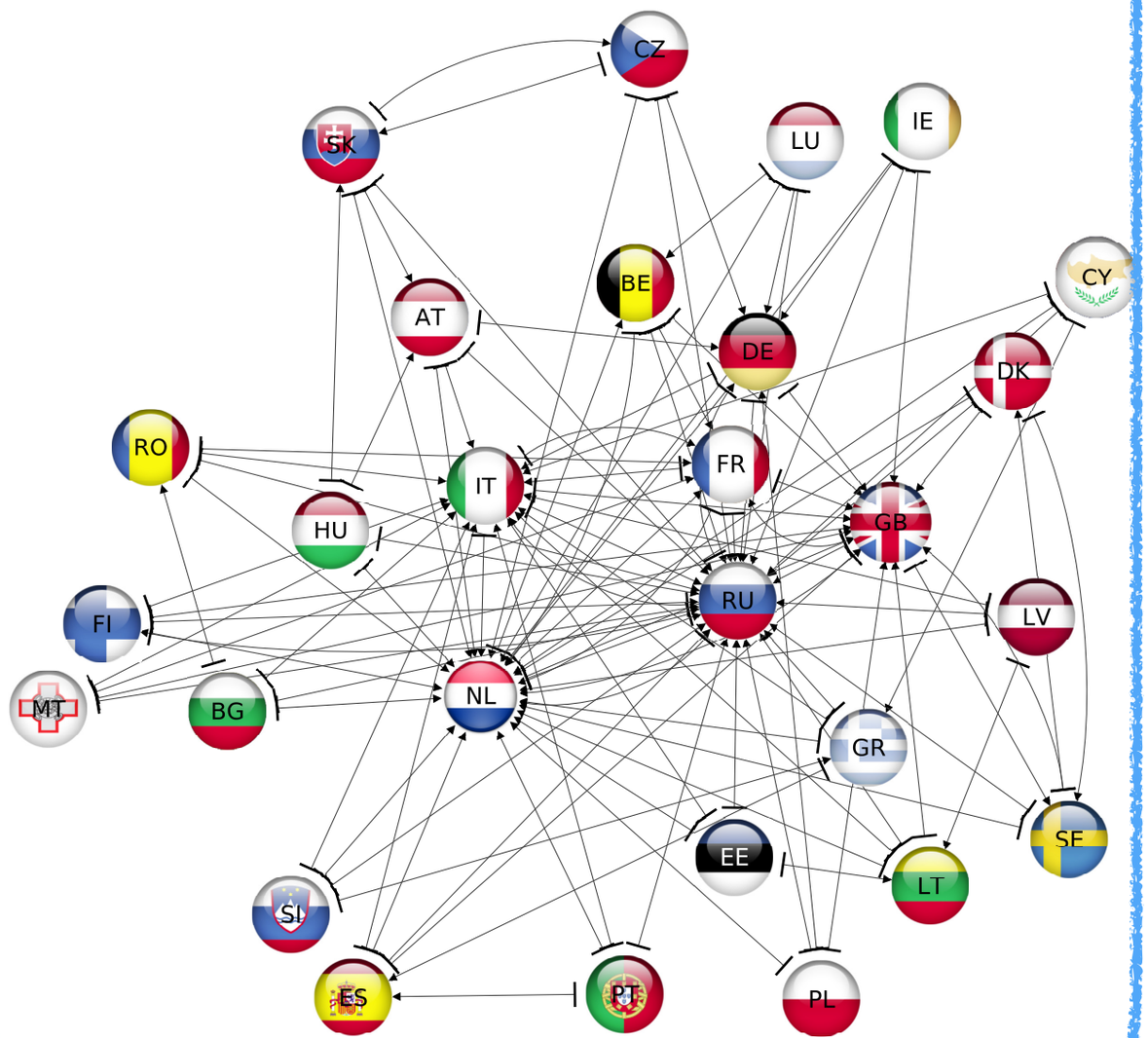
(W_{rr}~0.02 Wiki ex.)

Petroleum (27 Eu + Ru) 4 friend networks

G_R



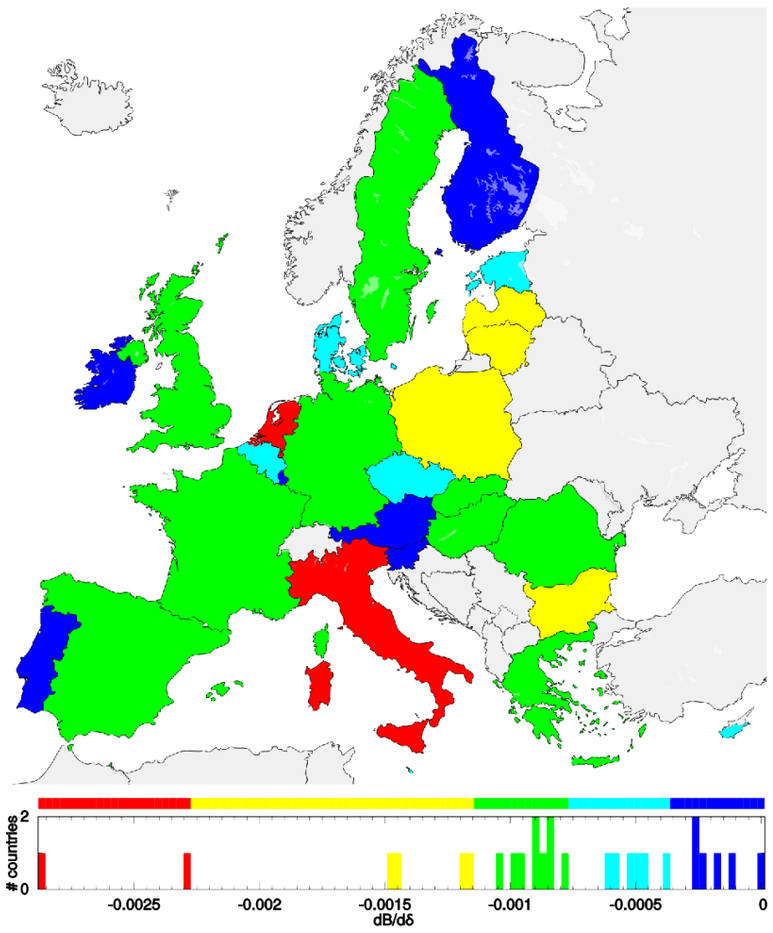
G_R^*



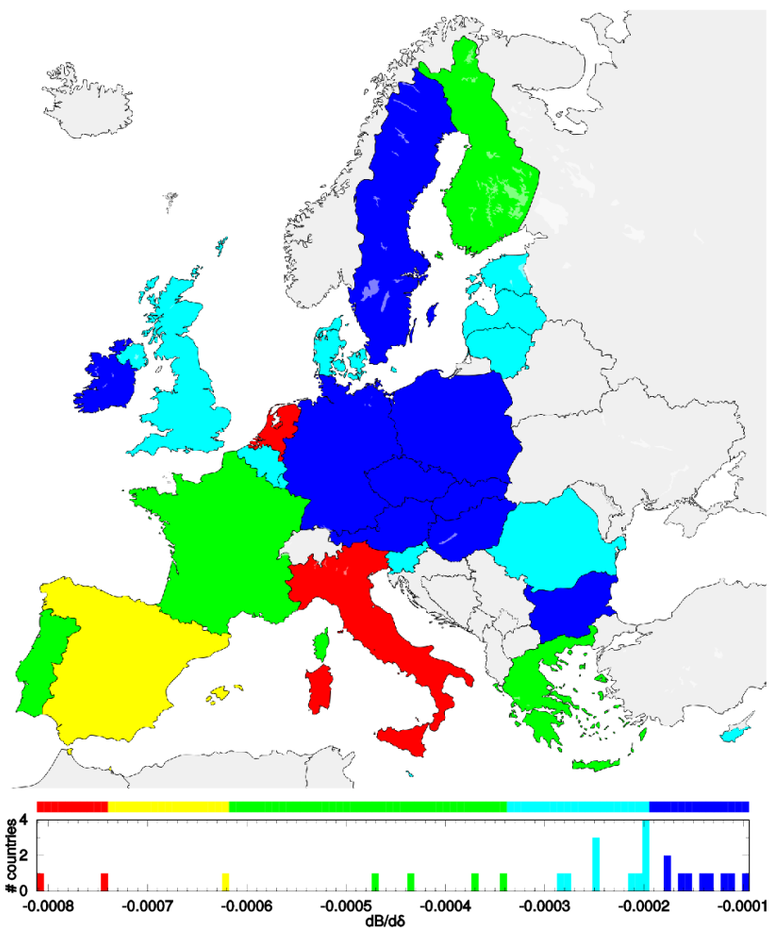
(plots made by Celestin Coquide)

Balance sensitivity to petroleum from:

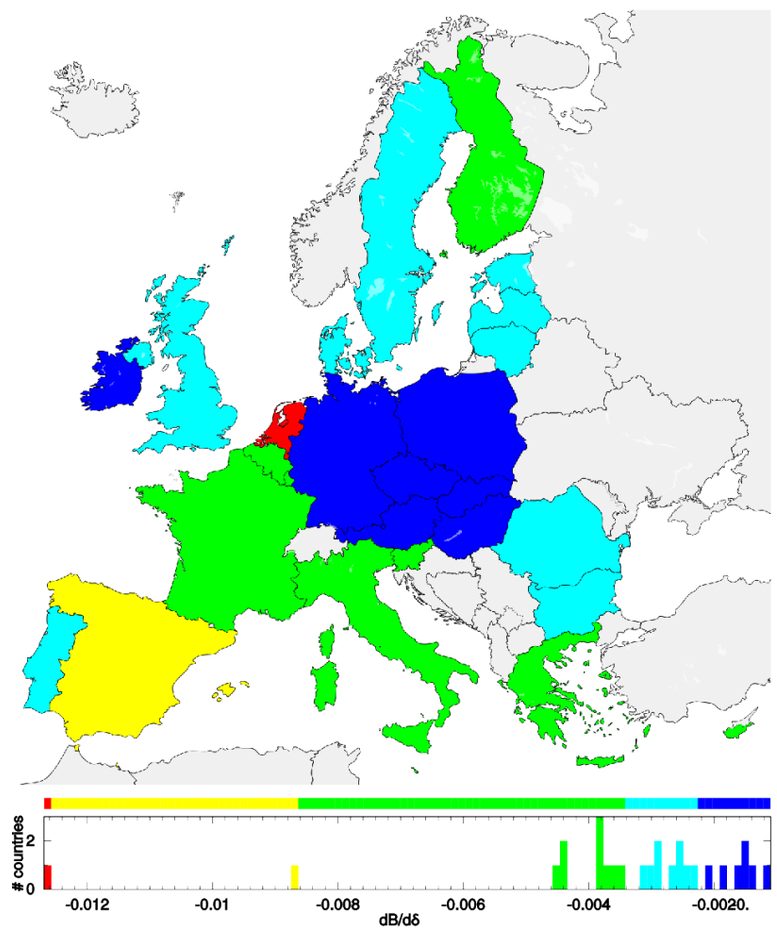
Russia



Saudi Arabia



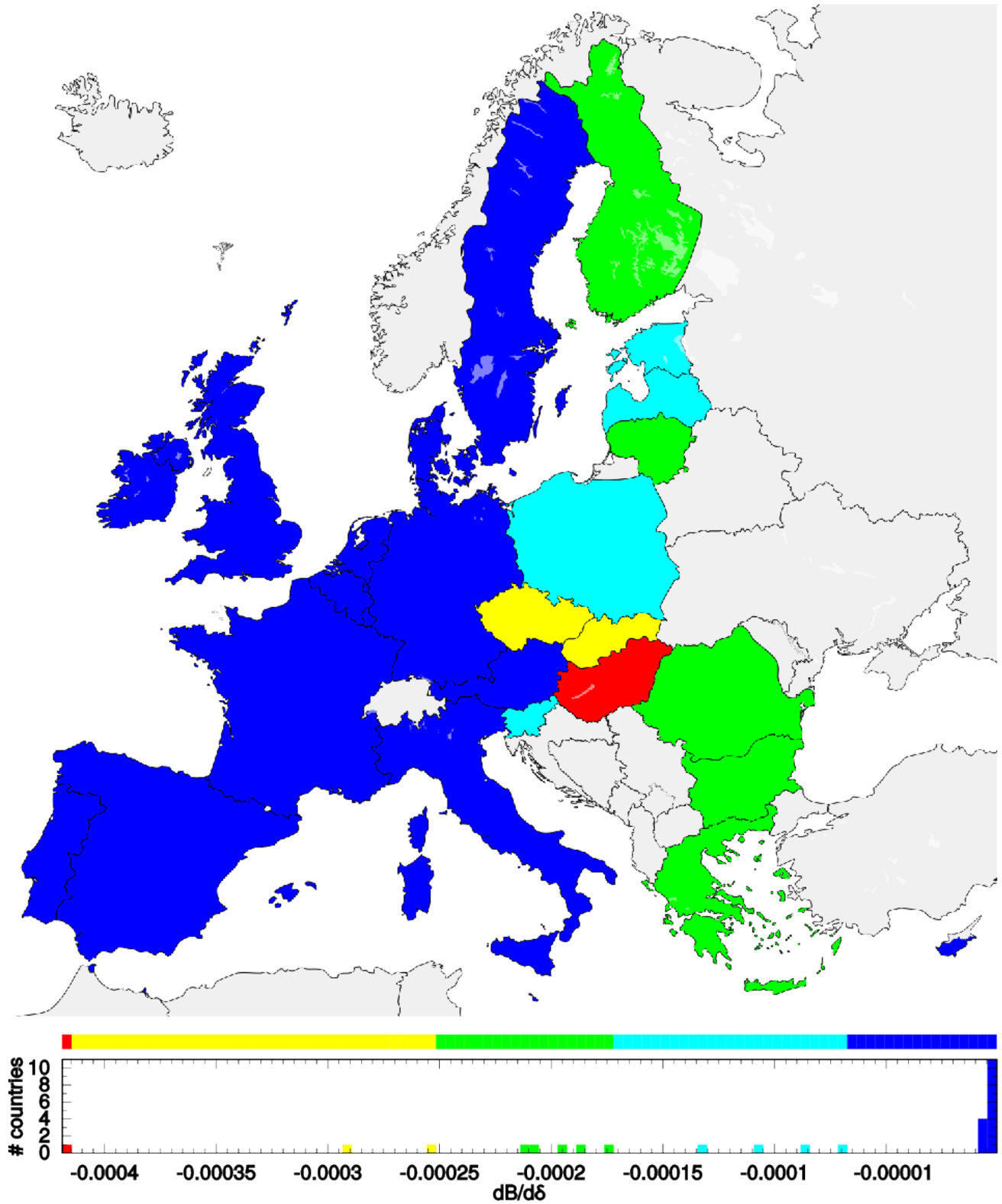
USA



(plots made by Celestin Coquide)

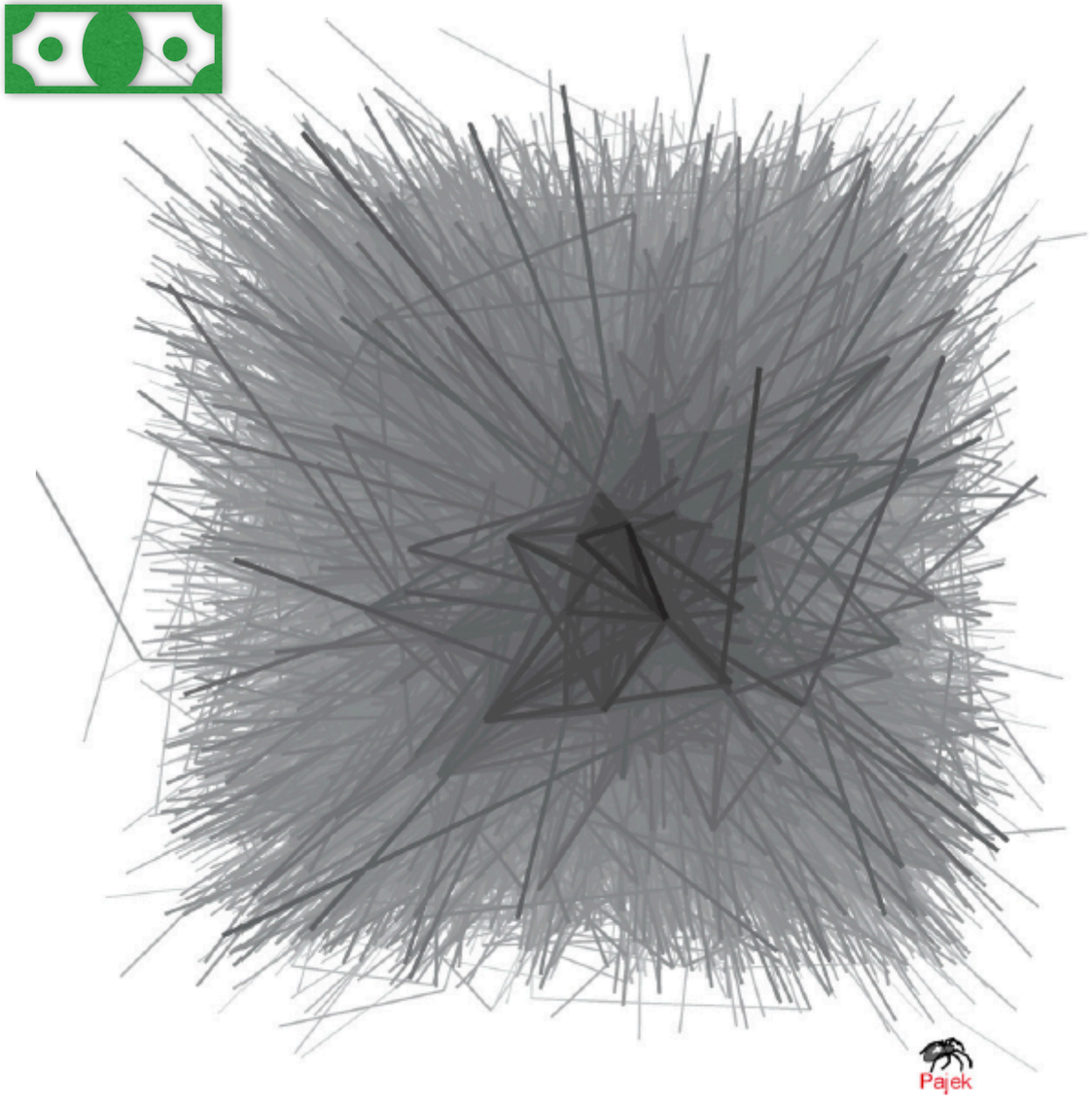
Balance sensitivity to gas from:

Russia



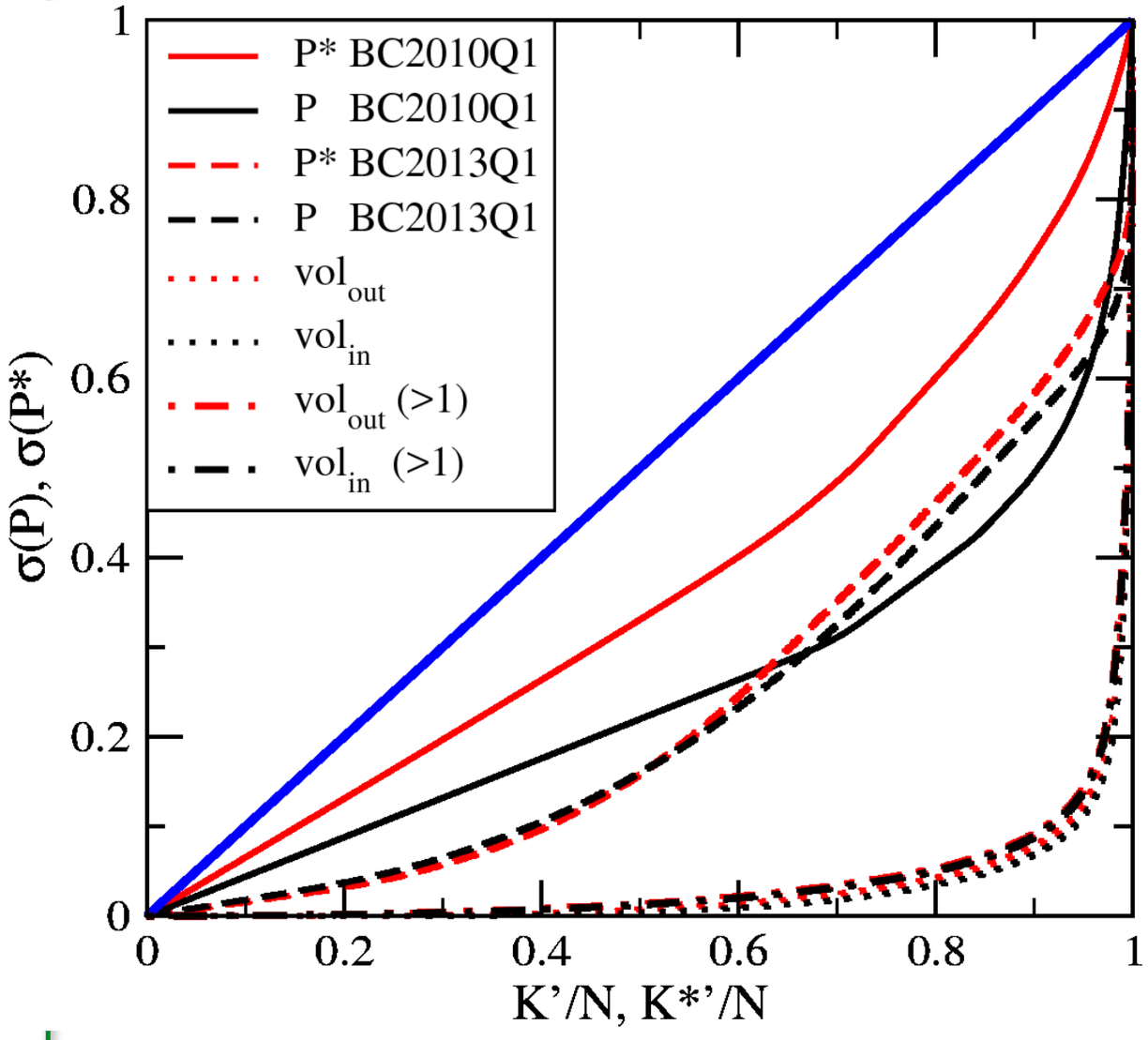
Bitcoin network

Interbank payment network



Fedwire interbank payment network. First day of Sample. 6600 nodes and over 70,000 undirected links [39].

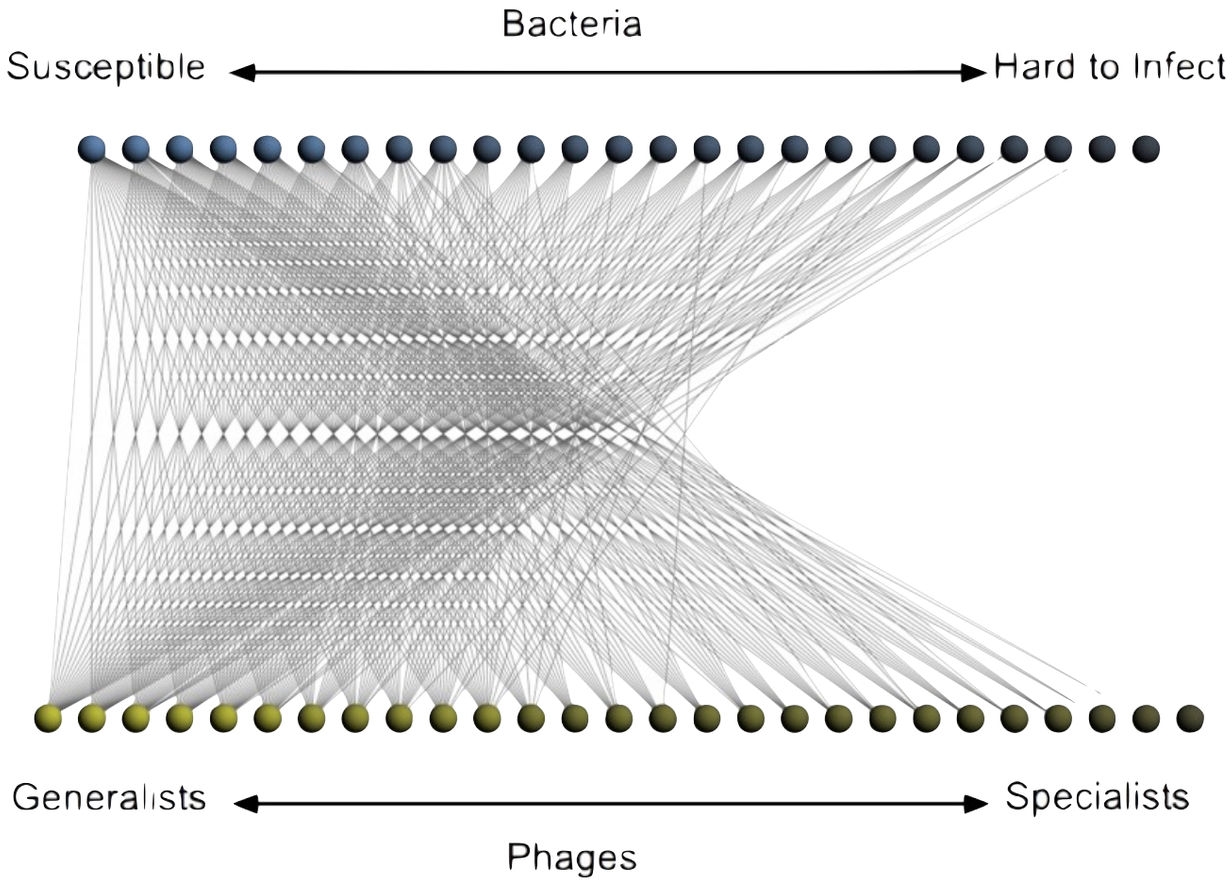
Soramaaki et al, Physica A 379, 317 (2007)



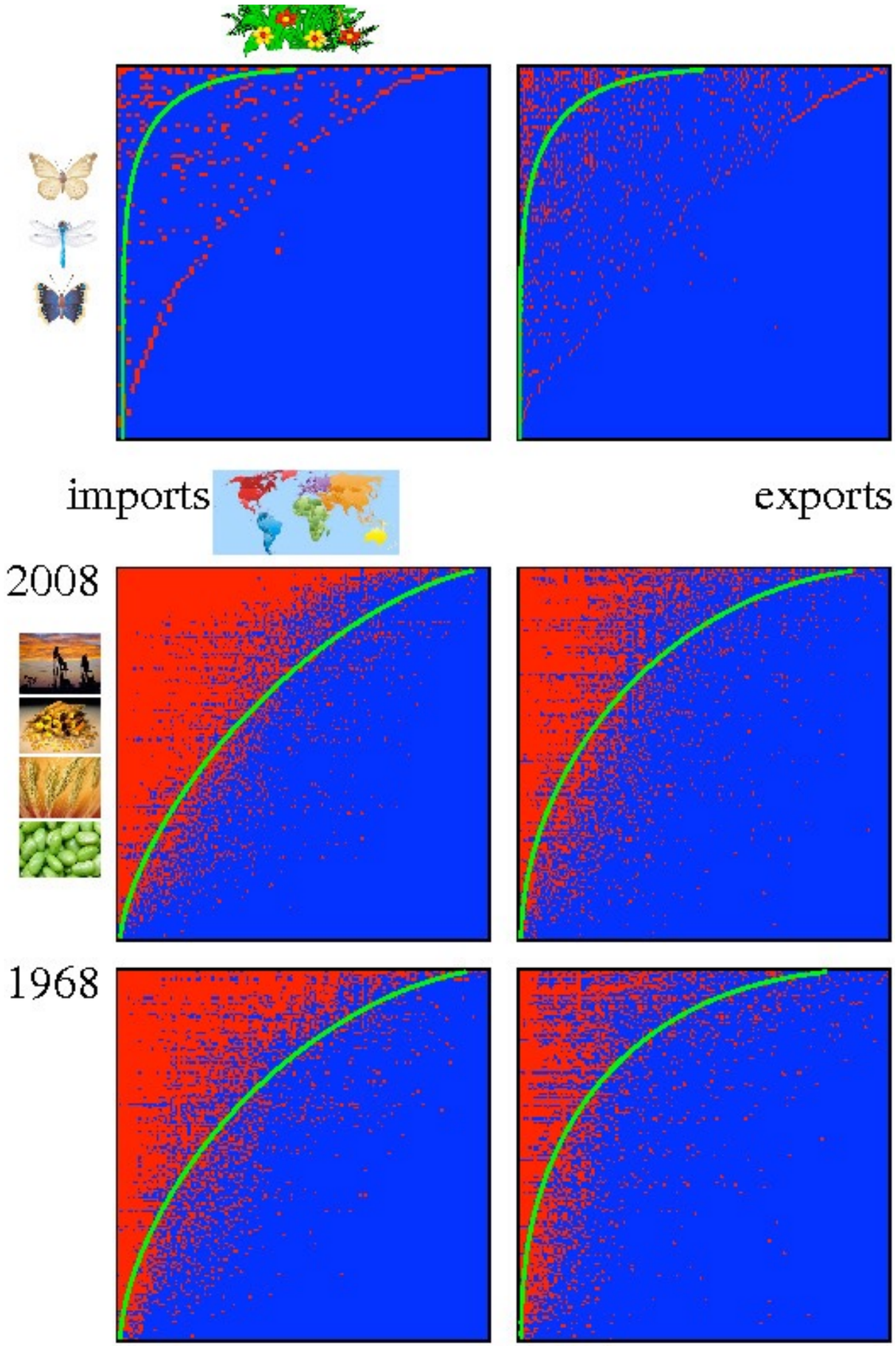
LE, K Frahm and D.L. Shepelyansky, Eur. Phys. J. B 91 (2018)

Ecological ranking for WTN

Nestedness



$$Q_{c,p}^{(i,e)} = \begin{cases} 1 & \text{if } m_{c,p}^{(i,e)} \geq \mu \\ 0 & \text{if } m_{c,p}^{(i,e)} < \mu \end{cases}$$



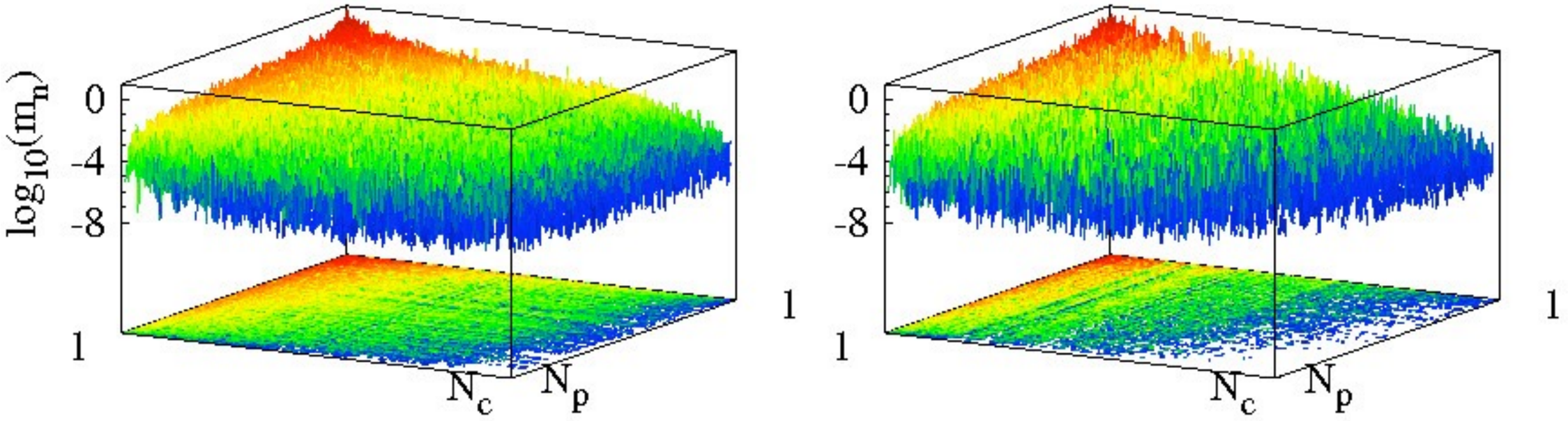
Nestedness

$$m^{(i,e)} = M^{(i,e)}/M_{\max}$$

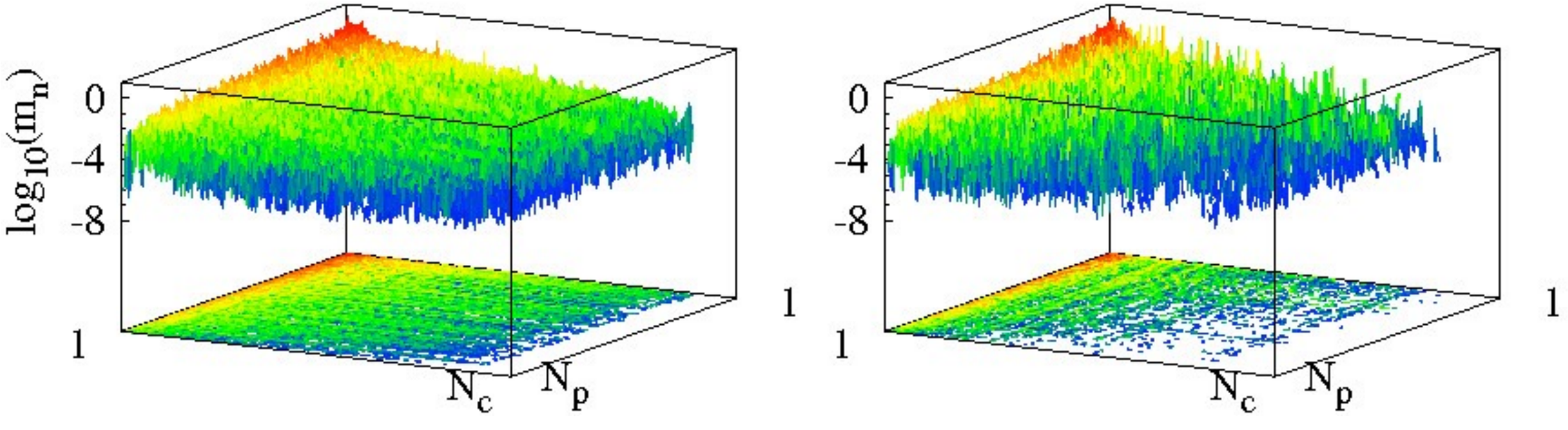
$$M_{p,c}^{(i)} = \sum_{c'=1}^{N_c} M_{c,c'}^{p}$$

$$M_{p,c}^{(e)} = \sum_{c'=1}^{N_c} M_{c',c}^{p}$$

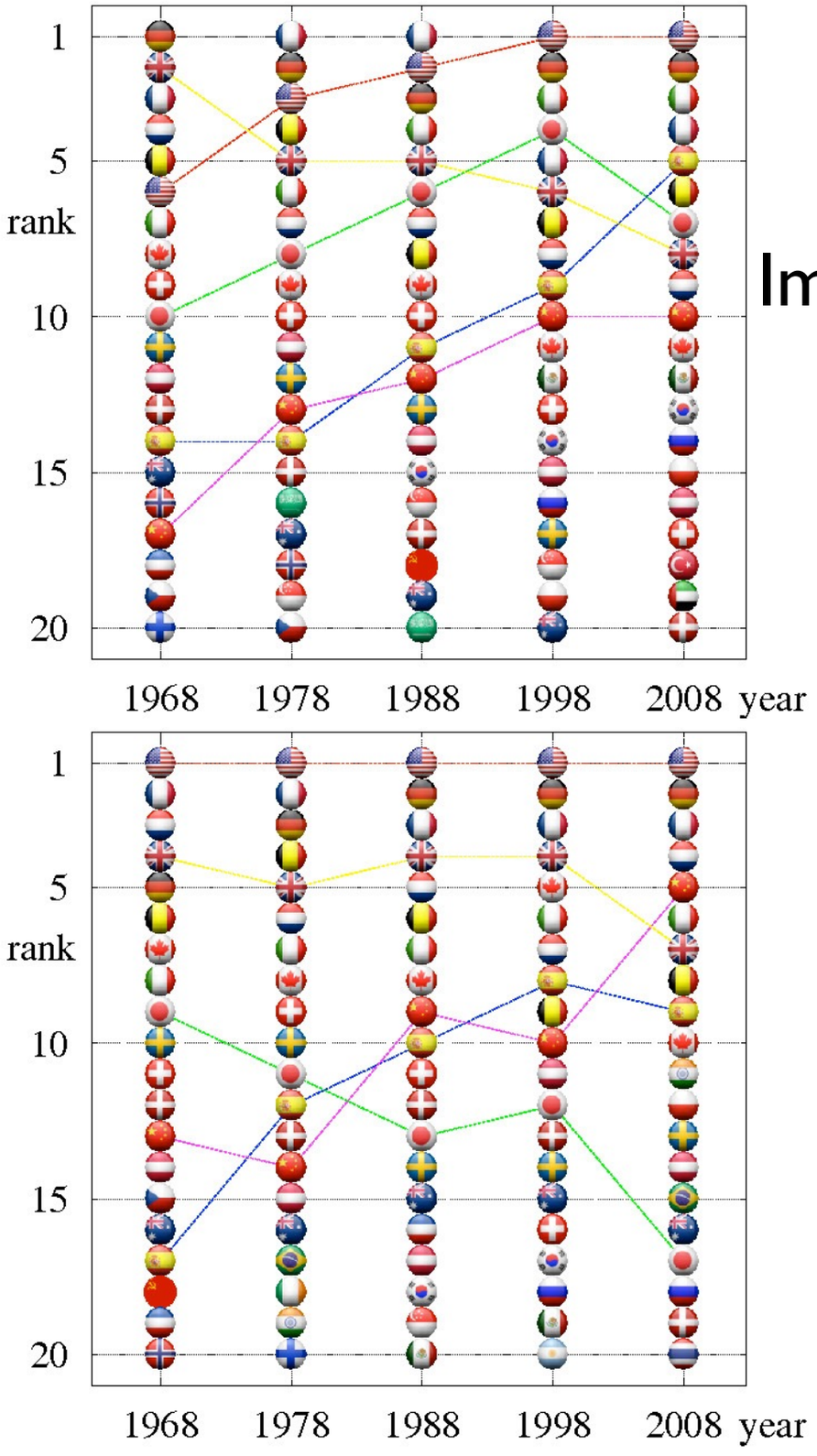
2008



1968

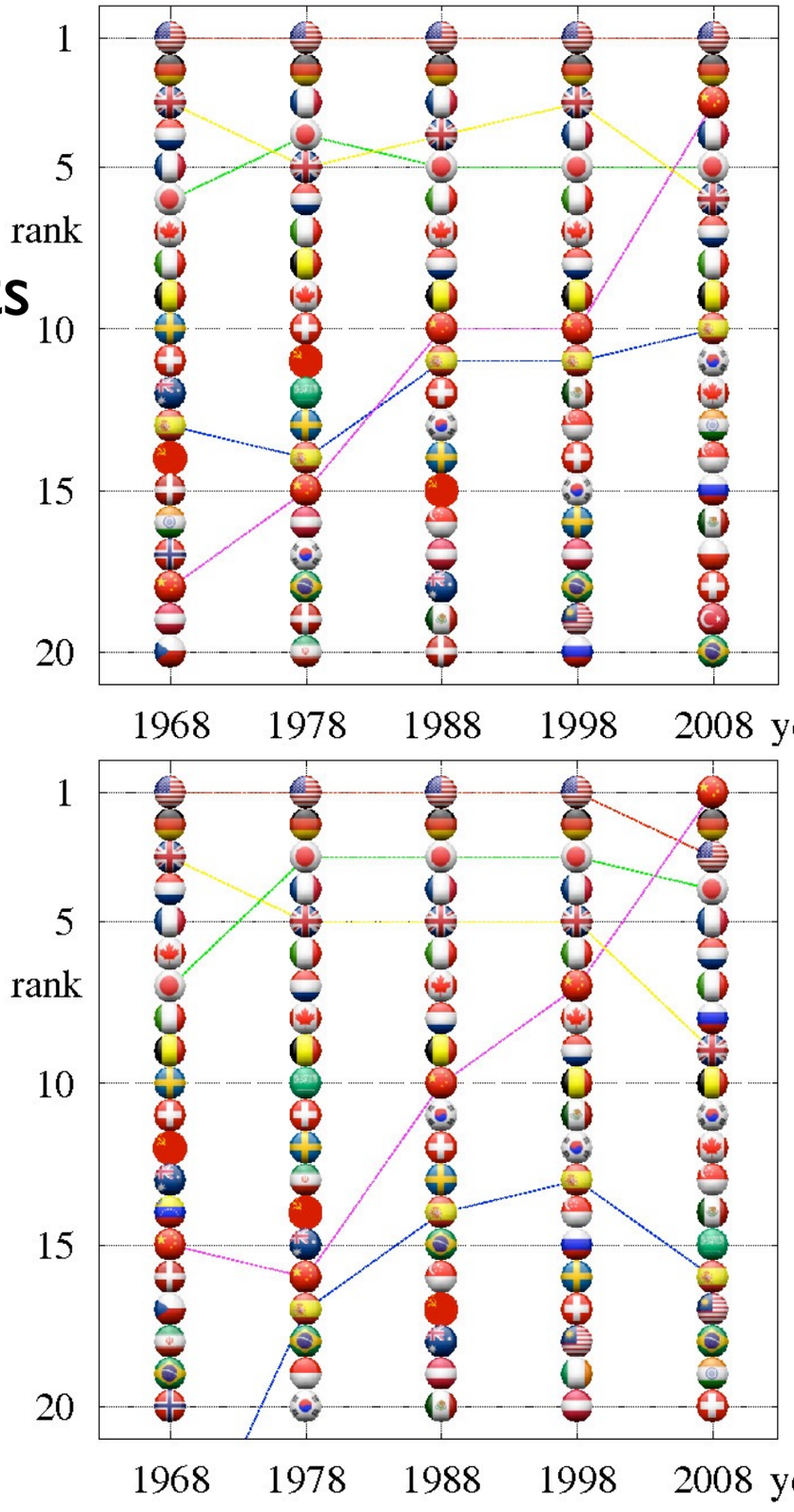


Ecolorank of countries



Imports

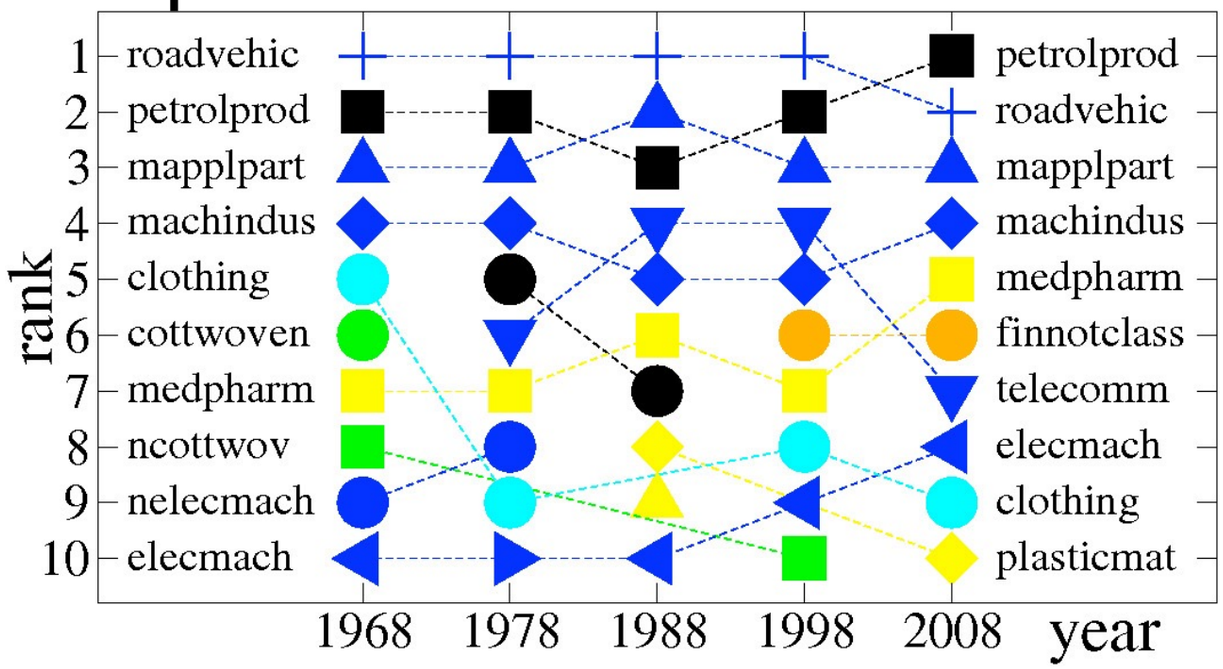
Exports



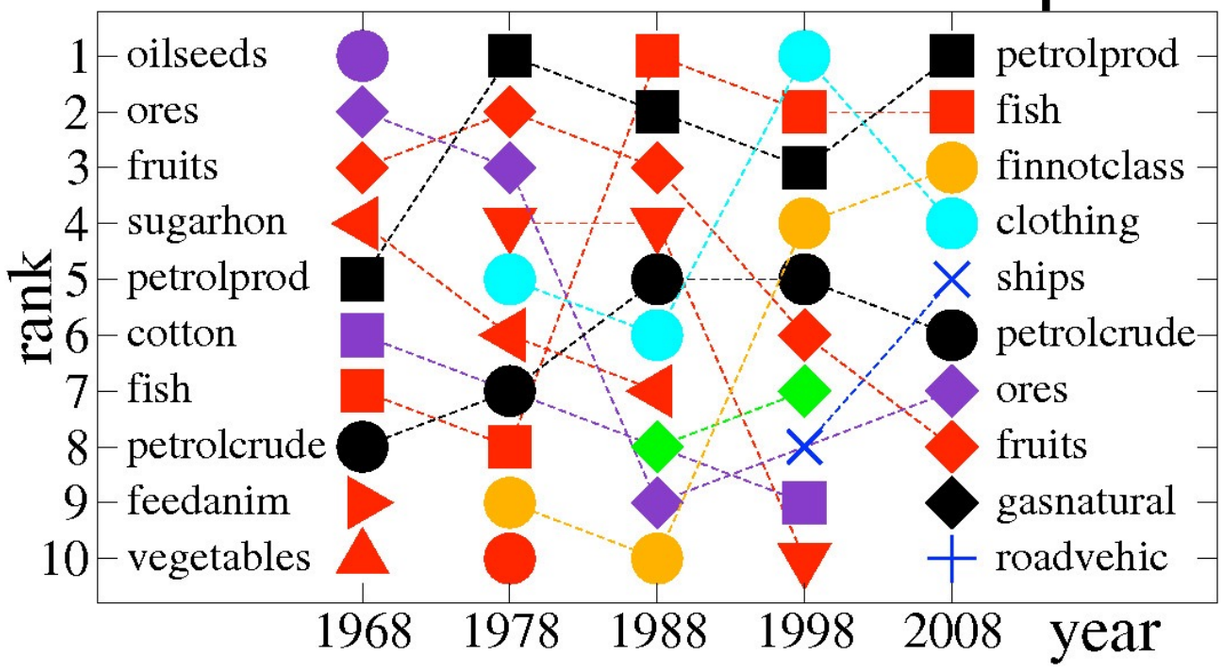
money ranking

Ecolorank of products

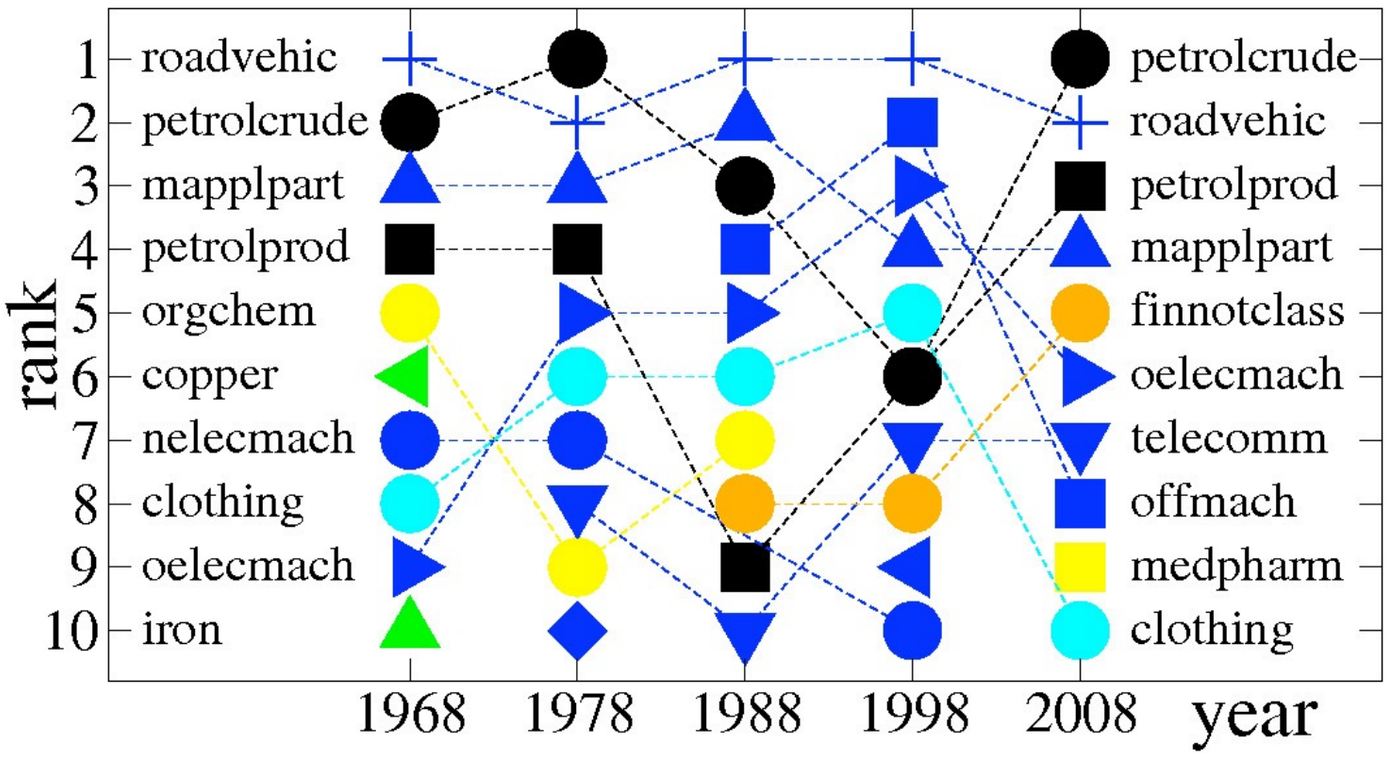
imports



exports



money rank



●	001	animals	Live animals
■	031	fish	Fish, fresh & simply preserved
♦	051	fruits	Fruit, fresh, and nuts excl. Oil nuts
▲	054	vegetables	Vegetables, roots & tubers, fresh or dried
◀	061	sugarhon	Sugar and honey
▼	071	coffee	Coffee
►	081	feedanim	Feed. stuff for animals excl. unmilled cereals
●	221	oilseeds	Oil seeds, oil nuts and oil kernels
■	263	cotton	Cotton
♦	283	ores	Ores & concentrates of non ferrous base metals
●	331	petrolcrude	Petroleum, crude and partly refined
■	332	petrolprod	Petroleum products
♦	341	gas	Gas, natural and manufactured
●	512	orgchem	Organic chemicals
■	541	medpharm	Medicinal & pharmaceutical products
♦	581	plasticmat	Plastic materials,regenerd. cellulose & resins
▲	599	chemmat	Chemical materials and products, nes
●	652	cottwoven	Cotton fabrics, woven ex. narrow or spec.fabrics
■	653	ncottwov	Text fabrics woven ex narrow, spec, not cotton
♦	667	pearlsprec	Pearls and precious and semi precious stones
▲	674	iron	Universals, plates and sheets of iron or steel
◀	682	copper	Copper
●	711	nelecmach	Power generating machinery, other than electric
■	714	offmach	Office machines
♦	718	machindus	Machines for special industries
▲	719	mapplpart	Machinery and appliances non electrical parts
◀	722	elecmach	Electric power machinery and switchgear
▼	724	telecomm	Telecommunications apparatus
►	729	oelecmach	Other electrical machinery and apparatus
+	732	roadvehicles	Road motor vehicles
×	735	ships	Ships and boats
●	841	clothing	Clothing except fur clothing
●	931	finnotclass	Special transactions not classd. accord.to kind

Conclusions

- Google matrix of the WTN (democratic in countries, global network properties):
  1) one product of all comm. (Nc)
  2) multiprod (Nc x Np)
  2d-ranking, spectrum, communities in eigenstates, correlation between P-C, comparison with I-E, new tool for trade analysis
- Dynamics, crisis, sensitivity to price variation
- reduced Google matrix on WTN
- Nestedness based ranking of WTN
- Bitcoin network

Thank you

Fractal Weyl law in complex networks

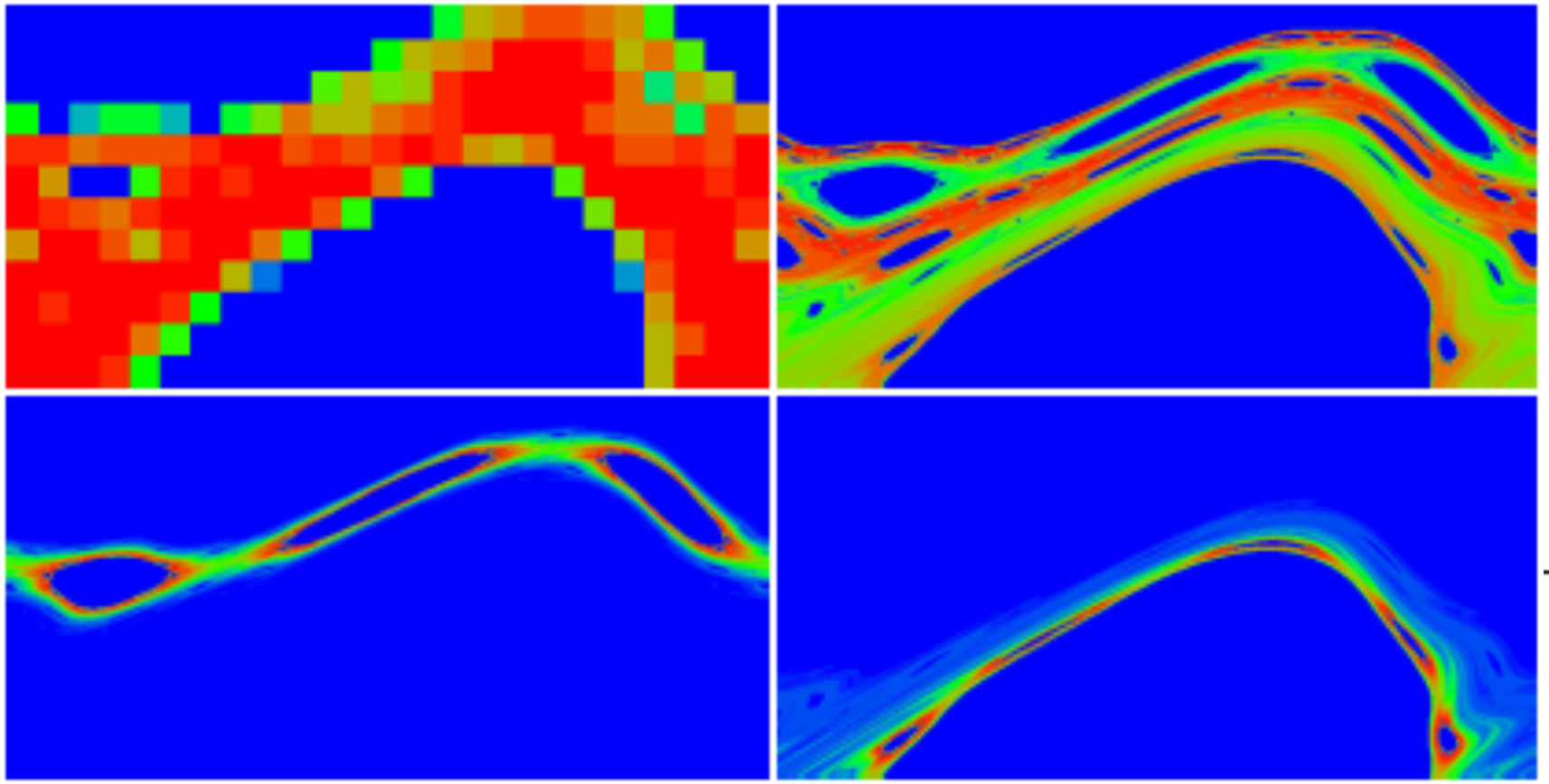
Fractal Weyl law

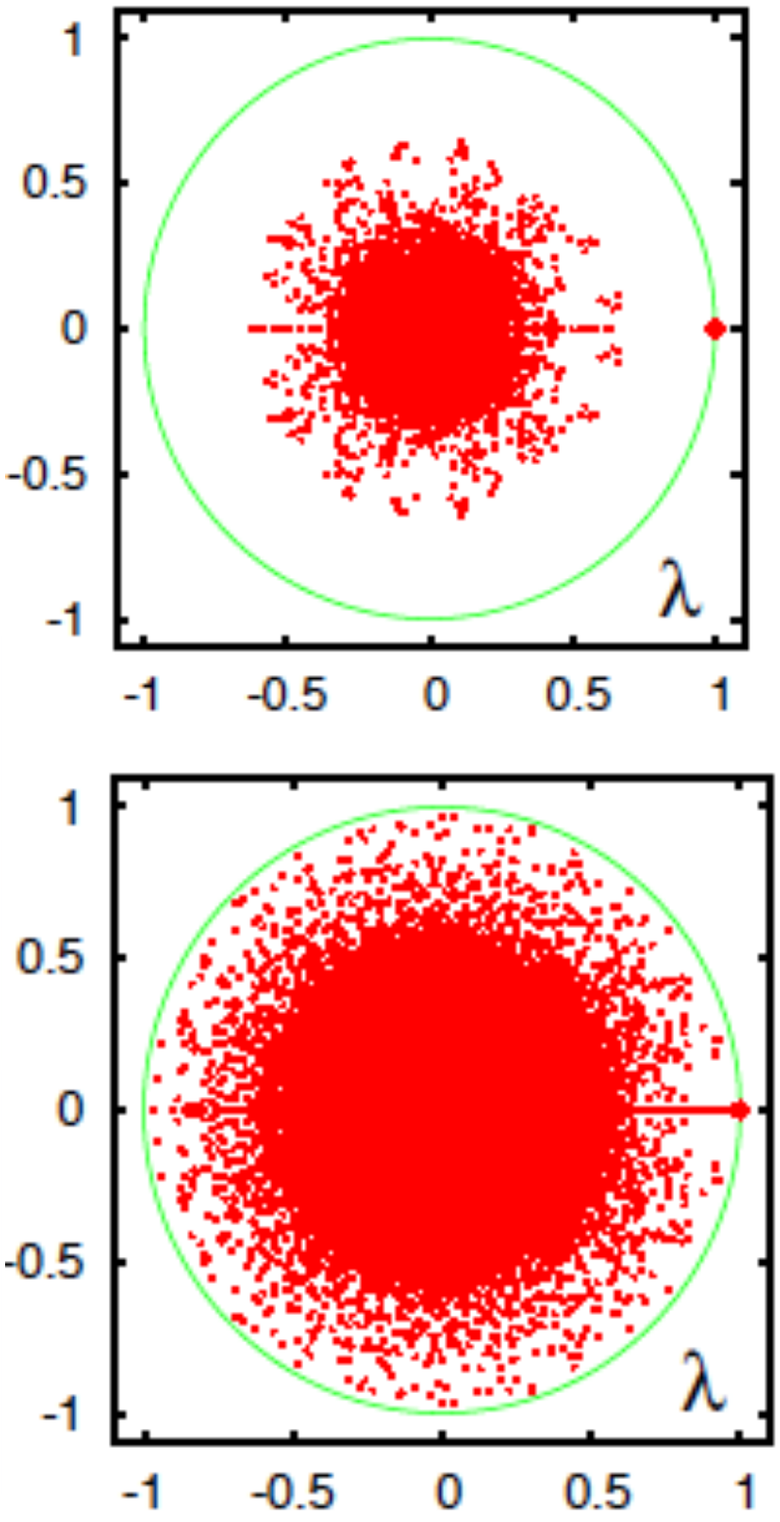
(Introduction by Nonnenmacher yesterday)

Chirikov standard map

$$\bar{p} = p + K \sin x$$
$$\bar{x} = x + \bar{p}$$

Ulam Method

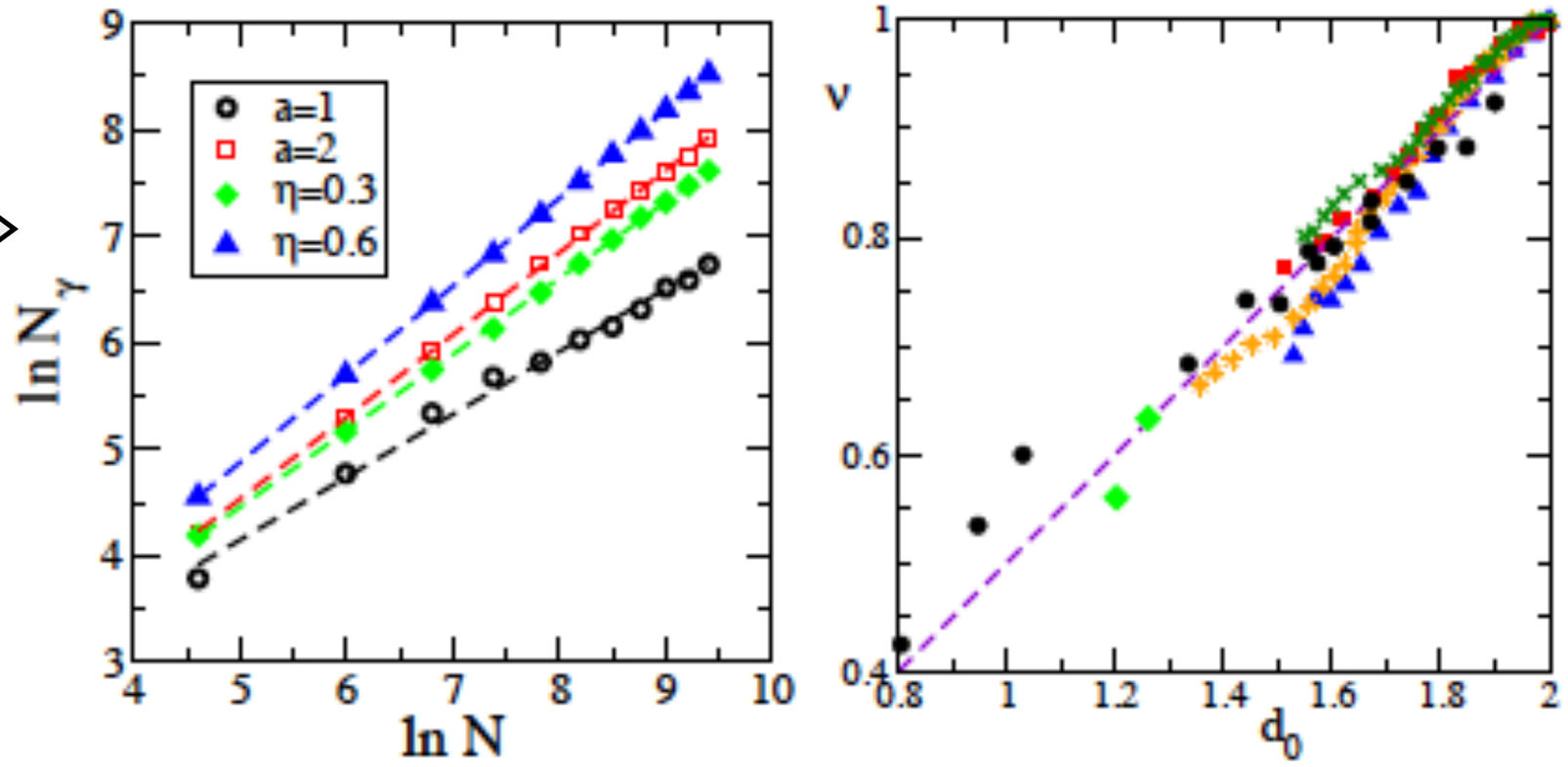


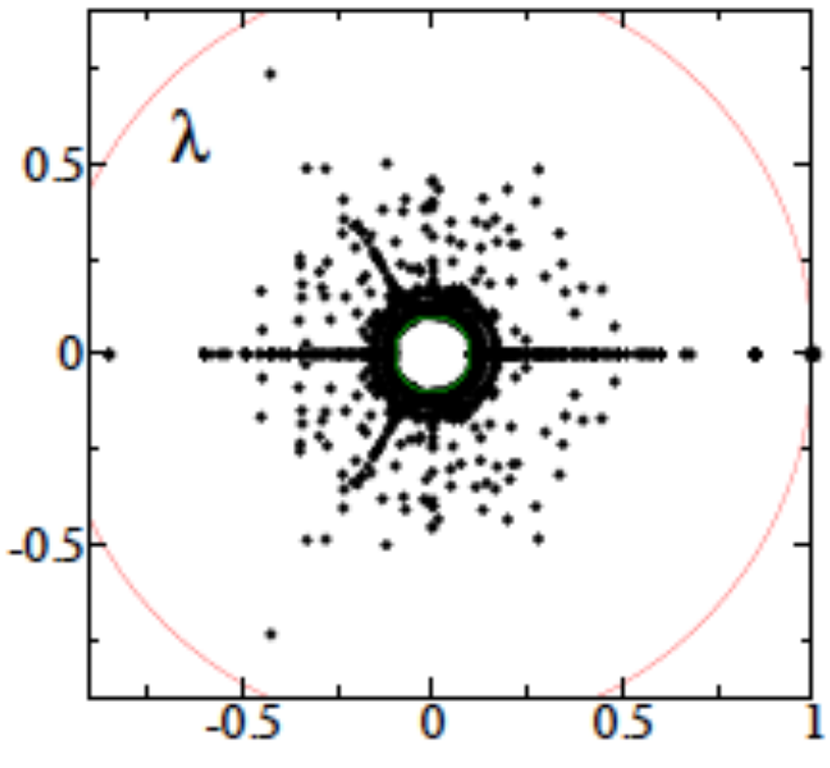


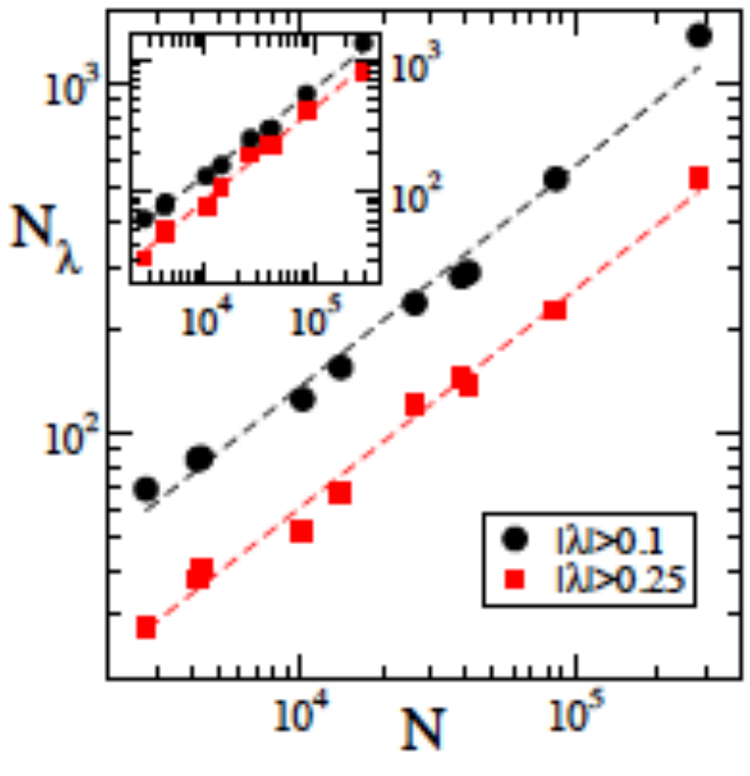
Fractal Weyl law

Standard map

L.E and Shepelyansky, Eur. Phys. J. B 75, 299 (2010)







Linux Kernel architecture network

L.E, Chepelianskii and Shepelyansky, Eur. Phys. J. B 79, 115 (2011)

product names

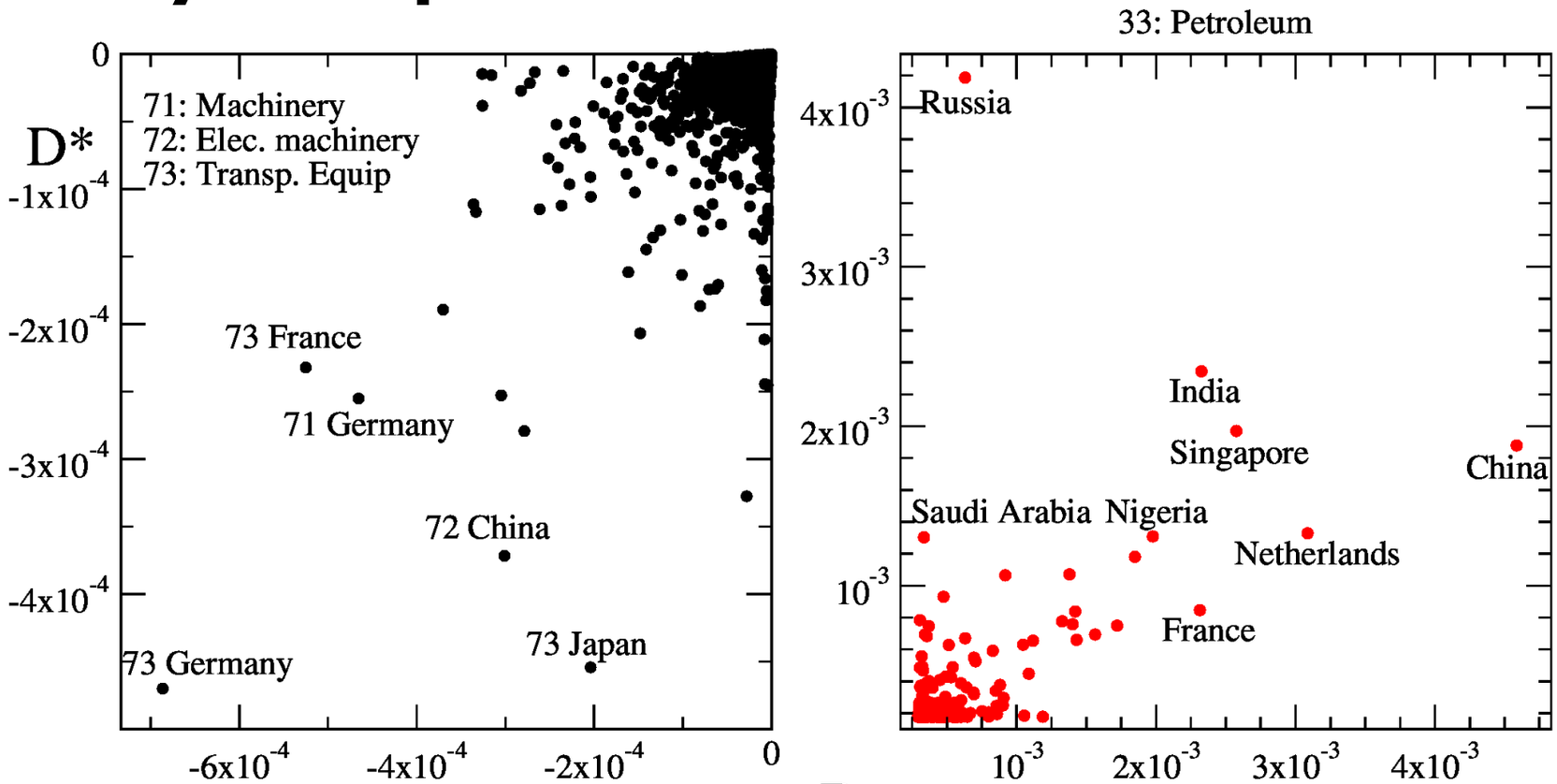
code	name	code	name
00	Live animals	54	Medicinal and pharmaceutical products
01	Meat and meat preparations	55	Perfume materials, toilet & cleansing preptions
02	Dairy products and eggs	56	Fertilizers, manufactured
03	Fish and fish preparations	57	Explosives and pyrotechnic products
04	Cereals and cereal preparations	58	Plastic materials, etc.
05	Fruit and vegetables	59	Chemical materials and products, nes
06	Sugar, sugar preparations and honey	61	Leather, lthr. Manufs., nes & dressed fur skins
07	Coffee, tea, cocoa, spices & manufacs. Thereof	62	Rubber manufactures, nes
08	Feed. Stuff for animals excl. Unmilled cereals	63	Wood and cork manufactures excluding furniture
09	Miscellaneous food preparations	64	Paper, paperboard and manufactures thereof
11	Beverages	65	Textile yarn, fabrics, made up articles, etc.
12	Tobacco and tobacco manufactures	66	Non metallic mineral manufactures, nes
21	Hides, skins and fur skins, undressed	67	Iron and steel
22	Oil seeds, oil nuts and oil kernels	68	Non ferrous metals
23	Crude rubber including synthetic and reclaimed	69	Manufactures of metal, nes
24	Wood, lumber and cork	71	Machinery, other than electric
25	Pulp and paper	72	Electrical machinery, apparatus and appliances
26	Textile fibres, not manufactured, and waste	73	Transport equipment
27	Crude fertilizers and crude minerals, nes	81	Sanitary, plumbing, heating and lighting fixt.
28	Metalliferous ores and metal scrap	82	Furniture
29	Crude animal and vegetable materials, nes	83	Travel goods, handbags and similar articles
32	Coal, coke and briquettes	84	Clothing
33	Petroleum and petroleum products	85	Footwear
34	Gas, natural and manufactured	86	Scientif & control instrum, photogr gds, clocks
35	Electric energy	89	Miscellaneous manufactured articles, nes
41	Animal oils and fats	91	Postal packages not class. According to kind
42	Fixed vegetable oils and fats	93	Special transact. Not class. According to kind
43	Animal and vegetable oils and fats, processed	94	Animals, nes, incl. Zoo animals, dogs and cats
51	Chemical elements and compounds	95	Firearms of war and ammunition therefor
52	Crude chemicals from coal, petroleum and gas	96	Coin, other than gold coin, not legal tender
53	Dyeing, tanning and colouring materials		

Sensitivity to price variation

$$D = dP/d\delta = \Delta P/\delta$$

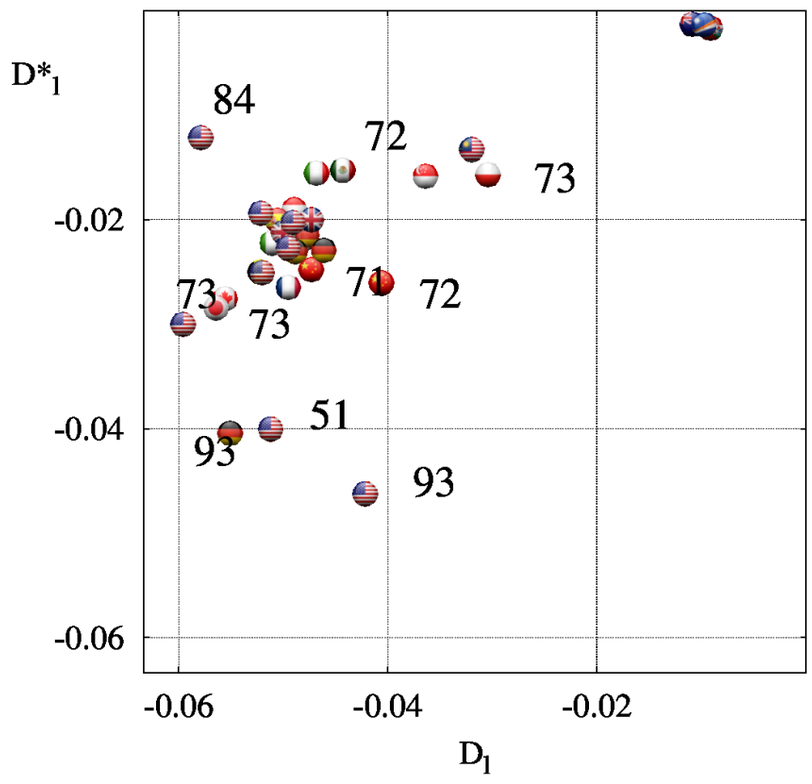
$$D^* = dP^*/d\delta = \Delta P^*/\delta.$$

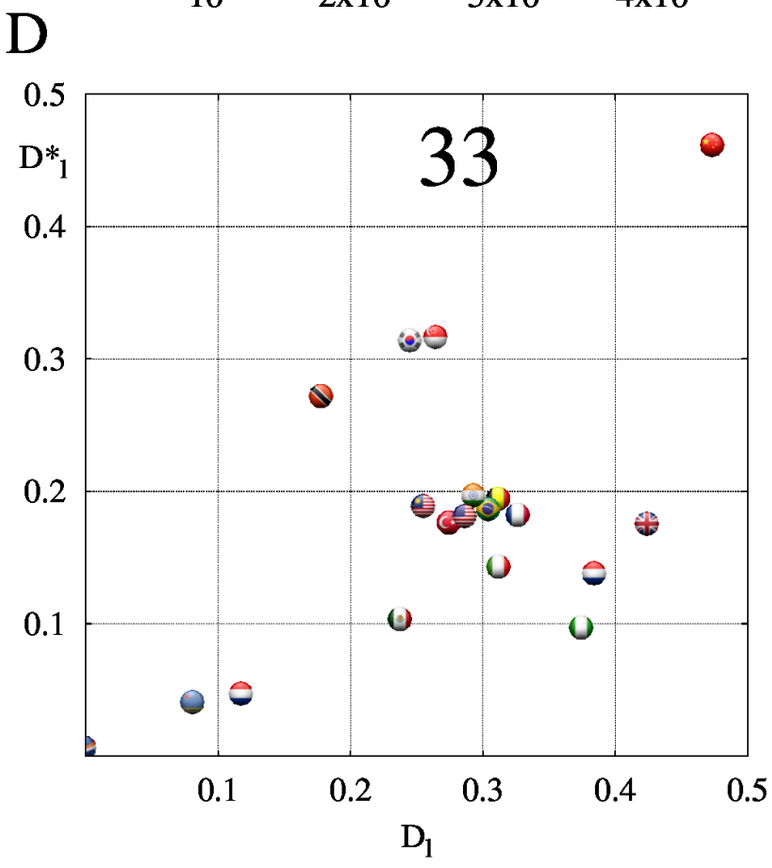
$$\delta' = 0.01, 0.03, 0.05$$



$$D_l = D/P \text{ and } D_l^* = D^*/P^*$$

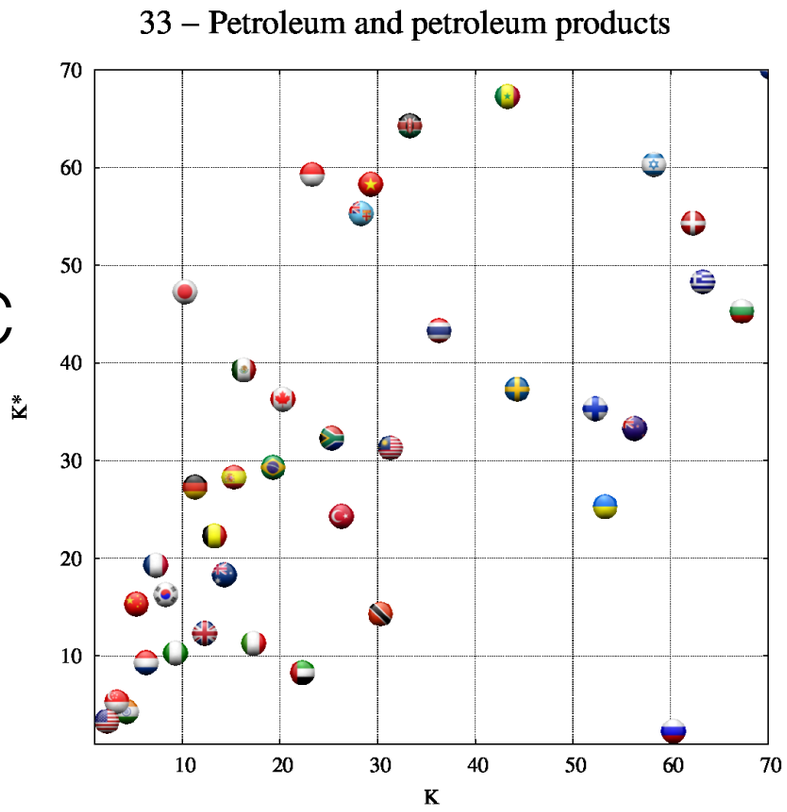
$$K_2 \leq 50,$$

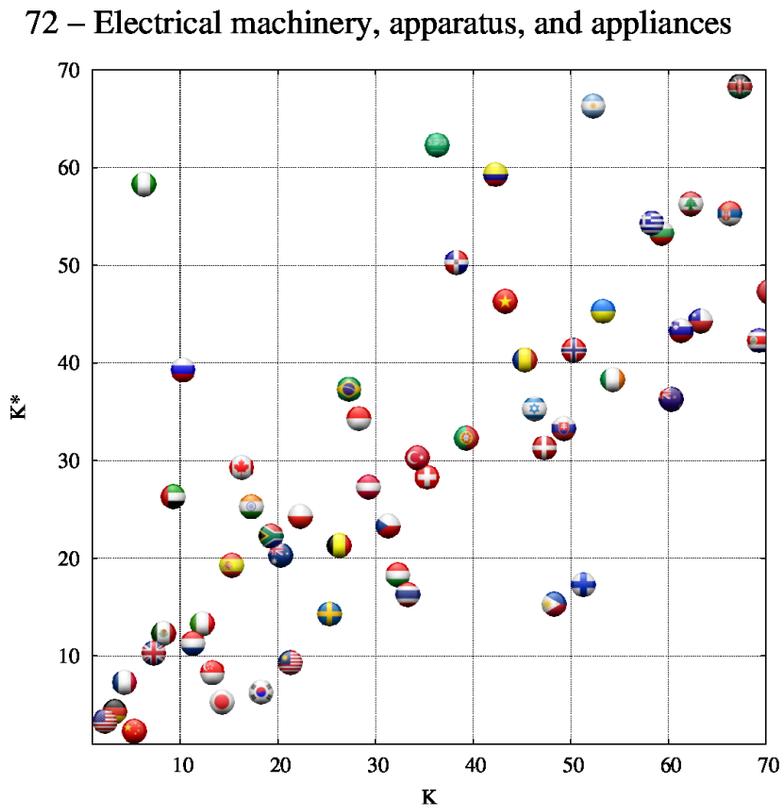


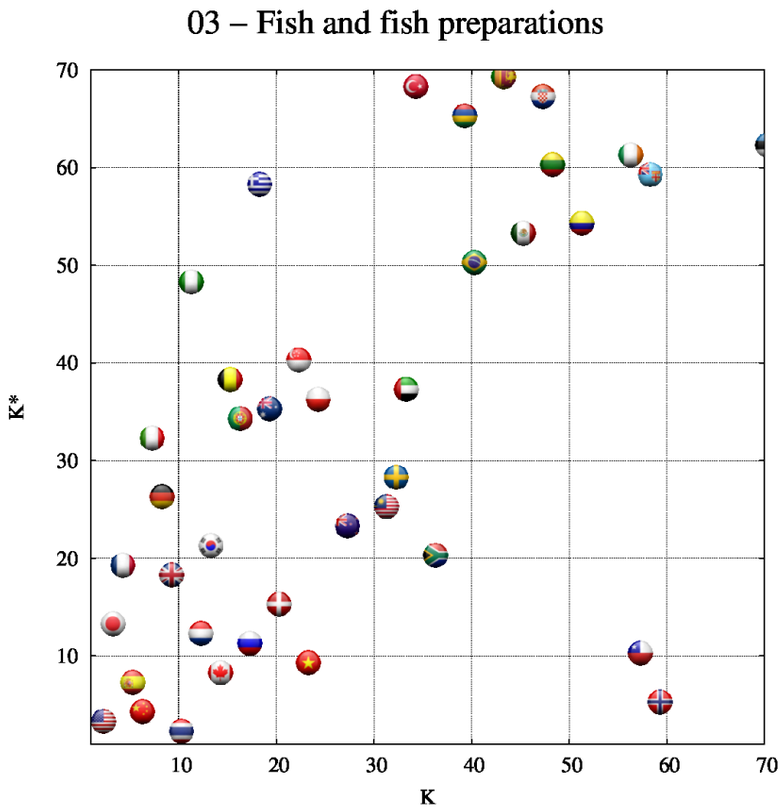


2d reduced ranks

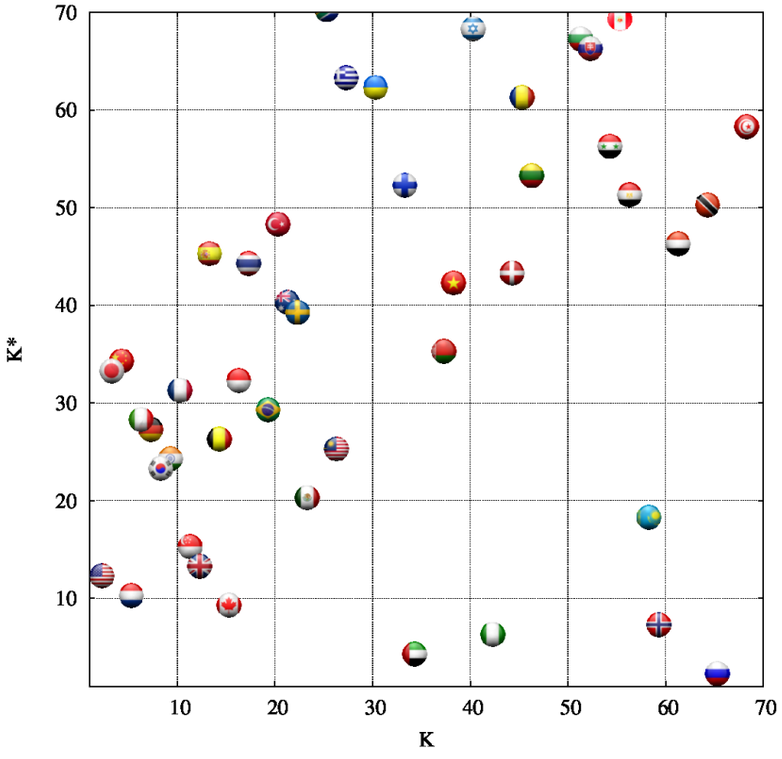
P-C

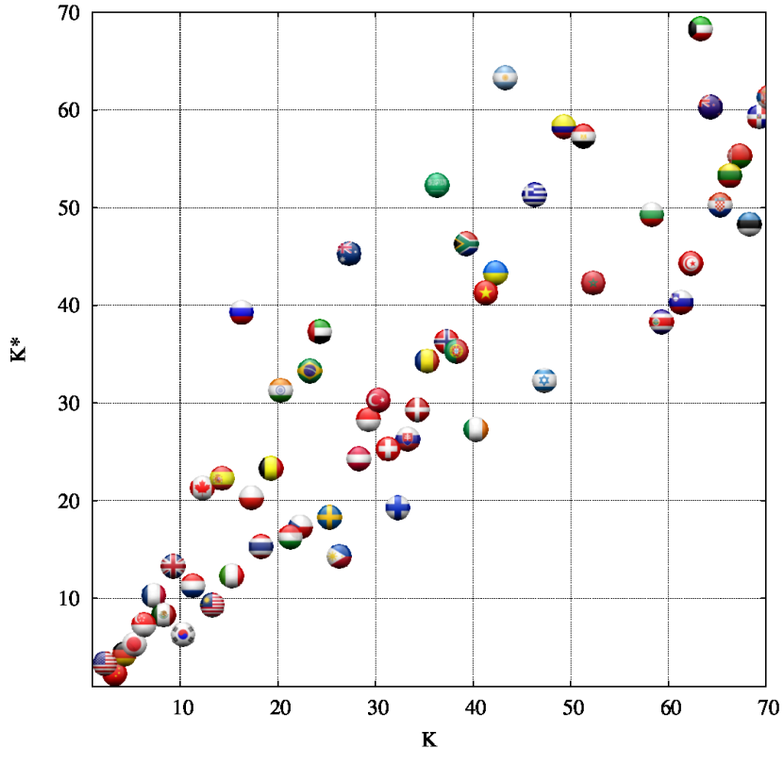


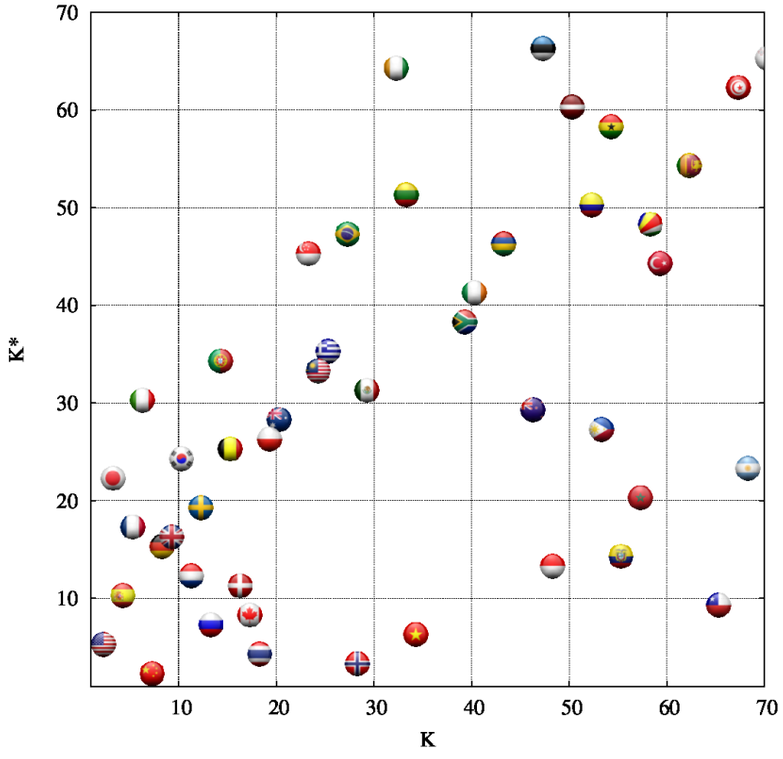




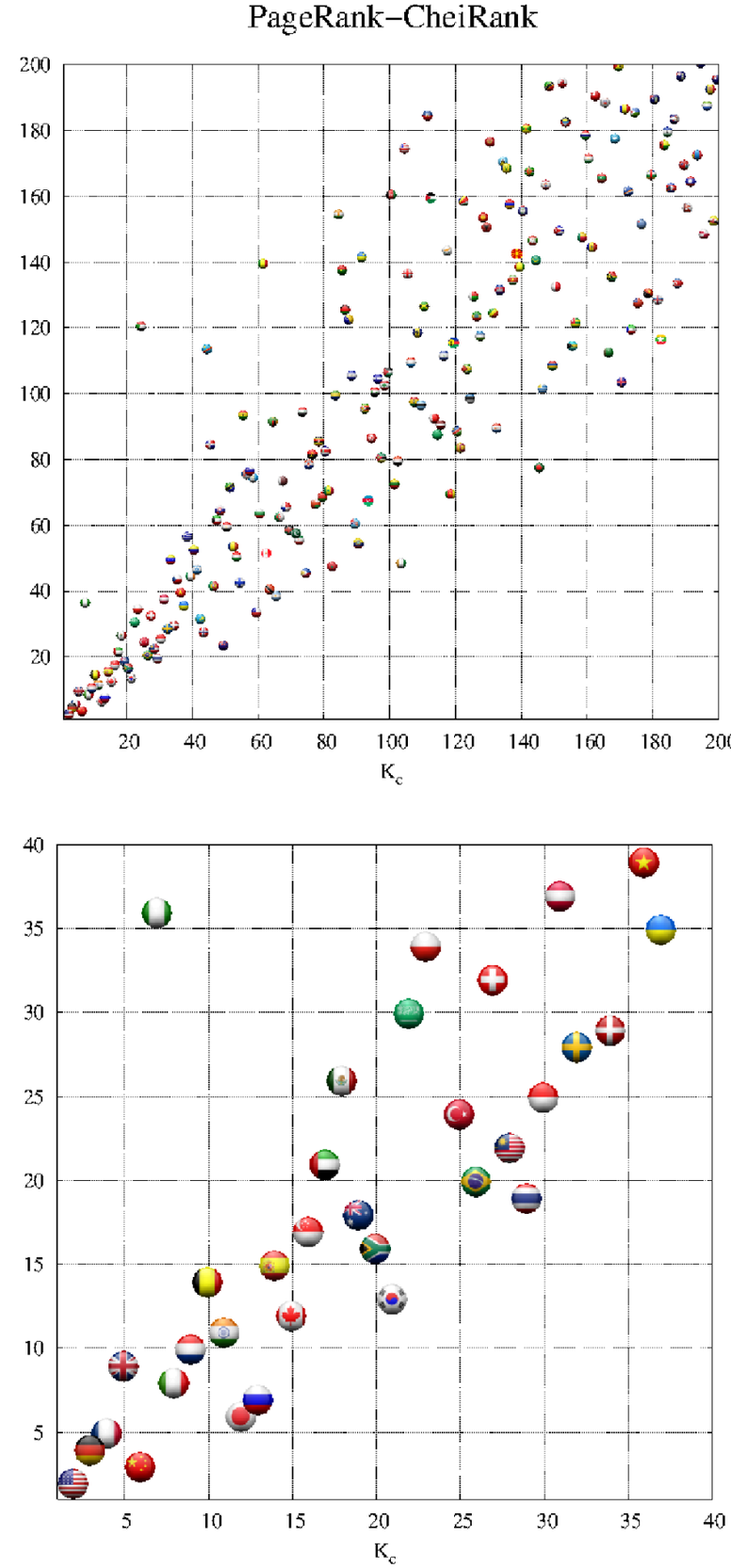
I-E

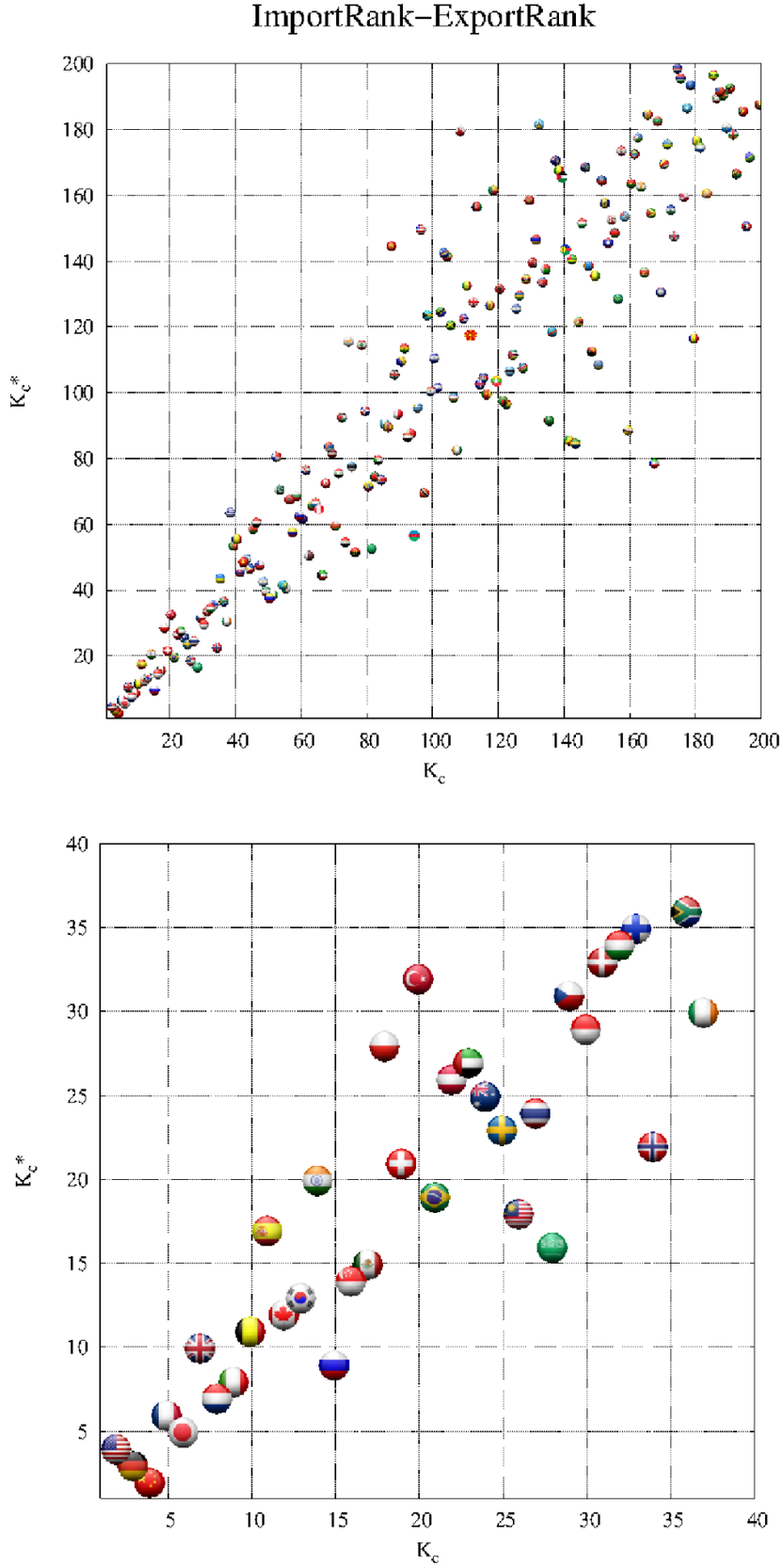


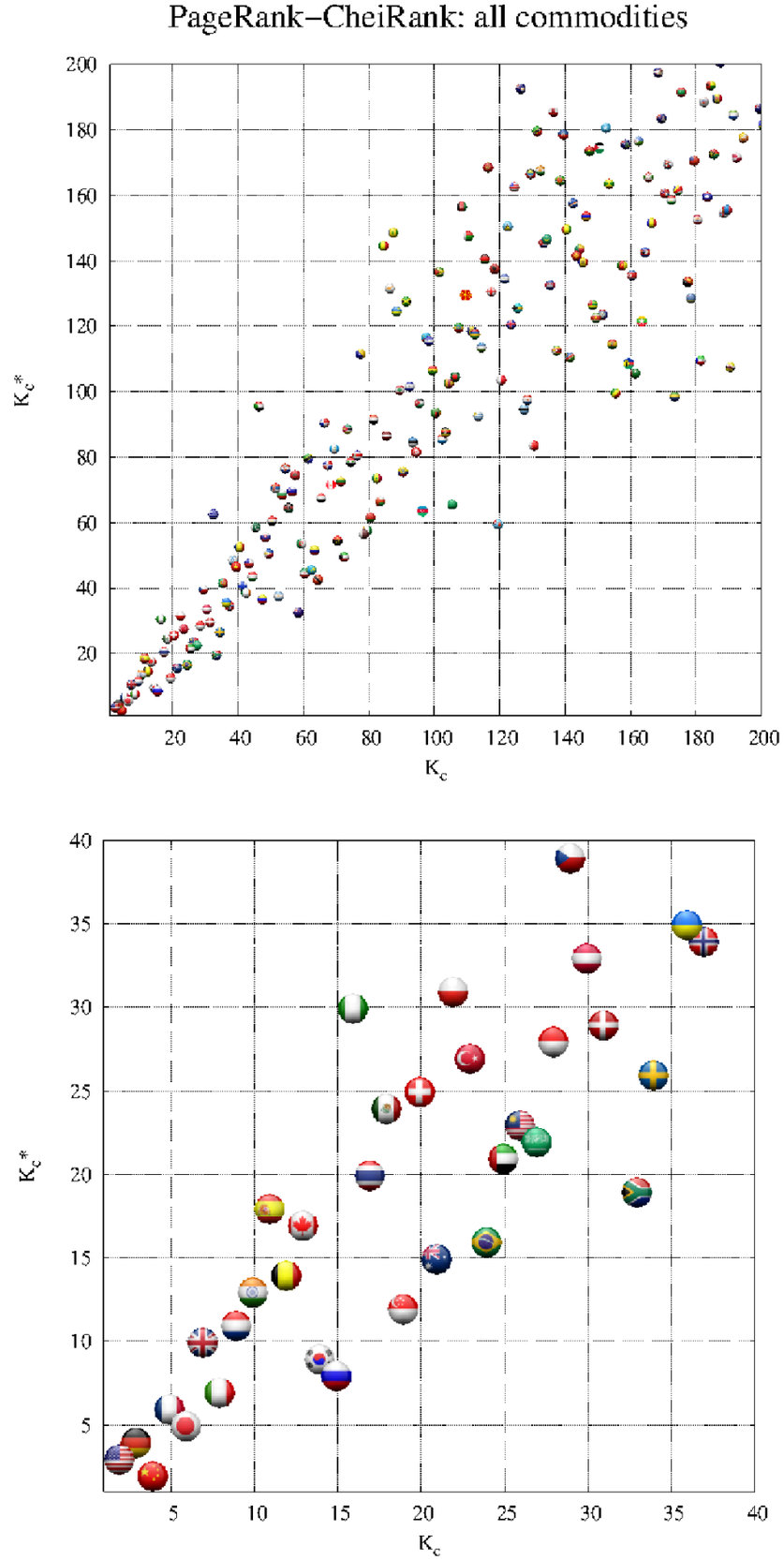




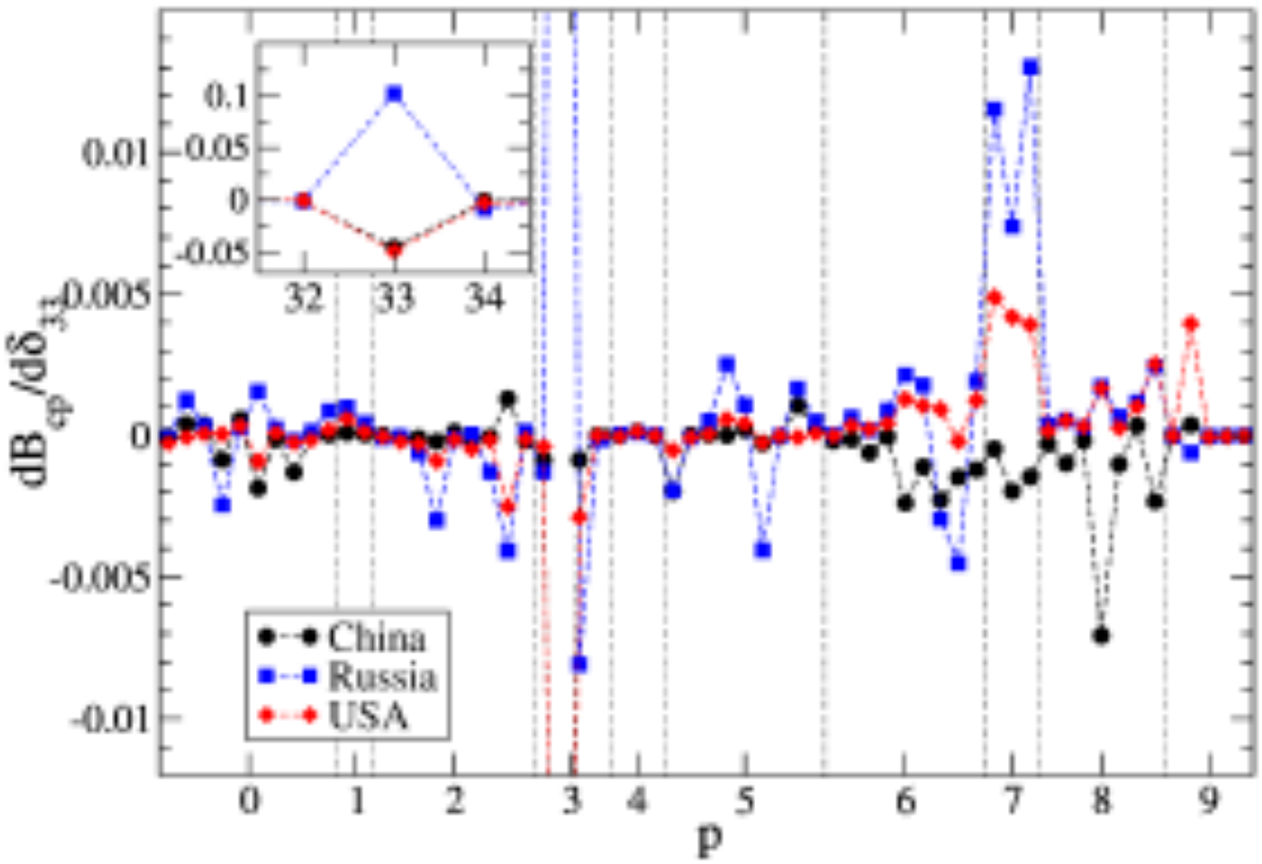
2d ranking of countries (multiproducts)

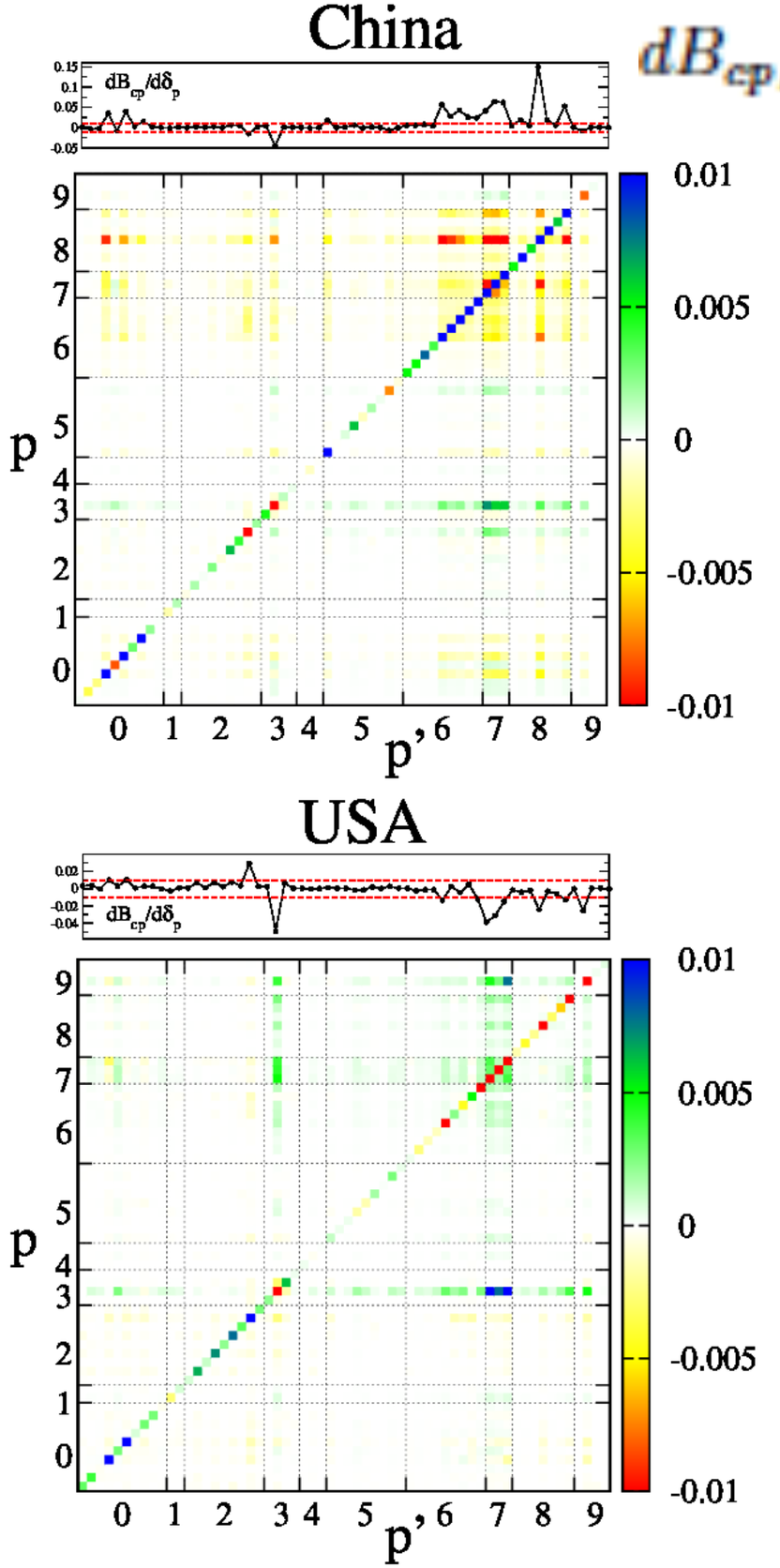




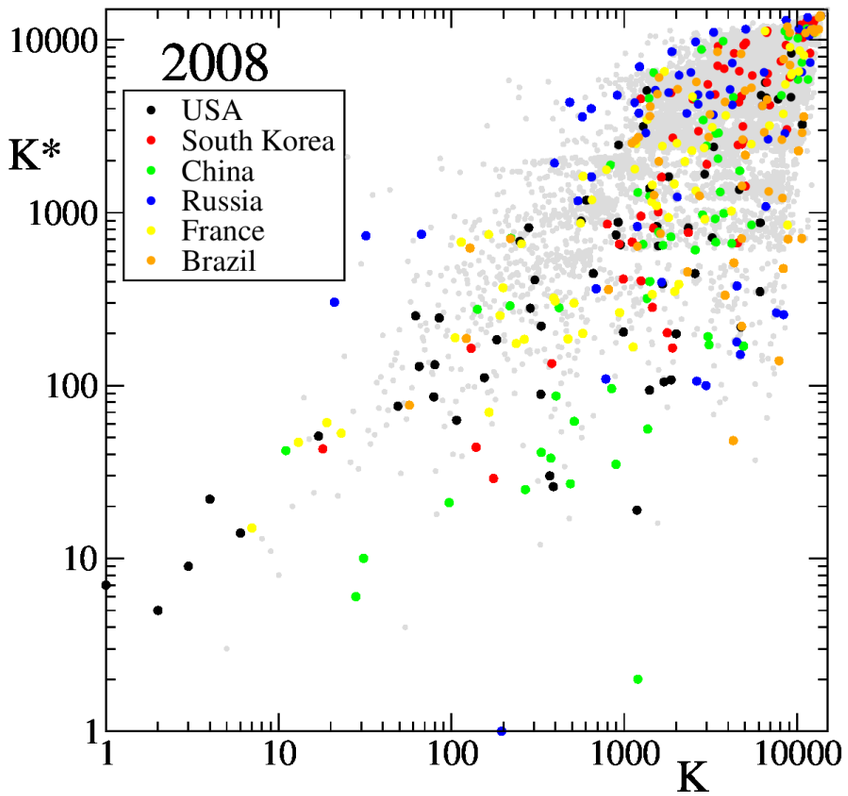


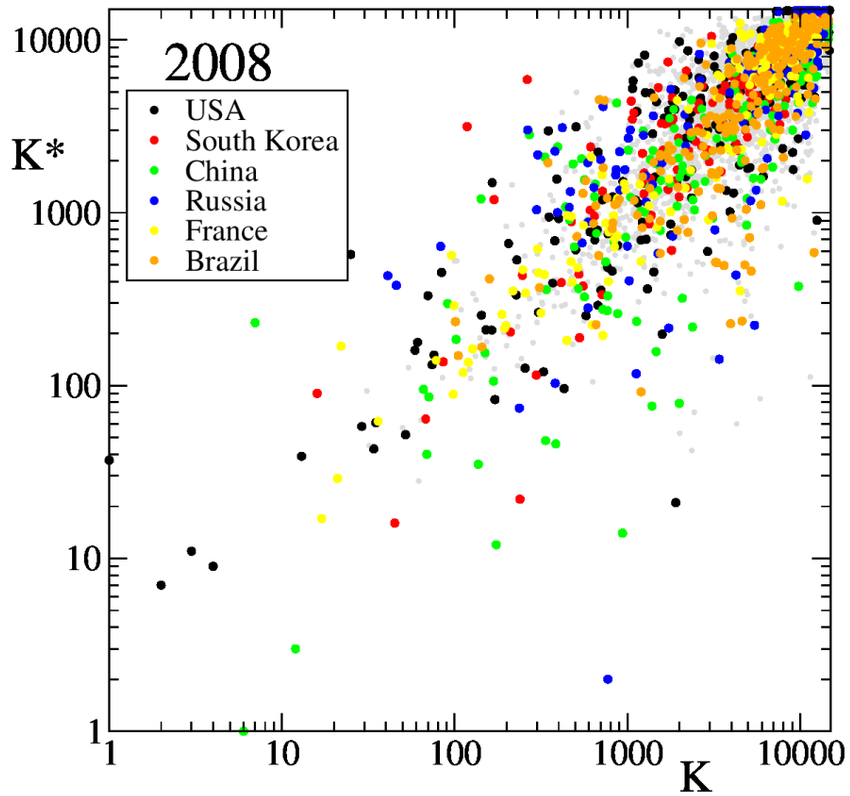
Sensitivity to price variation II

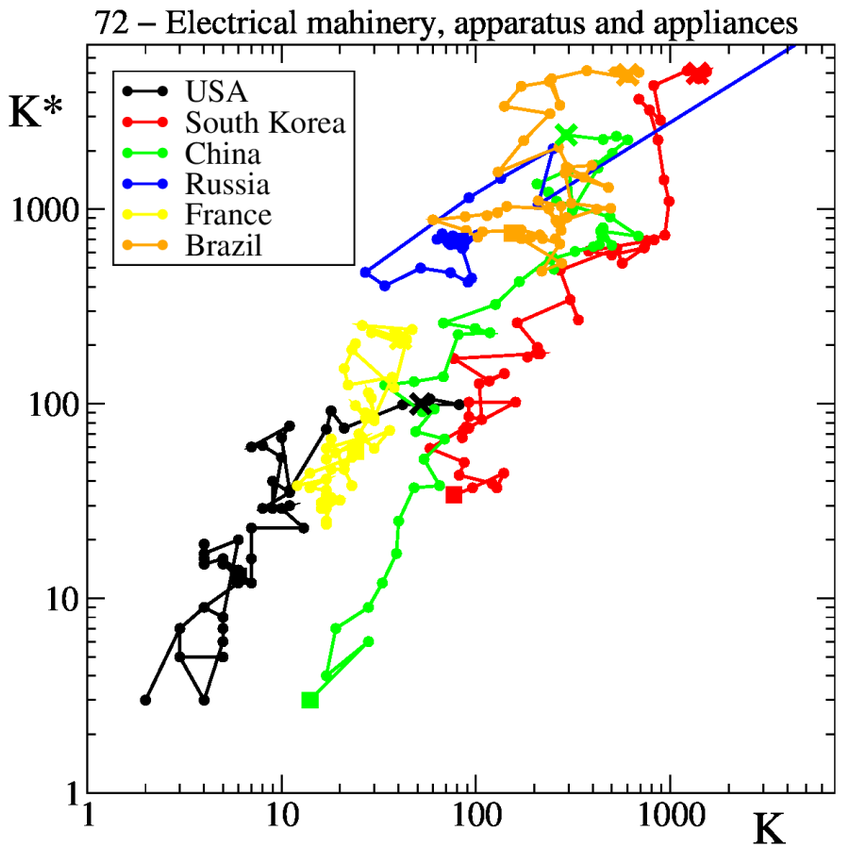


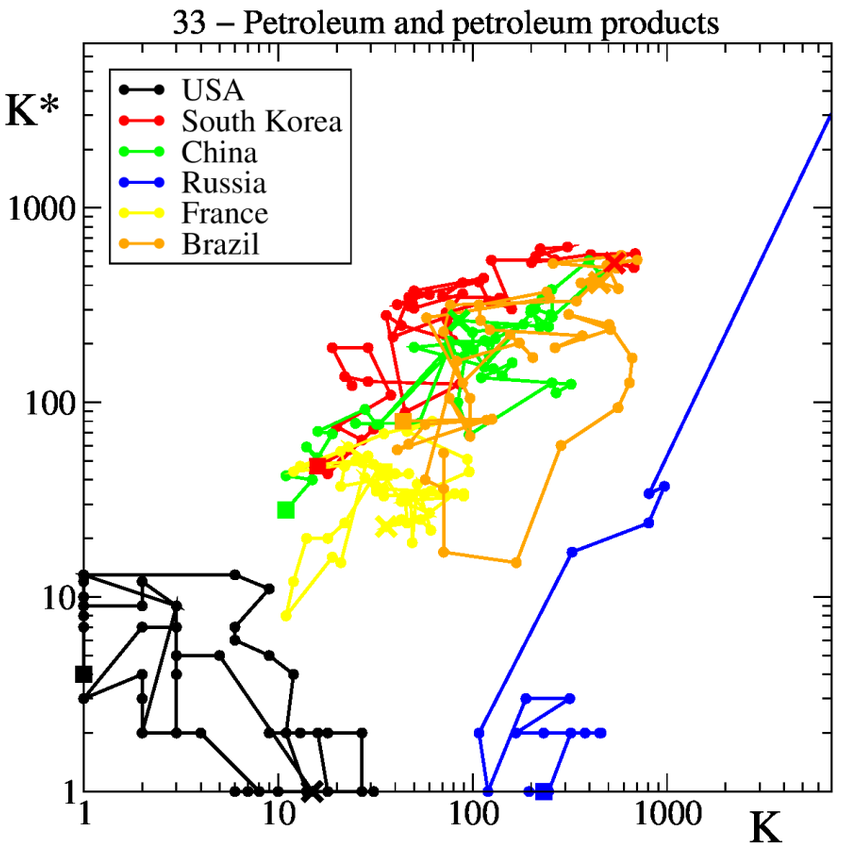


2d global ranks

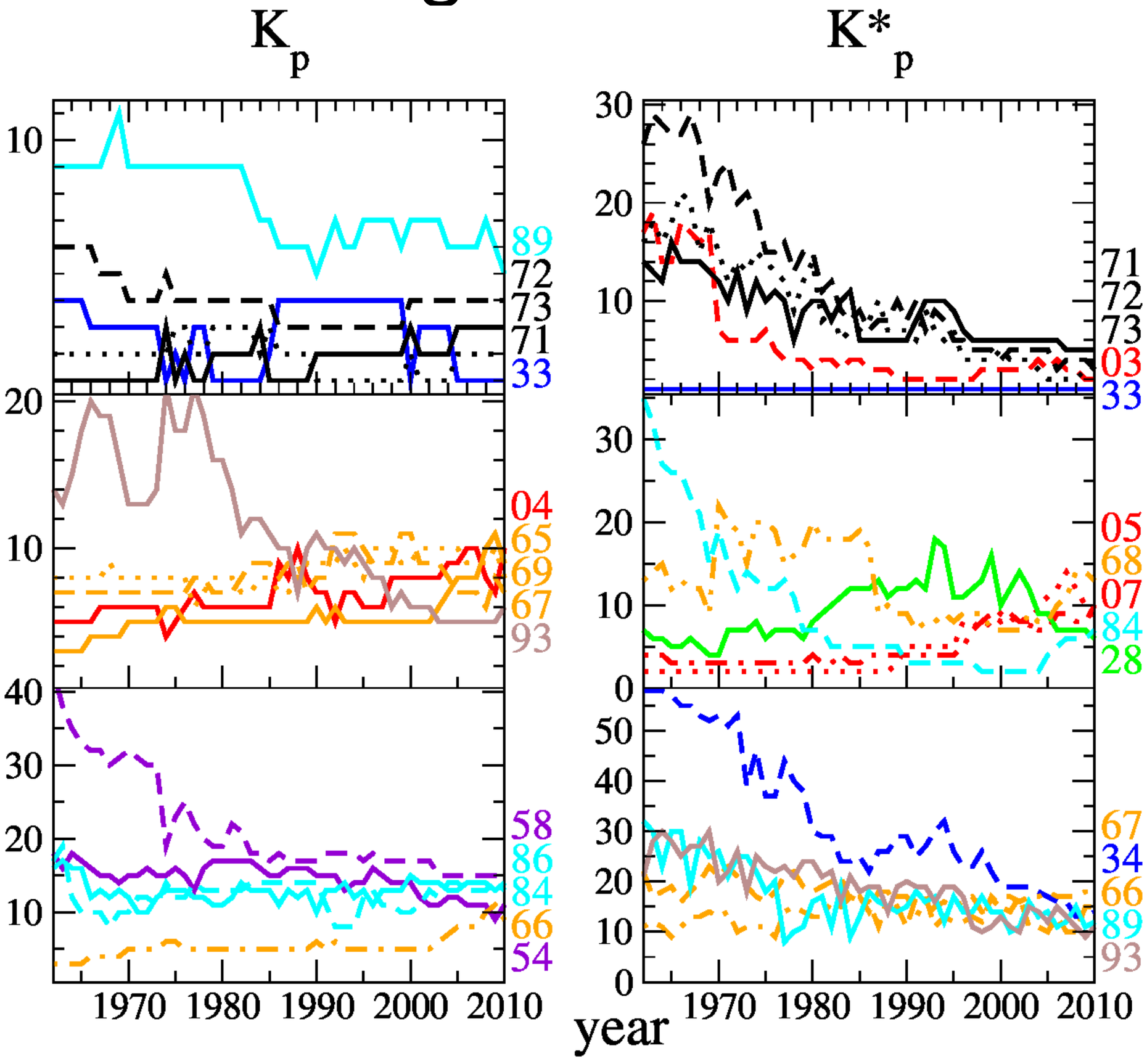




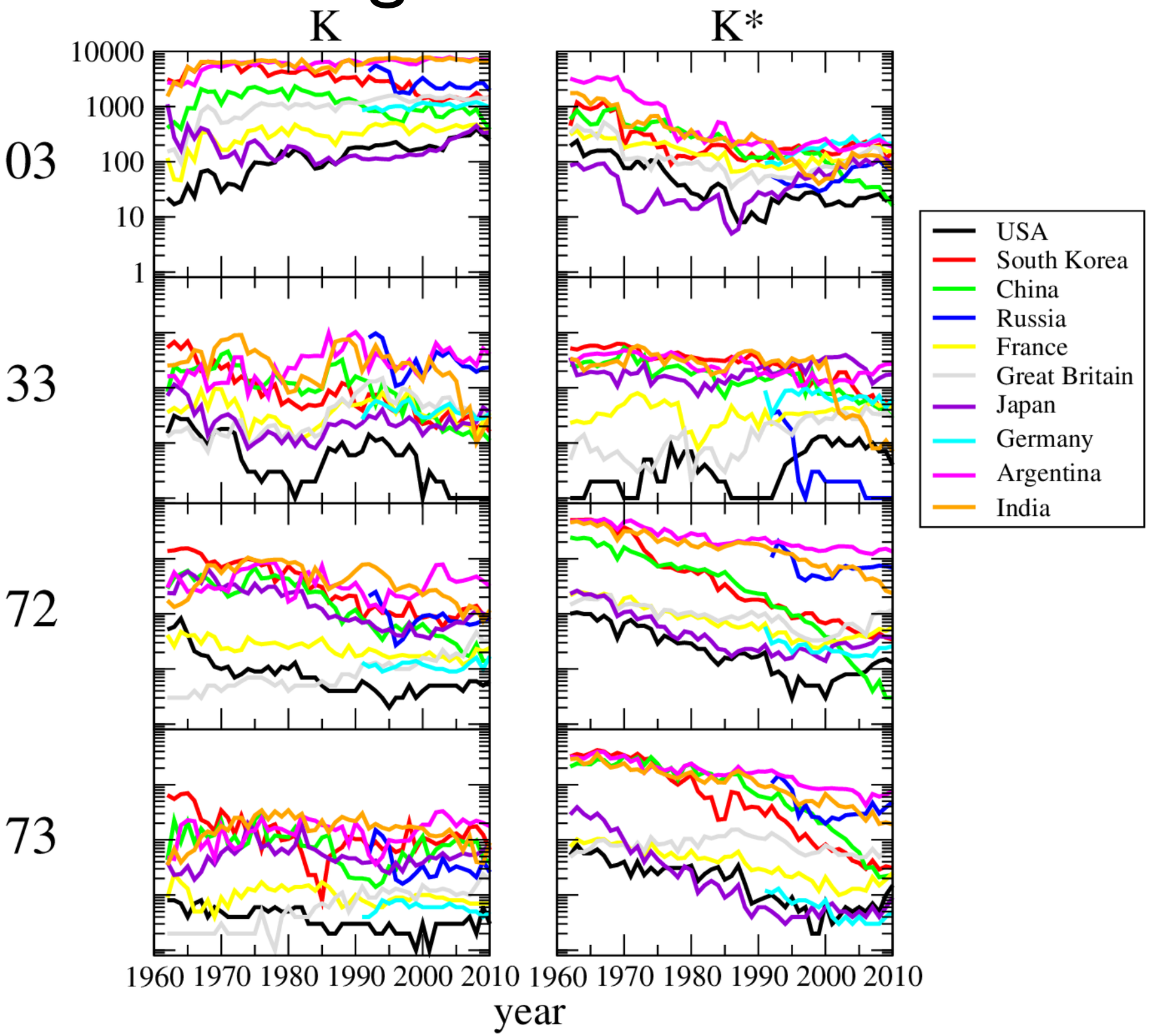




2d global ranks



2d global ranks

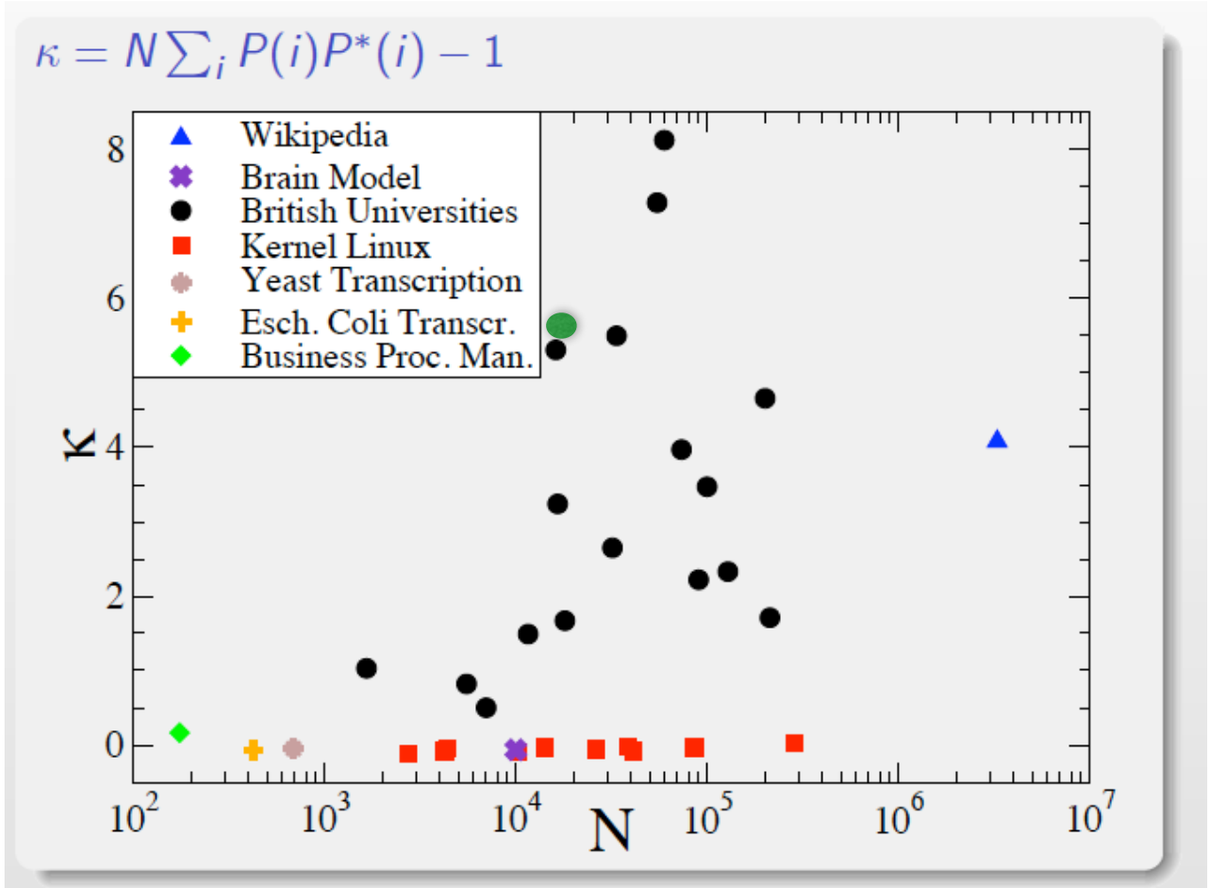


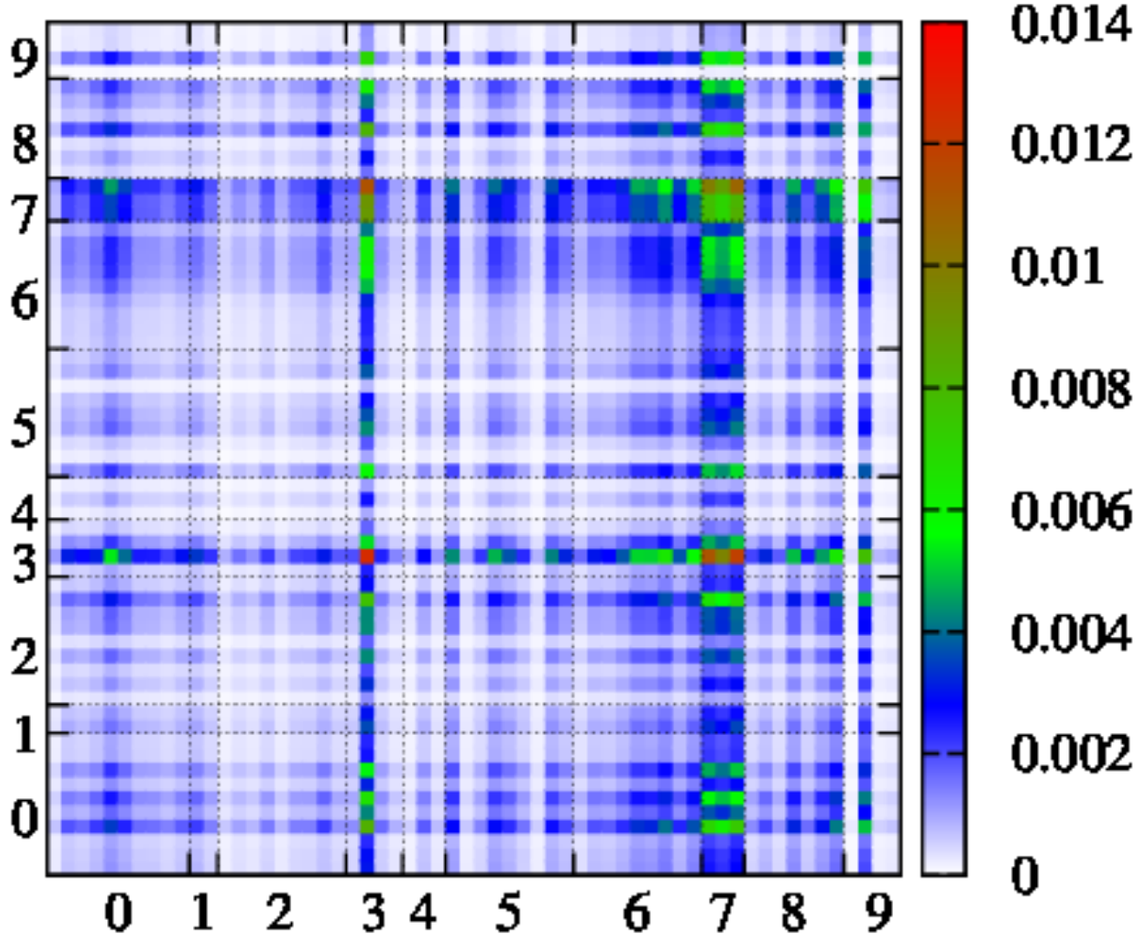
Tab. 1: List of countries: 27 from EU (blue) and the 10 best oil exporters (red) ordered by decreasing PageRank left (CheiRank right) value for petroleum product.

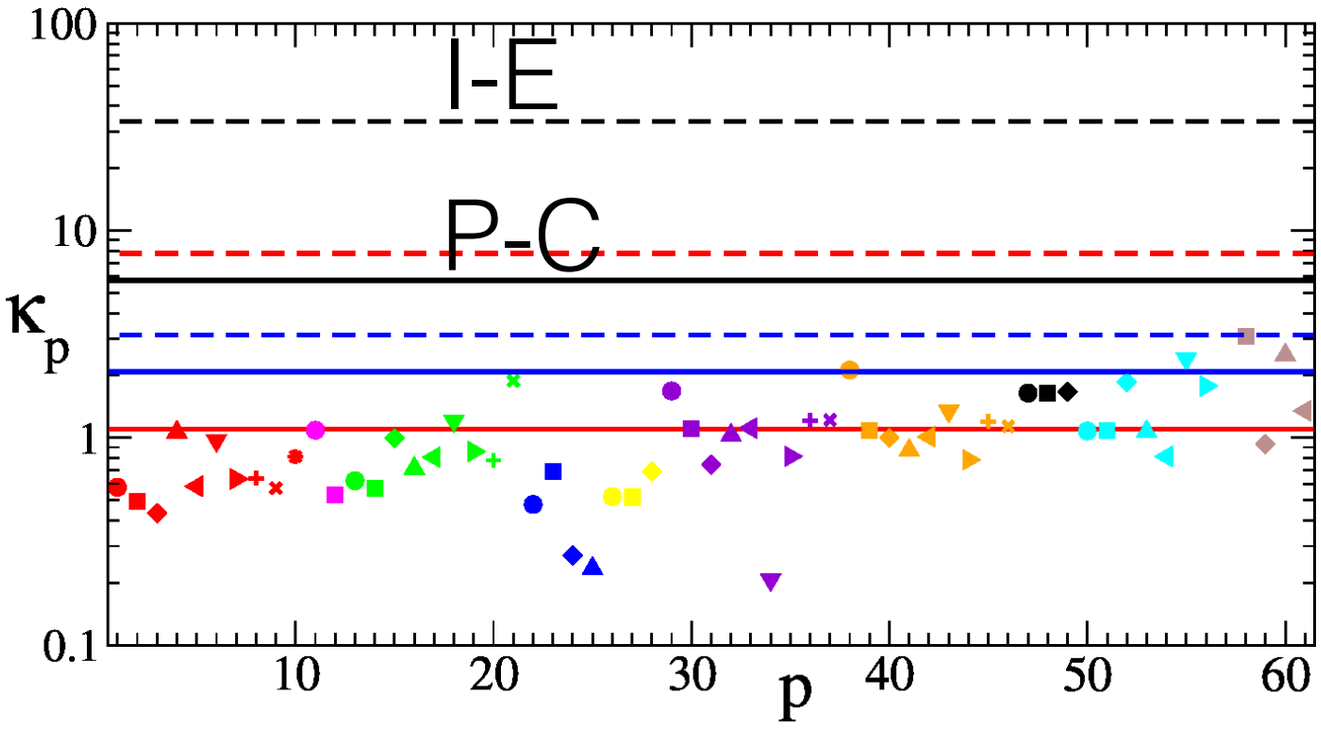
COUNTRY NAME	ISO-2
Netherlands	NL
France	FR
Nigeria	NG
Germany	DE
United Kingdom	GB
Belgium	BE
Spain	ES
Italy	IT
Canada	CA
United Arab Emirates	AE
Austria	AT
Sweden	SE
Slovenia	SI
Poland	PL
Finland	FI
Russia	RU
Czech Republic	CZ
Denmark	DK
Greece	GR
Luxembourg	LU
Bulgaria	BG
Cyprus	CY
Romania	RO
Angola	AO
Hungary	HU
Slovakia	SK
Portugal	PT
Republic of Ireland	IE
Saudi Arabia	SA
Estonia	EE
Malta	MT
Latvia	LV
Venezuela	VE
Kazakhstan	KZ
Lithuania	LT
Iraq	IQ
Kuwait	KW

COUNTRY NAME	ISO-2
Russia	RU
Saudi Arabia	SA
Kazakhstan	KZ
United Arab Emirates	AE
Netherlands	NL
Nigeria	NG
Italy	IT
United Kingdom	GB
Venezuela	VE
France	FR
Kuwait	KW
Belgium	BE
Germany	DE
Spain	ES
Angola	AO
Finland	FI
Canada	CA
Sweden	SE
Iraq	IQ
Romania	RO
Bulgaria	BG
Greece	GR
Denmark	DK
Lithuania	LT
Portugal	PT
Poland	PL
Estonia	EE
Hungary	HU
Austria	AT
Malta	MT
Latvia	LV
Slovakia	SK
Czech Republic	CZ
Republic of Ireland	IE
Cyprus	CY
Slovenia	SI
Luxembourg	LU

PageRank CheiRank correlator







$$\kappa_{pp'} = N_c \sum_{c=1}^{N_c} \left[\frac{P(p+(c-1)N_p)P^*(p'+(c-1)N_p)}{\sum_{c'} P(p+(c'-1)N_p) \sum_{c''} P^*(p'+(c''-1)N_p)} \right] - 1$$

$$\kappa_p = \kappa_{pp'} \delta_{p,p'}$$