A elasto-brittle model for sea-ice flow

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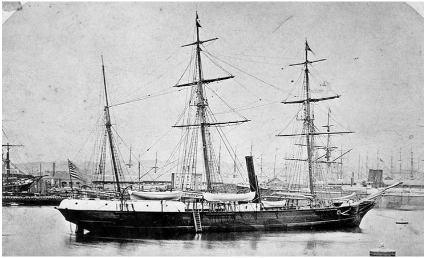
Motivations

[Polar region atlas, CIA, 1978]

- sea ice drift: due to ocean currents and wind
- accelerated: due to global heating

History



1881
Jeannette
G. W. de Long
Le havre

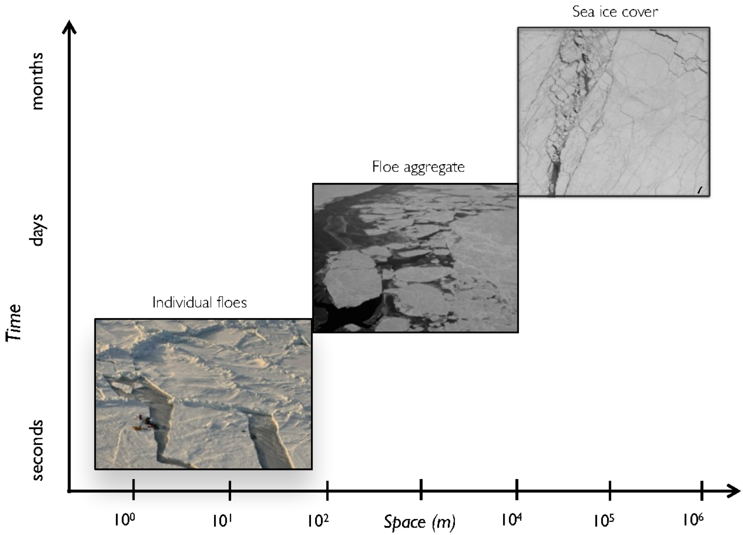
1893
Fram
F. Nansen
Bergen

Boats drift from Siberia to Spitzberg during 3 years

Outline

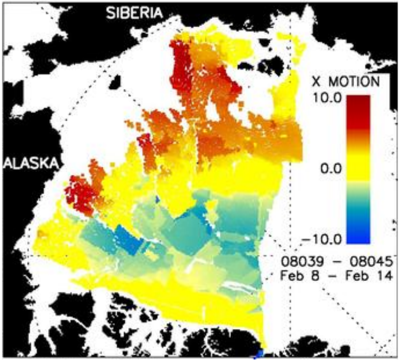
- Mathematical model
- Time & space discretization
- Test 1: constriction
- Test 2: Nares strait

Multi-scale damaged material

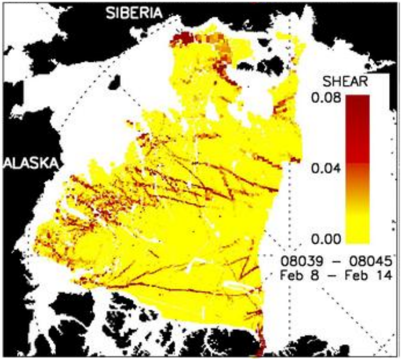


Effect: localization

motion



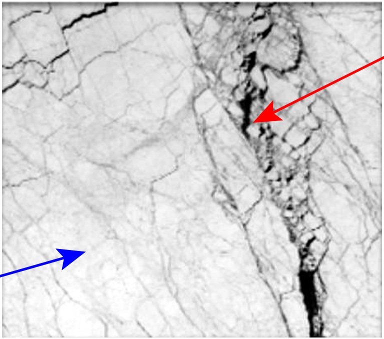
shear



Model: progressive damage & healing

$$0 \leqslant d(t, \boldsymbol{x}) \leqslant 1$$

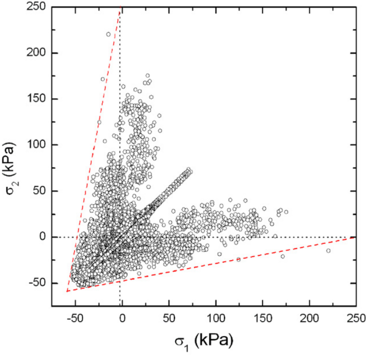
$d(t, \boldsymbol{x}) = 1$



$d(t, \boldsymbol{x}) = 0$

$$\dot{d} = f(\boldsymbol{\sigma})\frac{d}{t_d} - \frac{1}{t_h}$$

$f(\sigma)$: Mohr-Coulomb damage criterion



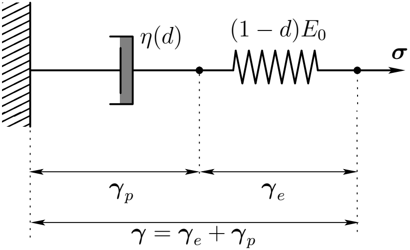
Sea ice experiments
[Weiss & Schulson, 2009]

$$f(\boldsymbol{\sigma}) = \max\left(0,\ 1 - \frac{\sigma_c}{\sigma_1 - q\sigma_2},\ \right)$$

where

$\boldsymbol{\sigma}$	: stress tensor
σ_1, σ_2	: principal stresses
σ_c, q	: material parameters

Material: viscoelastic Maxwell fluid

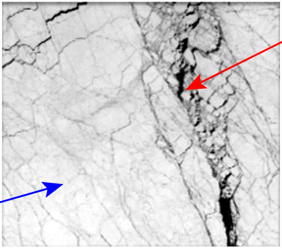


$$\begin{aligned} \boldsymbol{\sigma} &= E(d)\,\gamma_e \\ \dot{\gamma}_e + \frac{E(d)}{2\eta(d)}\,\gamma_e &= \dot{\gamma} \end{aligned}$$

where

$$\begin{array}{rcllcl} & & E(d) & = & (1-d)E_0 & : \text{elastic modulus} \\ & & \eta(d) & = & (1-d)\eta_0/d & : \text{viscosity} \\ \dot{\gamma} & = & D(\boldsymbol{u}) & = & (\nabla \boldsymbol{u} + \nabla \boldsymbol{u}^T)/2 & : \text{deformation rate} \\ & & \boldsymbol{u} & & & : \text{velocity field} \end{array}$$

d = phase field from solid to fluid



large deformations

viscous fluid

$d(t, \boldsymbol{x}) = 1$

bounded deformations

elastic solid

$d(t, \boldsymbol{x}) = 0$

$$
\begin{aligned}
\boldsymbol{\sigma} &= (1-d)E_0\,\gamma_e \\
\dot{\gamma}_e + \frac{E_0\ d}{2\eta_0}\gamma_e &= \dot{\gamma}
\end{aligned}
$$

where

$$\dot{\gamma} \ = \ D(\boldsymbol{u}) \ = \ (\nabla\boldsymbol{u} + \nabla\boldsymbol{u}^T)/2 \quad : \text{deformation rate}$$

Mathematical statement

(P): find γ_e, $\boldsymbol{u}$, d, ϕ and h such that

$$
\begin{array}{rcl}
\dot{\gamma}_e + \dfrac{E_0\ d}{\eta_0}\gamma_e - D(\boldsymbol{u}) & = & 0 \\
\rho\phi h\dot{\boldsymbol{u}} - \mathbf{div}(\phi h\, E(d)\gamma_e) + \rho\phi h f\,\boldsymbol{e}_z \wedge (\boldsymbol{u} - \boldsymbol{u}_w) & = & \phi \boldsymbol{f}_w \\
\dot{d} - f(E(d)\gamma_e)\dfrac{d}{t_d} + \dfrac{1}{t_h} & = & 0 \\
\dot{\phi} + \mathrm{div}(\phi\boldsymbol{u}) & = & 0 \\
\dot{h} & = & 0
\end{array}
$$

$+$ IC $+$ BC

$$
\begin{array}{rcl}
\text{notations}\quad \dot{\gamma}_e & = & \dfrac{\partial\gamma_e}{\partial t} + \ \boldsymbol{u}.\nabla\gamma_e - \nabla\boldsymbol{u}\ \gamma_e - \gamma_e\nabla\boldsymbol{u}^T \\
\dot{\varphi} & = & \dfrac{\partial\varphi}{\partial t} + \ \boldsymbol{u}.\nabla\varphi
\end{array}
$$

Semi-implicit time discretization

- step 1: find $\gamma_{e,n+1}$ and $\boldsymbol{u}_{n+1}$ such that

$$
\begin{aligned}
\frac{\gamma_{e,n+1}-\gamma_{e,n}}{\Delta t} + \boldsymbol{u}_n.\nabla\gamma_{e,n} - \nabla\boldsymbol{u}_n\,\gamma_{e,n} - \gamma_{e,n}\nabla\boldsymbol{u}_n^T + \frac{E_0\,d_n}{\eta_0}\gamma_{e,n+1} &= 2D(\boldsymbol{u}_{n+1}) \\
\rho\phi_n h_n\frac{\boldsymbol{u}_{n+1}-\boldsymbol{u}_n}{\Delta t} - \mathbf{div}(\phi_n h_n \gamma_{e,n+1}) + \rho\phi_n h_n f\,\boldsymbol{e}_z \wedge (\boldsymbol{u}_{n+1}-\boldsymbol{u}_w) &= \phi_n \boldsymbol{f}_w
\end{aligned}
$$

- step 2: find d_{n+1}, ϕ_{n+1} and h_{n+1} such that

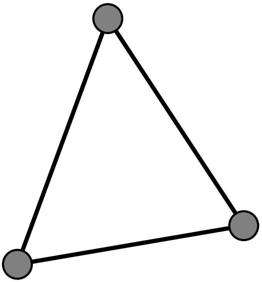
$$
\begin{aligned}
\frac{d_{n+1}-d_n}{\Delta t} + \boldsymbol{u}_{n+1}.\nabla d_{n+1} - f(E(d_n)\gamma_{e,n+1})\frac{d_{n+1}}{t_d} + \frac{1}{t_h} &= 0 \\
\frac{\phi_{n+1}-\phi_n}{\Delta t} + \boldsymbol{u}_{n+1}.\nabla\phi_{n+1} + \phi_{n+1}\,\mathrm{div}\,\boldsymbol{u}_{n+1} &= 0 \\
\frac{h_{n+1}-h_n}{\Delta t} + \boldsymbol{u}_{n+1}.\nabla h_{n+1} &= 0
\end{aligned}
$$

- step 3: mass redistribution

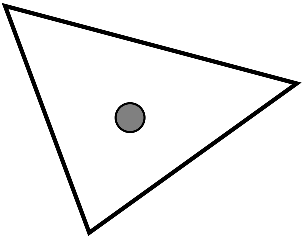
$$
\begin{aligned}
h_{n+1} &:= \max(1, \phi_{n+1})\ h_{n+1} \\
\phi_{n+1} &:= \min(1, \phi_{n+1})
\end{aligned}
$$

FEM approximation

$\boldsymbol{u}$: P_1



$\boldsymbol{\sigma}, d, \phi, h$: P_0 discontinuous

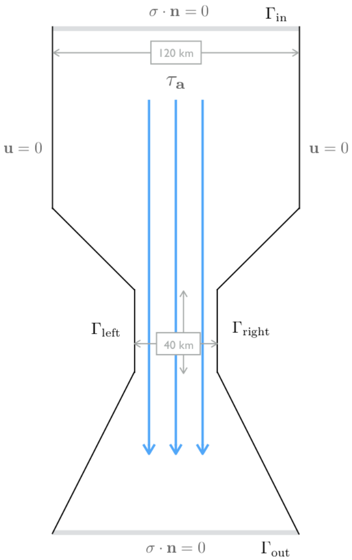


Implementation:

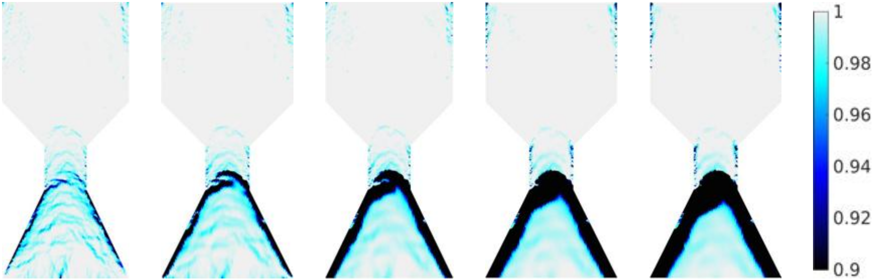
Rheolef C++ library

http://www-ljk.imag.fr/membres/Pierre.Saramito/rheolef

Test 1: constriction

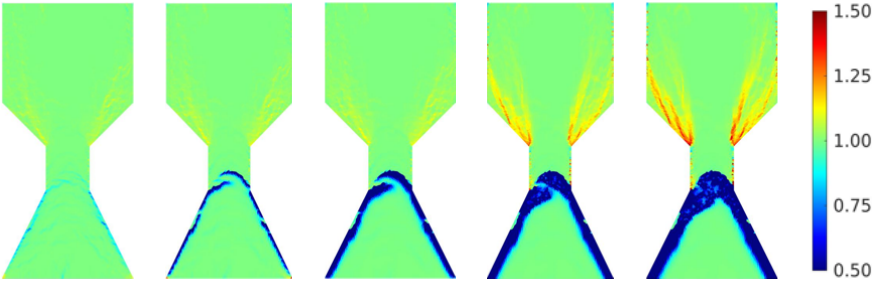


Test 1: constriction



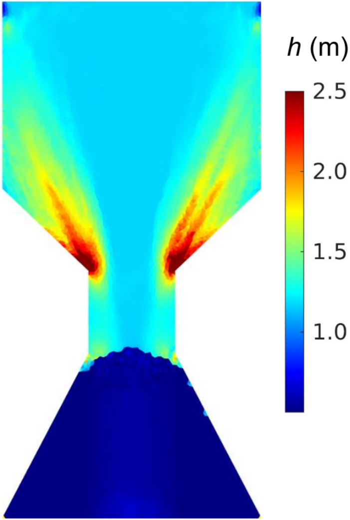
ϕ = ice concentration, in white-blue

Test 1: constriction

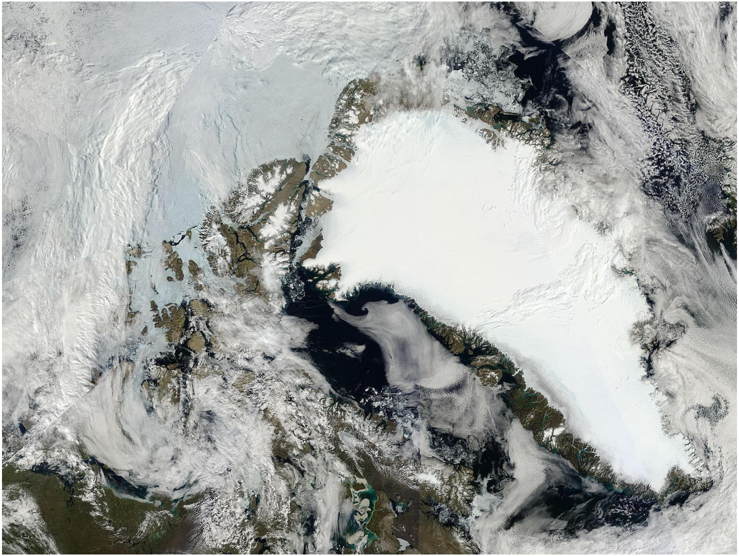


$h =$ ice height, in blue-red

Test 1: constriction (later)



Test 2: Nares strait

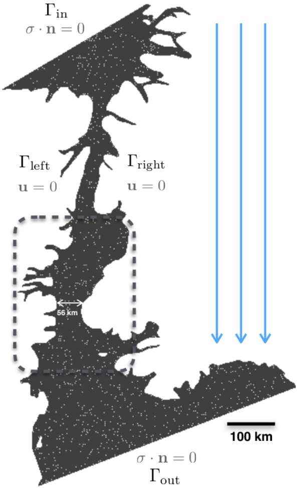


Moderate resolution imaging spectroradiometer reflectance

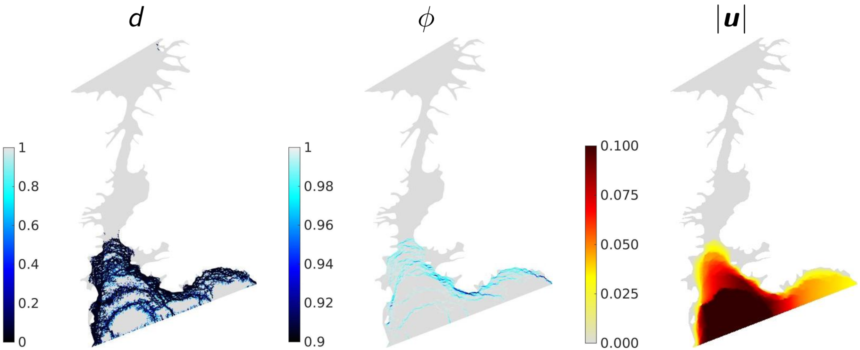
Test 2: Nares strait

02/07/2010

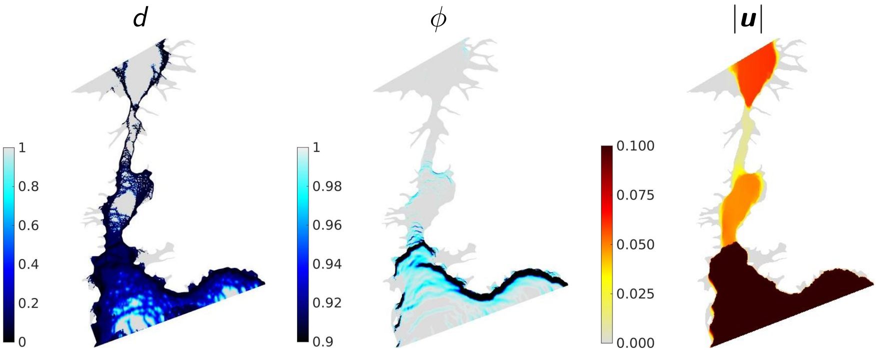
07/05/2016



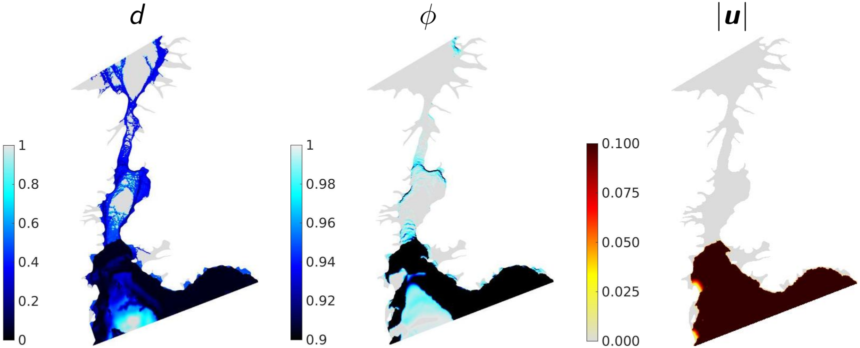
Test 2: Nares strait (6h)



Test 2: Nares strait (20h)

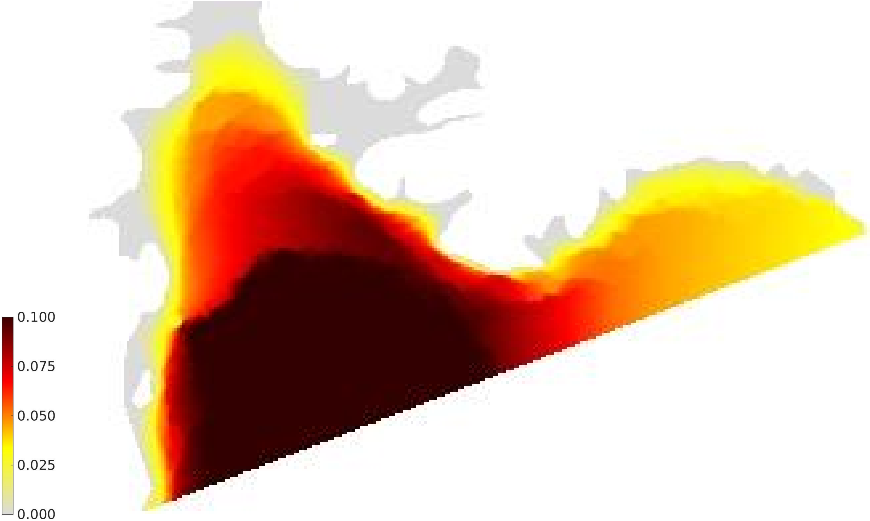


Test 2: Nares strait (72h)



Test 2: Nares strait: zoom (6h)

Velocity is piecewise constant :



More reading

papers: Dansereau, Weiss, Saramito, Lattes, Coche, Cryosphere, 2017.

Dansereau, Weiss, Saramito, Lattes, Cryosphere, 2016.

book: Saramito (2017). *Complex fluids*, SMAI & Springer

code: Saramito (2017).
Rheolef FEM C++ library
Free software (GPL licence)
http://www-ljk.imag.fr/membres/Pierre.Saramito/rheolef