Model fluids versus real fluids

How can we compare experimental and modelling results on complex fluids?

Guillaume Chambon
Viscoplastic fluids

➢ Yield stress (solid-fluid transition)
Formation of deposits
Models: depth-averaged...

Operational model (LAVE2D, Irstea)

Fernandez-Nieto et al., JCP, 2018
Models: ... vs DNS

Fouling layers

Drop encapsulation

Roustaei & Frigaard, JNNFM, 2013

Maleki et al., JFM, 2015
A first case study

Dam break experiments

A model viscoplastic fluid: Carbopol

\[ \gamma = 0 \quad \text{si} \quad \tau < \tau_c \]

\[ \tau = \tau_c + K \gamma^n \quad \text{si} \quad \tau \geq \tau_c \]

LHE, EPFL

Independent measurements of rheological parameters

Herschel-Bulkley law
Comparison with 3 flow models of increasing complexity:

• Kinematic-wave model
  (assumption of locally uniform flow)

\[
\frac{\partial h}{\partial t} + f'(h) \frac{\partial h}{\partial x} = 0,
\]

with

\[
f'(h) = Ah(h - h_c)^{1/n} \quad \text{and} \quad A = \left(\frac{\rho g \sin \theta}{\mu}\right)^{1/n}.
\]

• Advection-diffusion model
  (account for longitudinal pressure gradient in frame of lubrication approximation)

\[
\frac{\partial h}{\partial t} + nk \frac{\partial}{\partial x} \left[ \left( \tan \theta - \frac{\partial h}{\partial x} \right)^{1/n} \frac{h(1 + n) + nh_c}{(n + 1)(2n + 1)} Y_0^{1+1/n} \right] = 0,
\]

with \( Y_0 = \max \left( 0, h - h_c \left| 1 - \cos \theta \frac{\partial h}{\partial x} \right|^{-1} \right). \)

• 2-equation shallow-water model
  (empirical rheological closure of Coussot)

\[
\frac{\partial h}{\partial t} + \frac{\partial h \ddot{u}}{\partial x} = 0,
\]

\[
\frac{\partial h \ddot{u}}{\partial t} + \frac{\partial h \ddot{u}^2}{\partial x} + gh \cos \theta \frac{\partial h}{\partial x} = gh \sin \theta - \frac{\tau_b}{\rho}
\]

\[
\tau_b = \tau_c (1 + 1.93G^{3/10}) \quad \text{with} \quad G = \left( \frac{\mu}{\tau_c} \right)^3 \frac{\ddot{u}}{h}
\]
Kinematic-wave model:

$\theta = 25^\circ$

$\theta = 15^\circ$
Advection-diffusion model:

\[
\theta = 25^\circ
\]

\[
\theta = 15^\circ
\]
Kinematic-wave model:

\[ \theta = 25^\circ \]

\[ \theta = 15^\circ \]
Conclusions of the authors:

• Simple models are not outperformed by more sophisticated models

• Best agreement with data is obtained with the simple kinematic-wave model, in particular for front position at large times

• Predictions of shallow-water model are particularly poor, notably at large times
Velocity profiles:
(based on measured values of $\partial_x h$)

$\theta = 25^\circ$

$\theta = 15^\circ$

Flow conditions within the head significantly depart from lubrication conditions
Comparisons with a Newtonian fluid:

- **velocity profiles**
  - \( \Delta x = -75.3 \text{ mm} \) \( t = 16.4 \text{ s} \)
  - \( \Delta x = -50.3 \text{ mm} \) \( t = 16.3 \text{ s} \)
  - \( \Delta x = -40.3 \text{ mm} \) \( t = 15.8 \text{ s} \)
  - \( \Delta x = -25.3 \text{ mm} \) \( t = 15.6 \text{ s} \)
  - \( \Delta x = -6.6 \text{ mm} \) \( t = 15.3 \text{ s} \)
  - \( \Delta x = -1.6 \text{ mm} \) \( t = 15.2 \text{ s} \)

- **front position**
  - \( \Delta y \) vs. \( t \)

- **free-surface shape**
  - \( h \) vs. \( t \)
A second example

Flow in an expansion-contraction (cavity) numerical solution based on a regularized viscoplastic model

\[ \tau = \left[ 1 - \exp \left( -\frac{\eta_0 \dot{\gamma}}{\tau_c} \right) \right] \left( \tau_c + K \dot{\gamma}^n \right) \]

de Souza Mendes et al., JNNFM, 2007
Experimental flow patterns

Decreasing values of $\tau_c$
Model comparison

- Significant overestimation of the size of the yielded zone
- Prediction of symmetric yield surfaces
Shortcomings and open questions:

• Origin of the discrepancies for viscoplastic fluids?
  - model assumptions
    • thin-layer assumptions
    • rheological closures
  - numerical methods
    • numerical schemes
    • regularization
  - comparison methods
    • yield surfaces
    • “cumulative” discrepancies (case of velocity profiles)

• What about experimental uncertainties?
  - dynamical measurements (PIV, fronts, etc.)
  - 3D effects (front)
  - rheological characterization
    • repeatability
    • more complex rheological trends of the fluids

• How to circumvent these difficulties?
  - How credible are rheometrical measurements?
  - Are “real” yield-stress materials really viscoplastic?
  - How to account for experimental uncertainties in the comparisons with models?
  - Possible to “measure” plug zones?
Rheometrical measurements

Laboratory rheometers

Simple-shear (viscosimetric) flow

\[
D(\mathbf{\dot{v}}) = \frac{1}{2} \begin{pmatrix}
0 & \dot{\gamma} & 0 \\
\dot{\gamma} & 0 & 0 \\
0 & 0 & 0
\end{pmatrix} \Rightarrow \Sigma = \begin{pmatrix}
\sigma_{xx} & \sigma_{xy} & 0 \\
\sigma_{xy} & \sigma_{yy} & 0 \\
0 & 0 & \sigma_{zz}
\end{pmatrix}
\]

\[
\tau = \sigma_{xy} = f(\dot{\gamma})
\]

\[
N_1 = \sigma_{xx} - \sigma_{yy} = g_1(\dot{\gamma})
\]

\[
N_2 = \sigma_{yy} - \sigma_{zz} = g_1(\dot{\gamma})
\]

- stress-imposed or strain-rate imposed
- in the former case: \( \dot{\gamma}_{\text{min}} \approx 10^{-3} - 10^{-2} \text{ s}^{-1} \)
Cone and plate device

Raw measurements:
- $\Omega$: rotation velocity
- $\Gamma$: torque
- $(F_N$: normal force)

\[ \dot{\gamma} \approx \frac{\Omega r}{\theta_0 r} = \frac{\Omega}{\theta_0} \Rightarrow \text{constant} \]

\[ \Gamma \approx \int_0^R 2\pi r^2 \tau \, dr \Rightarrow \tau = \frac{3\Gamma}{2\pi R^3} \]
Parallel-plates device

\[ \dot{\gamma}(r) \approx \frac{\Omega r}{H} \Rightarrow \text{heterogeneous} \]

\[ \Gamma = \int_0^R 2\pi r^2 \tau(r) dr \Rightarrow \tau \approx \frac{3\Gamma}{2\pi R^3} \]

\[ \Rightarrow \tau(R) = \frac{\Gamma}{2\pi R^3} \left( 3 + \frac{\dot{\gamma}(R)}{\Gamma} \frac{\partial \Gamma}{\partial \dot{\gamma}(R)} \right) \]

numerical differentiation is required
Couette (concentric cylinder) device

\[ \tau(r) = \frac{\Gamma}{2\pi Hr^2} \]

\[ \Omega = \int_{r_1}^{r_2} \frac{\dot{\gamma}(r)}{r} \, dr \Rightarrow \text{inverse problem} \]

- small gap approximation \((r_2 - r_1 \ll r_1)\): \(\Omega \approx \dot{\gamma} \frac{r_2 - r_1}{r_1} \)

- in general:
  - series expansion
  - Tikhonov regularization
  - Wavelet-vaguelette decomposition
  - ...

shear-rate heterogeneity

\( \text{LHE, EPFL} \)

\( \text{Ovarlez et al., 2009} \)
Experimental procedures

Strain-rate (or stress) “ramps”: flow curve

- Flow curve
- Yield stress (extrapolation: fitting)
- Influence of $t_{step}$ (steady-state)?
Experimental procedures

Constant strain rate: flow startup

- Yield stress
- Pre-yielding behavior
Experimental procedures

Constant stress: creep test

Clay

Gel

Coussot et al., J. Rheol., 2006

➢ Yield stress
Perturbative factors in rheometry

Free-surface perturbations

\[ \tau \approx \frac{3\Gamma}{2\pi R^3} \]

strong influence of geometry errors
Wall-slip

(a) Solid surface

(b) Solid surface

use of rough tools

influence on flow curve (Carbopol)

\( \tau \) (Pa)

\( \dot{\gamma} \) (s\(^{-1}\))
Shear-banding, cracking

\[ h = \frac{V}{\dot{\gamma}_c} \]

\[ \dot{\gamma} = \dot{\gamma}_c \]

\[ \dot{\gamma} \rightarrow 0 \]

\[ \dot{\gamma}_{macro} = \frac{V}{H} \]

critical shear rate below which flow becomes heterogeneous

⇒ related to thixotropic behavior of the material

Ovarlez et al., JNNFM, 2012

apparent plateau in flow curves (bentonite)

Coussot, 2006
Steady-state

- Time to reach equilibrium increases when $\dot{\gamma}$ decreases.
- Need to be accounted for when measuring the flow curve: $t_{\text{step}} > t_{\text{eq}}$

Moan et al., J. Rheol., 2003
Steady-state


Moller et al., *EPL*, 2009

*The yield stress—a review or ‘παντὸς ρεῖ’—everything flows?*

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Received 30 March 1998; received in revised form 4 May 1998
Confinement effects

Géraud et al., EPJE, 2013

Effects relative to microstructure size (cooperativity length $\xi$)
Conclusion: measurement repeatability

Rheometry of viscoplastic fluids is an art!

Typical uncertainty levels on HB parameters (for a given protocol!):

- $\tau_c$ and $K$: $\pm 5 - 10\%$
- $n$: $\pm 1\%$
Complex rheological trends of yield-stress materials

Thixotropy: dependence on flow/deformation history

“simple” yield-stress fluid


Thixotropic yield-stress fluids

Up and down ramps

Da Cruz et al., PRE, 2002
Complex rheological trends of yield-stress materials

**Thixotropy**: ageing

- Bentonite suspension: dam break with increasing rest times

_Coussot et al., Phys. Fluids, 2005_
Viscosity bifurcation

Creep tests:

existence of a critical shear rate below which no steady flows are possible

\[ \gamma = \frac{\tau}{\eta} \]


Ragouilliaux et al., Rheol. Act., 2006
Consequences on flow behavior

Shear localization:

\[
\Omega > 25 \text{ rpm} \quad \Omega < 25 \text{ rpm}
\]

Cement paste:

Ovarlez et al., 2009

Couette cell:

Cone and plate:

Ovarlez et al., Rheol. Act., 2009
Consequences on flow behavior

“Catastrophic” fluidization:

A rather generic behavior

Creep tests on clay materials sampled in different landslides

Carrière et al., Landslides, 2018

thixotropy observed as soon as Brownian effects and/or attractive interactions between constituents exist
A simple toy model

competition between ageing and shear rejuvenation

\[
\begin{aligned}
\frac{d\lambda}{dt} &= \frac{1}{T} - \alpha \lambda \dot{\gamma} \\
\eta &= \eta_0(1 + \lambda^n)
\end{aligned}
\]

\[
\dot{\gamma}_c = \frac{(n - 1)^{1/n}}{\alpha T}
\]

\[
\tau_c = \frac{\eta_0}{\alpha T} \lambda_0^{n-1}
\]

Coussot et al., J. Rheol., 2002

Coussot et al., Phys. Fluids, 2005
Transition from simple to thixotropic yield-stress behaviour

Carbopol samples

M. Dinkgreve, PhD, 2018

Carbopol microgel microstructure

- **Strong influence of preparation protocol!**

  - (a) Gently stirred: Large cross-linked sponges
  - (b) Strongly stirred: Smaller structures; Brownian effects

M. Dinkgreve, PhD, 2018
Elasticity

Carbopol samples

Shear Stress (Pa) vs. Shear Rate (1/s)

Herschel-Bulkley fit gives:
\[ \tau_y = 53.7 \pm 2.7 \text{ Pa} \]

Dinkgreve et al., Rheol. Acta, 2017

Elastic pre-yielding deformation
Elasticity

Stress relaxation experiments (Carbopol)

Significant elastic strains even above the yield stress

Piau, JNNFM, 2007
Influence on the flow: unsteady flows

Impact of yield stress fluids on a hydrophobic surface

Kaolin clay

Carbopol

Luu & Forterre, JFM, 2009
Elasto-viscoplastic models

\[ \lambda \dot{\tau} + \max \left( 0, \frac{|\tau_d| - \tau_0}{|\tau_d|} \right) \tau - 2\eta_m D(\mathbf{v}) = 0, \]

Saramito, JNNFM, 2007

Large amplitude oscillations: model

Experiments (Carbopol)
Drop rebound

Shows importance of elastic (reversible) deformations above yielding

*Luu & Forterre, JFM, 2009*
Influence on the flow: steady flows

Flow around a sphere (low $Re$)


Ahonguio et al., JNNFM, 2014

strong fore-aft asymmetry of the velocity field
Influence on the flow: steady flows

Expansion-contraction geometry

Numerical simulations for increasing Deborah number

Asymmetry of the yield surface is explained by elasticity
Normal stresses

Ahonguio et al., JNNFM, 2014

Very few data on normal stresses in yield-stress materials
Complex rheological trends of yield-stress materials

More: towards thixotropic elasto-viscoplastic models?

A unified approach to model elasto-viscoplastic thixotropic yield-stress materials and apparent yield-stress fluids

Paulo R. de Souza Mendes · Roney L. Thompson
Complex rheological trends of yield-stress materials

3D rheology

Herschel-Bulkley law in simple shear:

\[
\begin{align*}
\dot{\gamma} &= 0 \quad \text{if} \quad |\tau| < \tau_c \\
\tau &= \tau_c + K \dot{\gamma}^n \quad \text{if} \quad |\tau| \geq \tau_c
\end{align*}
\]

3D extrapolation:

\[
\begin{align*}
\dot{\gamma}_{ij} &= 0 \quad \text{if} \quad |\tau| < \tau_c \\
\tau_{ij} &= \tau_c \frac{\dot{\gamma}_{ij}}{|\gamma|} + K \dot{\gamma}_{ij}^n \quad \text{if} \quad |\tau| \geq \tau_c
\end{align*}
\]

\[
|\tau| = \sqrt{\sum \frac{1}{2} \tau_{ij}^2} \quad |\dot{\gamma}| = \sqrt{\sum \frac{1}{2} \dot{\gamma}_{ij}^2}
\]

von-Mises criterion
Combination of shear and squeeze flow

Carbopol and emulsions

Yield criterion

\[ \sqrt{\tau_{r\theta}^2 + \tau_{rz}^2} = \tau_c \]

For \( \dot{\gamma}_{r\theta} \ll \dot{\Gamma}_{sq} \): \( \tau_{r\theta} \propto \dot{\Gamma}_{sq} \dot{\gamma}_{r\theta} \)

3D rheology

Ovarlez et al., Nature Mat., 2010
Elongational behavior

Balmforth et al., JNNFM, 2010

Carbopol

Yarin et al., J. Rheol., 2004

• Good validity of 3D rheology for Carbopol and simple yield-stress fluids
• What for more complex fluids?
Free surface flow in steady uniform regime

Conveyor belt channel

Chambon et al., JFM, 2014
Carbopol
Assessing measurement accuracy

Flow height

Velocity profiles (linear PIV)
Theoretical predictions in steady uniform regime

\[
    u(y) = \begin{cases} 
    u_0 \left[ 1 - \left(1 - \frac{y}{h - h_c}\right)^{(n+1)/n} \right] - u_b, & y < h - h_c \\
    u_0 - u_b, & y \geq h - h_c 
    \end{cases}
\]

\[
    u_0 = \frac{n}{n+1} \left( \frac{\rho g \sin \theta}{K} \right)^{1/n} (h - h_c)^{(n+1)/n}
\]

\[
    h_c = \frac{\tau_c}{\rho g \sin \theta}
\]

Relation between \( h \) and \( u_b \): \( u_b = u_0 \left( 1 - \frac{n}{2n+1} \frac{h-h_c}{h} \right) \)
Height-velocity relation

Kaolin
**Height-velocity relation**

- **Systematic discrepancy**

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**Carbopol**

![Graphs showing height-velocity relation for different Carbopol samples (C_01, C_07, C_08, C_09). The graphs compare theoretical predictions with experimental data, indicating systematic discrepancies.]
Velocity profiles

(a) Theor. pred. (measured params., no slip)
- Theor. pred. (measured params., with slip)
- Theor. pred. (corrected params., no slip)
- Theor. pred. (corrected params., with slip)

Exp. data: $u_b = 201 \text{ mm s}^{-1}$
Exp. data: $u_b = 102 \text{ mm s}^{-1}$
Exp. data: $u_b = 53 \text{ mm s}^{-1}$
Need for a systematic correction of Carbopol rheological parameters:

- +10% on $\tau_c$
- +20% on $K$
Rheological parameter correction

Origin of the correction: scale effects?
Conclusions

• Uncertainties on rheological measurements
  
  ➢ significant influence on theoretical predictions (height, velocity, etc.)

• If possible, infer rheological parameters from the flow itself
Front internal dynamics

High-resolution space-time velocimetry (STV)

Freydier et al., JNNFM, 2018
Velocity fields

Longitudinal velocity

Vertical velocity
Velocity profiles

- **Longitudinal velocity**
  - $x_f (mm)$
  - $u (\text{mm.s}^{-1})$

- **Vertical velocity**
  - $-v (\text{mm.s}^{-1})$
Evolution of the plug zone

- Unsheared plug progressively thins, and disappears, in surge tip
Order-0 model (lubrication)

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]

\[
e\frac{Re}{Fr^2} \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = \frac{Re}{Fr^2} \tan \theta - e \frac{Re \partial p}{Fr^2 \partial y} + Bi \frac{\partial \sigma_{xy}}{\partial x} + Bi \frac{\partial \sigma_{xx}}{\partial x}
\]

\[
e^3 \frac{Re}{Fr^2} \left( \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = -e \frac{Re \partial p}{Fr^2 \partial y} + e^2 Bi \frac{\partial \sigma_{xy}}{\partial x} + e^2 Bi \frac{\partial \sigma_{yy}}{\partial y}
\]

\[
\begin{cases}
  u^{(0)} = u_p \left[ 1 - \left( 1 - \frac{y}{h-h_p} \right)^{\frac{n+1}{n}} \right] & \text{for } y < h - h_p \\
  u^{(0)} = u_p & \text{for } y \geq h - h_p
\end{cases}
\]

with
\[
u_p = \frac{n}{n+1} \Lambda^{1/n} (h-h_p)^{(n+1)/n}
\]

\[
h_p = \frac{Bi}{\Lambda} \quad \Lambda = \lambda - \frac{\epsilon Re}{Fr^2} \frac{\partial_x h}{x}
\]
“Raw” comparison

Renormalization to account for experimental uncertainties on

- rheological parameters: $h \rightarrow h/H_N$
- average velocity (3D effects): $u \rightarrow u/\bar{u}$
Non-dimensionalized comparison

$\chi_f^*$ decreases

Good agreement until $\chi_f \approx 1$
Surface velocity

\[ \theta = 15.3^\circ; G = 0.35; Fr = 0.06 \]

\[ \theta = 11.9^\circ; G = 0.17; Fr = 0.38 \]
Order-1 model ($\epsilon^1$)

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]

\[
\epsilon \text{Re} \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = \frac{\text{Re}}{Fr^2} \tan \theta - \epsilon \frac{\text{Re}}{Fr^2} \frac{\partial p}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + \epsilon Bi \frac{\partial \sigma_{xx}}{\partial x}
\]

\[
\epsilon^3 \text{Re} \left( \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = -\epsilon \frac{\text{Re}}{Fr^2} - \epsilon \frac{\text{Re}}{Fr^2} \frac{\partial p}{\partial y} + \epsilon Bi \frac{\partial \sigma_{xy}}{\partial x} + \epsilon Bi \frac{\partial \sigma_{yy}}{\partial y}
\]

\[
u = u^{(0)} + \epsilon u^{(1)} \text{ with:}
\]

\[
u^{(1)} = \text{Re} U_I(x, y, t) + Bi U_N(x, y, t)
\]

\[
u^{(1)} = \text{Re} U_I(x, h - h_p, t) + Bi [U_N(x, h - h_p, t) + U_{np}(x, y, t)]
\]

for $y < h - h_p$

for $y \geq h - h_p$

correction due to inertia

corrections due to normal stresses

plastic normal stresses in the pseudo-plug

\[|\sigma| = Bi \sqrt{\sigma_{xy}^2 + \sigma_{xx}^2} = Bi + O(\epsilon)\]

Balmforth & Craster, JNNFM, 1999
Expressions of the corrective terms

**Inertia:**

\[
U_I = \Lambda^{(3-n)/n} (h - h_p)^{(2n+3)/n} p u_1 \left( \frac{y}{h - h_p} \right) \partial_x h
\]

\[
+ \Lambda^{(3-2n)/n} (h - h_p)^{(3n+3)/n} p u_2 \left( \frac{y}{h - h_p} \right) \partial_x \Lambda
\]

\[
- \Lambda^{(2-2n)/n} (h - h_p)^{(2n+2)/n} p u_1 \left( \frac{y}{h - h_p} \right) \partial_t \Lambda
\]

**Normal stresses:**

\[
U_N = -\frac{\pi}{2} \Lambda^{(1-2n)/n} h_p \left( h - h_p \right)^{1/n} \left( 1 - \left( 1 - \frac{y}{h - h_p} \right)^{1/n} \right) \sgn(\partial_x u_p) \partial_x \Lambda
\]

\[
U_{np} = 2\Lambda^{(1-n)/n} \left( h - h_p \right)^{1/n} \sqrt{1 - \left( \frac{h - y}{h_p} \right)^2} \left[ \partial_x h + \frac{1}{n + 1} (h + nh_p) \frac{\partial_x \Lambda}{\Lambda} \right] \sgn(\partial_x u_p)
\]
Model comparison

Longitudinal velocity profiles

- Close to tip:
  - shear throughout the fluid layer
  - surface velocity is (generally) larger

- Non-differentiable matching at pseudo-plug interface...
Surface velocity

\[ \theta = 15.3^\circ; \ G = 0.35; \ Fr = 0.06 \]
Surface velocity

\[ \theta = 15.3^\circ; G = 0.29; Fr = 0.13 \]
Surface velocity

\[
\theta = 15.3^\circ; \ G = 0.24; \ Fr = 0.28
\]
Global overview of all experiments

\[ E = |u_{p,\text{predicted}} - u_{p,\text{measured}}| \]

at \( x_f^* = 1 \)

- \( \epsilon^1 \) approximation improves quantitative agreement in the tip region
- Gain remains marginal for large values of \( Fr \)
  - Inertial correction terms need to be improved?
Shear rate

Average shear rate in the pseudo-plug zone

\[ \theta = 15.3^\circ; G = 0.31; Fr = 0.06 \]

\[ \theta = 15.3^\circ; G = 0.27; Fr = 0.18 \]
Apparent plug

\( \theta = 15.3^\circ; G = 0.27; Fr = 0.18 \)

\( \theta = 15.3^\circ; G = 0.20; Fr = 0.39 \)

- Apparent plug based on \( \dot{\gamma}_c \) (order 1)
- Pseudo-plug (lubrication)

Order-1 explains collapse of the unsheared region at the front
Conclusions

• Accounting for experimental uncertainties through proper non-dimensionalization of variables

• Experimental measurement of plug zones

• Quantitative comparisons of subtle features (e.g., pseudo-plug shear rate)

• Viscoplastic rheology enhances differences between $\epsilon^0$ and $\epsilon^1$ models!
Comparison with a Newtonian fluid

Predictions of $\varepsilon^0$ and $\varepsilon^1$ models are virtually indistinguishable.
Flow over a cavity

Poiseuille plug-flow

$H_{up}$

flow zone

streamlines + PIV

solid-liquid interface

dead zone

Chambon, Vigneaux, Marly, Luu, Philippe, JNNFM, subm, 2018
Viscoplastic boundary layer theory: large $Bi$

- **Oldroyd's (1947)** self-similar solution:
  $$\delta \propto Bi_\ell^{-1/3} \ell$$

- **Piau (2002)**

- **Balmforth & Craster’s (2017)** generalization (assume symmetric velocity profiles in the BL):
  $$y_E \propto Bi_\ell^{-1/3} \ell$$

\[ \frac{\partial_y \tau_{xy}}{2} + \frac{\partial_x \tau_{xx}}{\kappa} = 0 \]

Viscous + Plastic \quad Plastic

\[ Bi_\ell = \frac{\tau_c \ell}{K U} \]
Yield surface position

\[ \frac{|y_{int}|}{D} \]

\[ Hb_D = \frac{\tau_c}{K \left( \frac{D}{U_{up}} \right)^n} \]

Control of \( y_{int} \)?
Direct numerical simulations

- Augmented Lagrangian
- Highly-accurate numerical scheme

- Bingham rheology ($n = 1$)
  \[
  \begin{aligned}
  \dot{\gamma} &= 0 \quad \text{si } \tau < \tau_c \\
  \tau &= \tau_c + \eta \dot{\gamma} \quad \text{si } \tau \geq \tau_c
  \end{aligned}
  \]

Marly & Vigneaux, JNNFM 2017
First qualitative comparison: stress fields

Numerical simulations

Experiments
First qualitative comparison: interface shape

\[ A_{\text{tot}} = \int_{0}^{D} |y_{\text{int}}(x_{1/2})| dx \]

Numerical simulations

Experiments
Yield surface position

Numerical simulations

Experiments

\[ \frac{|y_{int}|}{D} \]

\[ D \text{ (cm)} \]

\[ B_D \]

\[ Hb_D \]
Velocity and strain-rate profiles

- Non-symmetric profiles
- 3 flow zones
  - Upper plug
  - Poiseuille-like layer
  - Boundary layer

Numerical simulations
Velocity and strain-rate profiles

Experiments

Shear stress: \( \tau = \tau_c + K |D(u)|^n \)

Poiseuille-like zone
Oldroyd’s boundary layer equation

\[ \partial_y \tau_{xy} \]

\[ \partial_x \tau_{xx} \]

\[ 2 \partial_x \tau_{xx} + \partial_y \tau_{xy} \]
New scaling for BL thickness

\[ B_{cav} = \frac{\tau_c}{\eta} \frac{D}{U_s} \]

\[ H b_{cav} = \frac{\tau_c}{K} \left( \frac{D}{U_s} \right)^n \]
Generalized Oldroyd’s scaling

Numerical simulations

Experiments

\[ \frac{\delta_{BL}}{D} \propto B_{cav}^{-1/3} \]

\[ \frac{\delta_{BL}}{D} \propto Hb_{cav}^{-1/(n+2)} \]
Boundary layer – PL zone interface

Slip velocity $U_s$

Numerical simulations

Experiments
Boundary layer – PL zone interface

Maximum shear stress $\tau_m$

- **Numerical simulations**
  - Graph showing $\tau_m$ vs $\tau_w$ (Pa)

- **Experiments**
  - Graph showing $\tau_m$ vs $\tau_w$ (Pa)

- **Boundary condition of PL zone essentially controlled by upward flow**
Boundary layer – PL zone interface

Stress ratio $\tau_m/\tau_w$

Numerical simulations

Experiments

$$B_{up} = \frac{\tau_c H_{up}}{\eta U_{up}}$$

$$Hb_{up} = \frac{\tau_c}{K} \left( \frac{H_{up}}{U_{up}} \right)^n$$
PL zone thickness

Numerical simulations

Experiments
Conclusions

• Experiment / model comparison based mainly on qualitative trends and scaling laws

• Allows in-depth exploration of viscoplasticity-related features

• Opens interesting prospects for the extension of existing viscoplastic boundary layer theories