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Lie group analysis of an optimization problem for a portfolio with an illiquid asset

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Working in the Merton's optimal consumption framework with continuous time we consider an optimization problem for a portfolio with an illiquid, a risky and a risk-free asset. Our goal in this paper is to carry out a complete Lie group analysis of PDEs describing value function and investment and consumption strategies for a portfolio with an illiquid asset that is sold in an exogenous random moment of time T with a prescribed liquidation time distribution. The problem of such type leads to three dimensional nonlinear Hamilton-Jacobi-Bellman (HJB) equations. Such equations are not only tedious for analytical methods but are also quite challenging form a numeric point of view. To reduce the three-dimensional problem to a two-dimensional one or even to an ODE one usually uses some substitutions, yet the methods used to find such substitutions are rarely discussed by the authors.

We use two types of utility functions: general HARA type utility and logarithmic utility. We carry out the Lie group analysis of the both three dimensional PDEs and are able to obtain the admitted symmetry algebras. Then we prove that the algebraic structure of the PDE with logarithmic utility can be seen as a limit of the algebraic structure of the PDE with HARA-utility as $\gamma \to 0$. Moreover, this relation does not depend on the form of the survival function $\overline{\Phi}(t)$ of the random liquidation time T.
We find the admitted Lie algebra for a broad class of liquidation time distributions in cases of HARA and log utility functions and formulate corresponding theorems for all these cases.

We use found Lie algebras to obtain reductions of the studied equations. Several of similar substitutions were used in other papers before whereas others are new to our knowledge. This method gives us the possibility to provide a complete set of non-equivalent substitutions and reduced equations.

We also show that if and only if the liquidation time defined by a survival function $\overline{\Phi}(t)$ is distributed exponentially, then for both types of the utility functions we get an additional symmetry. We prove that both Lie algebras admit this extension, i.e. we obtain the four dimensional L_4^{HARA} and L_4^{LOG} correspondingly for the case of exponentially distributed liquidation time.
We list reduced equations and corresponding optimal policies that tend to the classical Merton policies as illiquidity becomes small.

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