Functional Equations in LIMoges (FELIM) 2018

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Functional Equations in LIMoges (FELIM) 2018 is the eleventh in a series of annual international gatherings for researchers in functional equations. This conference, held annually at the University of Limoges since 2008, aims to present recent advances in symbolic or symbolic-numeric algorithms which treat systems of linear or nonlinear, ordinary or partial, differential equations, (q-)difference equations,... Additionally, FELIM emphasizes on the development state of related software implementations and the publicity of such codes. Topics include, but are not limited to, ordinary differential equations, difference equations, partial differential equations; integration of dynamical systems; methods for local solving (formal, symbolic-numeric, modular); methods for global solving or simplification (e.g., decomposition, factorisation); applications and software applications.

The conference website with links to the previous editions can be found at:

https://indico.math.cnrs.fr/event/3105/

1 Invited talks abstracts

Guillaume Chèze (Université de Toulouse, France)
Symbolic Computations of First Integrals for Polynomial Vector Fields

`Wednesday, March 28, 10:00-11:00`

In this talk I will present a work done in collaboration with Thierry Combot (Université de Bourgogne, France). We have shown how to generalize to the Darbouxian, Liouvillian and Riccati case the extactic curve introduced by J. Pereira. With this approach, we get new algorithms for computing, if it exists, a rational, Darbouxian, Liouvillian or Riccati first integral with bounded degree of a polynomial planar vector field. We give probabilistic and deterministic algorithms. The arithmetic complexity of our probabilistic algorithm is in $\tilde{\mathcal{O}}(N^{\omega+1})$, where N is the bound on the degree of a representation of the first integral and $\omega \in [2; 3]$ is the exponent of linear algebra. This result improves previous algorithms.

Galina Filipuk (University of Warsaw, Poland)
Orthogonal Polynomials, Differential and Difference Equations

`Thursday, March 29, 9:30-10:30`

In this talk I shall overview some recent results on the connection of recurrence coefficients of semi-classical orthogonal polynomials to nonlinear differential and difference equations.

Elizabeth L Mansfield (University of Kent, UK)
Introduction to Moving Frames and their Applications

`Friday, March 30, 9:30-10:30`

In this talk I will introduce the modern version of the Lie group based moving frame, and the calculus of invariants that the frame makes possible. I will discuss a particular application to Noether's conservation laws. Time permitting, I will talk about matching smooth and approximate laws by matching the smooth and discrete frames at the heart of each invariant calculus.

2 Contributed talks abstracts

Sergei A. Abramov[1] (Dorodnicyn Computing Centre, Federal Research Center "Computer Science and Control" of the Russian Academy of Sciences, Department of Computational Mathematics and Cybernetics, Moscow State University, Moscow, Russia)
On Unimodular Matrices of Difference Operators

`Thursday, March 29, 11:30-12:00`

[1]Supported by RFBR grant 16-01-00174-a.

We consider matrices of difference operators $L \in \mathrm{Mat}_n(K[\sigma])$, where K is a difference field of characteristic 0 with an automorphism σ. We discuss approaches to compute the dimension of the space of those solutions of the system of equations $Ly = 0$ that belong to an adequate extension of K. On the base of one of those approaches, we propose a new algorithm for computing $L^{-1} \in \mathrm{Mat}_n(K[\sigma])$ whenever it exists. We investigate the worst-case complexity of the new algorithm, counting both arithmetic operations in K and shifts of elements of K. This complexity turns out to be smaller than in the earlier proposed algorithms for inverting matrices of difference operators.

Islam Boussaada (IPSA & L2S, Université Paris Saclay, CNRS, Centrale Supelec, UP SUD, France)
On the Dominancy of Multiple Spectral Values for Time-Delay Systems with Applications

`Wednesday, March 28, 16:00-16:30`

Recent results on multiplicity induced-dominancy for spectral values in reduced-order time-delay systems will be presented. Application in stabilizing delayed controller design will be parametrically discussed. Various examples illustrate the applicative perspectives of the approach.
This is a joint work with Silviu Niculescu (IPSA & L2S, Université Paris Saclay, CNRS, Centrale Supelec, UP SUD, France).

Thierry Combot (Université de Bourgogne, France)
Generalized Picard Fuchs Equations for Planar Vectors Fields

`Wednesday, March 28, 11:30-12:00`

We consider a rational vector field $X(x, y)$ in the plane admitting a symbolic first integral $I(x, y)$. We now want to solve the corresponding differential system with respect to time. To this we need to consider an integral of a rational function R on a generic curve $I(x, y) = h$. However, when I is non rational, the integral can be very complicated in the complex domain, the curve having infinite genus and thus a priori infinitely many cycles generating monodromy. Thus we want to express the integral under the form $F(x, y)$ where F is a bivariate holonomic function. This condition appears to be equivalent for the tangential Malgrange pseudo group to be finite dimensional. In this case the length of loops satisfies a differential equation in h, the Picard Fuchs equation. We design an algorithm to compute the holonomic function F and the Picard Fuchs equation when it exists. As such expressions are exceptional rather than generic, we will present additional existence conditions and examples.

Renat Gontsov (Institute for Information Transmission Problems of RAS, Moscow, Russia)
On the Convergence of Formal Dulac Series Satisfying an Algebraic ODE

`Thursday, March 29, 14:30-15:00`

We consider an n-th order ODE

$$F(x, y, \delta y, \dots, \delta^n y) = 0, \tag{1}$$

where $F = F(x, y_0, y_1, \dots, y_n)$ is a polynomial of $n+2$ complex variables and δ is the derivation $x(d/dx)$. Assume (1) has a formal *Dulac series* solution φ of the form

$$\varphi = \sum_{k=0}^{\infty} p_k(\ln x)\, x^k, \qquad p_k \in \mathbb{C}[t].$$

Such series appear rather often as formal solutions of algebraic ODEs (of the Abel equations, Emden-Fowler type equations, Painlevé equations, *etc.*). We propose the following sufficient condition of convergence.

Let the series φ formally satisfy the equation under consideration:

$$F(x, \Phi) := F(x, \varphi, \delta\varphi, \dots, \delta^n\varphi) = 0,$$

and let for each $j = 0, \dots, n$ one have

$$\frac{\partial F}{\partial y_j}(x, \Phi) = a_j x^m + b_j(\ln x) x^{m+1} + \dots,$$

where $a_j \in \mathbb{C}$, $b_j \in \mathbb{C}[t]$, and $m \in \mathbb{Z}_+$ is common for all j. If $a_n \neq 0$ then for any open sector S of sufficiently small radius, with the vertex at the origin and of the opening less than 2π, the series φ converges uniformly in S. (In the case of all $p_k =$ const, one has a formal *power series* solution φ of (1) and the well known sufficient condition of the convergence of such a solution obtained by B. Malgrange.)

For example, the third, fifth and sixth Painlevé equations possess formal solutions in Dulac series. Their convergence (obtained recently by S. Shimomura for the Painlevé V and VI) can be proved by applying the above result.

This is a joint work with Irina Goryuchkina (Keldysh Institute of Applied Mathematics of RAS, Moscow, Russia).

François Ollivier (CNRS, France)

Decidability of the Membership Problem for a Finitely Generated Differential Ideal

`Wednesday, March 28, 16:30-17:00`

In this talk, we show that the membership problem for finitely generated ordinary differential ideal $I \subset F[x_1, \dots, x_n]$, where F is an effective differential field of characteristic 0 is decidable. Let F be an effective differential field of characteristic 0 and $R := F[x_1, \dots, x_n]$. Ritt has shown that the membership problem for a prime differential ideal, and so to a perfect differential ideal of R, is decidable if the factorization on $F[x]$ is effective (Ritt, 1950). Using regular representations (Boulier et al. 1997), factorization is no longer requested.

Gallo et al. 1991 show that the membership problem is undecidable for arbitrary ordinary differential ideals. And Umirbaev 2016 proves that the membership problem for finitely generated partial differential ideal, i.e. with a least 2 derivations, is undecidable, using an analogy with 2 tapes Minsky machines. We show that the membership problem for finitely generated ordinary differential ideal $I \subset R$, is decidable. This result is a consequence of the following theorem, relying on the notion of differential Gröbner (Carrà Ferro 1987) or Standard bases (Ollivier 1990).

Theorem 1. *We denote by R_e the algebra of polynomials of order at most e and use a lexicographic ordering on monomials with respect to an orderly ordering on derivatives. Let $I = [\Sigma] \subset R$, where Σ is finite. For any integer $e \geq \mathrm{ord}(\Sigma)$, let $B \subset I$ be autoreduced, if all S-polynomial between a derivative of order at most $e + 1$ and a derivative of order at most e of 2 polynomials in B are reduced to 0, then any polynomial $P \in R_e$ belongs to I iff it is differentially reduced to 0 by B.*

Jordy Palafox (LMAP, Université de Pau et des Pays de l'Adour, France)
Ecalle's Arborification and its Applications

`Wednesday, March 28, 15:00-15:30`

The tree-like structures and formal series on trees/forests appear in many fields of mathematics, in numerical analysis from the Butcher's work on Runge-Kutta schemes (see [2]), in dynamical systems with the linearization of analytic vector fields (see [5]) or in stochastic calculus with the differential equation driven by a rough signal (see [6]) or the stochastic partial derivative equations (see [1]). In the 70's, Jean Ecalle introduced for the problem of normalization of local analytic vector fields or diffeomorphisms, the mould formalism and the arborification method (see [3], [4]). The mould calculus allows to extract universal coefficients from normalizing formal series and the arborification to prove the convergence with small divisors. In this talk, we will give a presentation of the method of arborification and its diverse properties. We illustrate it on the Bruyno theorem of analytic linearization of vector fields satisfying a Bryuno arithmetic condition. We will also show how this method allows us to rediscover the Butcher's results in numerical analysis and its role in the study of stochastic differential equations.
This is a joint work with Jacky Cresson (LMAP, Université de Pau et des Pays de l'Adour, France) and Dominique Manchon (LMBP CNRS, Université Clermont-Auvergne, France).

References

[1] Y.Bruned, M.Hairer, L.Zambotti, *Algebraic renormalization of regularity structures*, Arxiv préprint : https://arxiv.org/pdf/1610.08468v2.pdf , 2017.

[2] J. C. Butcher, *Numerical methods for Ordinary Differential Equations*, Third Edition, Wiley, 2016.

[3] J. Cresson, D. Manchon, J.Palafox, *Arborifcation, invariance and convergence of normalizing series*, 27 pages, préprint, 2017.

[4] J. Ecalle, *Singularités non abordables par la géométrie*, Ann. Inst. Fourier, 42(1-2),73-164, 1992.

[5] J. Ecalle et B. Vallet, *Correction and linearization of resonant vector fields and diffeomorphisms*, Math. Z., 229: 249-318, 1998.

[6] M. Gubinelli, *Ramification of rough paths*, Journal of Differential Equations 248, no.4, 693-721, 2010.

Marko Petkovšek (University of Ljubljana, Slovenia)
Definite Sums as Solutions of Linear Recurrences

`Thursday, March 29, 11:00-11:30`

We present an algorithm which, given $m \in \mathbb{N}\backslash\{0\}$, $a_1, a_2, \ldots, a_m \in \mathbb{N}\backslash\{0\}$, $b_1, b_2, \ldots, b_m \in \mathbb{Z}$, and a linear recurrence operator L with polynomial coefficients, returns a linear recurrence operator L' with polynomial coefficients such that for every sequence h,

$$L\left(\sum_{k=0}^{\infty}\prod_{i=1}^{m}\binom{a_i n + b_i}{k} h_k\right) = 0$$

implies $L'(h) = 0$. This can be viewed as a (very modest) first step towards solving the "inverse Zeilberger's problem": Given a linear recurrence with polynomial coefficients, find solutions in the form of a definite hypergeometric sum.

Alexander Prokopenya (Warsaw University of Life Science, Poland)
Dynamics of a Block Sliding on the Plane with Variable Coefficient of Friction

`Friday, March 30, 11:00-11:30`

It is known that a body sliding on a rough surface is acted on by the friction force that is parallel to the surface and is directed opposite to the velocity of the body. According to the Amontons-Coulomb law (see [1, 2]), the friction force does not depend on the area of contact of the body and the surface and is proportional to the normal reaction force.
In the case when the body contacts the surface in one or two points one can easily write out the equations of motion of the body because the points of application of the friction forces and the normal forces are known. But in case of finite size area of contact the normal force becomes inevitably a distributed one. Therefore, the equations of the body motion may be written only if a model of the normal force distribution is given.
In the present talk we analyze dynamics of a homogeneous rectangular block in the case when it slides on a rough horizontal plane. To obtain the equations of motion we consider the

following model of dry friction of the block and the plane. First, we assume that deformation of the block is negligible and it may be considered as a rigid body. Besides, the elastic properties of the plane are the same in all its points and does not depend on the coefficient of friction. In the framework of such a model one can consider that the density of the normal force is a linear function $N(x) = k\,x + b$, where x is a local coordinate measured along the block from its center of mass, and k, b are the two constants which may be found from the conditions of the block motion without rotation. Doing necessary symbolic and numerical calculations, we have analyzed motion of the system and demonstrated some peculiarities of the block sliding on the plane with variable coefficient of friction.

References

[1] Bo N.J. Persson. *Sliding friction*, Physical principles and applications. Springer-Verlag, Berlin, Heidelberg, 2000.

[2] Le x. Anh. *Dynamics of mechanical systems with Coulomb friction*, Springer-Verlag, Berlin, Heidelberg, 2003.

Vladimir Salnikov (CNRS, Université de La Rochelle, France)
From Modeling of Coupled Systems to Graded Geometry and Back to Numerics

`Friday, March 30, 11:30-12:00`

In this contribution I would like to discuss how some objects from generalized and graded geometry appear naturally in the qualitative analysis of non-conservative and/or coupled systems. Basically, this is a collection of examples that we have worked out in the context of so called port-Hamiltonian systems (following A. van der Schaft and B. Maschke) and implicit Lagrangian systems (following H. Yoshimura and J. Marsden).
It turns out that the concepts of Dirac structures and Lie algebroids provide an interesting interpretation of these notions. Then, using the machinery of graded geometry and Q-manifolds, we discuss some possible ways to construct efficient and reliable numerical methods, appropriate for simulation of such systems.

Camilo Sanabria (Universidad de los Andes, Bogota, Colombia)
An Algorithm for Computing Differential Equations for Invariant Curves

`Thursday, March 29, 15:00-15:30`

I will describe an algorithm based on the Picard-Vessiot theory that constructs, given any curve invariant under a finite linear algebraic group over the complex numbers, an ordinary linear differential equation whose Schwarz map parametrizes it.

Raquel Sánchez Cauce (Universidad Autónoma de Madrid, Spain)
Differential Galois Theory and Darboux Transformations for Integrable Systems

`Wednesday, March 28, 14:30-15:00`

In this talk we will introduce Bäcklund-Darboux transformations for integrable systems of AKNS type. We will apply the Differential Galois Theory of linear partial differential systems to the Bäcklund-Darboux transformations of the AKNS solitonic partial differential equations and we will prove that the differential Galois group of the transformed system is isomorphic to a subgroup of the differential Galois group of the initial system.
As an application, we will explicitly compute fundamental matrices for Schrödinger operator $L-E=-\partial^2+u-E$ for all values of the energy $E\in\mathbb{C}$ and for u the Adler-Moser potentials, which are rational solutions of the Korteweg-de Vries equation (KdV). We will also compute their differential Galois groups.
This is a joint work with Sonia Jiménez, Juan J. Morales and María Ángeles Zurro.

Jean-Claude Yakoubsohn (Université de Toulouse, France)
Fast Computation of SVD

`Thursday, March 28, 16:00-16:30`

We show how approximate fastly the singular value decomposition of a matrix in the general case, i.e when there are multiple singular values.
This work is in collaboration with Joris Van Der Hoeven.