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The Skorokhod embedding problem and single jump martingales: a connection via change of time

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Let $\overline{N}_t = \sup_{s\leq t} N_s$ be a running maximum of a local martingale N. We assume that N is max-continuous, i.e. $\overline{N}$ is continuous. The Skorokhod embedding problem corresponds to a special case where N is a Brownian motion stopped at a finite stopping time τ. Consider the change of time generated by the running maximum: $\sigma_t := \inf\{s\colon \overline{N}_s > t\}$.
Then the time-changed process $M := N \circ \sigma$ has a simple structure:
$M_t = N_{\sigma_t} = t \wedge W - V1_{\{t\geq W\}}$,
where $W := \overline{N}_\infty$ and $V := \overline{N}_\infty - N_\infty$ (V is correctly defined on the set $\{\overline{N}_\infty < \infty\}$). Besides, $M_\infty = N_\infty$ and $\overline{M}_\infty = \overline{N}_\infty$. This simple observation explains how we can use single jump martingales M of the above form to describe properties of N. For example, N is a closed supermartingale if and only M is a martingale and the negative part of $W - V$ is integrable. Another example shows how to connect the Dubins-Gilat construction of a martingale whose supremum is given by the Hardy-Littlewood maximal function and the Azéma-Yor construction in the Skorokhod embedding problem.

Summary

We establish a connection between the sets of possible joint distributions of pairs $(N_\infty, \overline{N}_\infty)$ for different subclases of max-continuous local martingales N, in particular, for N corresponding to the Skorokhod embedding problem.

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