Advanced Methods in Mathematical Finance

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Recover Dynamic Utility from Monotonic Characteristic Processes

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In the real world, decision making under uncertainty is often viewed as an optimization problem under choice criterium, and most of theory focuses on the derivation of the "optimal decision" and its out-comes. But, poor information is available on the criterium yielding to these observed data. The interesting problem to infer the unknown criterium from the known results is an example of inverse problem. Here we are concerned with a very simple version of the problem: what does observation of the "optimal" out-put tell us about the preference, expressed in terms of expected utility; in Economics, this question was pioneered by the american economist Samuelson in1938.
Typically we try to reproduce the properties of the stochastic value function of a portfolio optimization problem in finance, which satisfies the first order condition $U(t, z)$). In particular, the utility process U is a strictly concave stochastic family, parametrized by a number $z \in R^+$ ($z \mapsto U(t, z)$), and the characteristic process $X^c = (X_t^c(x))$ is a non negative monotonic process with respect to its initial condition x, satisfying the martingale condition $U(t, X_t^c(x))$ is a martingale, with initial condition $U(0, x) = u(x)$. We first introduce the adjoint process $Y_t(u_x(x)) = U_x(t, X_t^c(x))$ which is a characteristic process for the Fenchel transform of U if and only if $X_t^c(x)Y_t(u_x(x))$ is a martingale. The minimal property is the martingale property of $Y_t(u_x(x))$ with the x-derivative of $X_t^c(x)$, which is sufficient to reconstruct U from $U_x(t, x) = Y_t(u_x((X_t^c)^{-1}(x)))$. Obviously, in general, without additional constraints, the characterization is not unique. Various example are given, in general motivated by finance or economics: contraints on a characteristic portfolio in a economy at the equilibrium, thoptimal portfolio for a in complete financial market, under strongly orthogonality between X and Y , the mixture of different economies….In any case, the results hold for general but monotonic processes, without semimartingale assumptions.

Summary

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