Transitions de phase et équations non locales

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Institut de mathématique Simion Stoilow de l'Académie Roumaine

Book of Abstracts

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La théorie d'Oseen-Frank des cristaux liquides/The Oseen-Frank theory of liquid crystals

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The lecture will discuss the classical Oseen-Frank theory of nematic liquid crystals, and some results with Epifanio Virga on energy-minimizing properties of universal solutions, and with Lu Liu on exterior problems.

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Un problème variationnel dégénéré/A degenerate variational problem

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Nous considérons un problème en calcul des variations pour des fonctions définies sur un ouvert borné et à valeurs scalaires, et pour un intégrande convexe qui n'est ni régulier ni strictement convexe. Nous décrivons les propriétés de régularité et d'unicité des solutions. Il s'agit d'un travail en collaboration avec Guy Bouchitté.

We present a scalar problem in the multiple integrals calculus of variations with a convex Lagrangian which is neither smooth nor strictly convex. We describe the regularity of the solutions and we address the uniqueness problem. This is a joint work with Guy Bouchitt\'e.

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Transitions de phase et problèmes non locaux pour écoulements compressibles/Phase transitions and nonlocal problems arising in compressible flows

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The aim of this talk is to present quantitative estimates for transport equations with rough, i.e. non-smooth, velocity fields. The final goal is to use those estimates to obtain new global existence results à la Leray on complex systems where the transport equations is coupled to other PDEs for instance as in fluid mechanics. We will explain for instance how it helps to treat phase transitions and nonlocal problems arising naturally in compressible flows.

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Sur les défauts du type ”anneau de Saturne” des cristaux liquides nématiques/On Saturn-ring defects in a nematic liquid crystal

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We consider energy minimizing configurations of a nematic liquid crystal, as described by the Landau-de Gennes model. We focus on an important model problem concerning a nematic surrounding a spherical colloid particle, with normal anchoring at the surface. For topological reasons, the nematic director must exhibit a defect (singularity), which may take the form of a point or line defect. We consider two physical regimes in which “Saturn-ring” configurations will be energetically favorable: the case of colloids of small radius, and the case of strong applied magnetic fields.

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Modèles sous-quadratiques pour les cristaux liquides nématiques/Subquadra models for nematic liquid crystals

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Nematic liquid crystals are matter in an intermediate phase between the solid and the liquid ones. The constituent molecules, while isotropically distributed in space, retain long-range orientational order. The classical variational theories for nematic liquid crystals are quadratic in the gradient and as a consequence, configurations with a singular line have infinite energy within these theories. On the other hand, line defects are commonly observed in these materials. Based on this observation, Ball and Bedford have proposed models with subquadratic growth in the gradient. In this talk, we discuss some properties of a subquadratic Landau-de Gennes model and its relations with the models that have been proposed by Ball and Bedford. The talk is based on a joint work with Giandomenico

Orlandi (University of Verona, Italy) and on a work in progress with Apala Majumdar (University of Bath, UK) and Bianca Stroffolini (University of Naples Federico II, Italy).

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Existence et approximation pour des modèles variationnels de rupture fragile/Existence and approximation for variational models of brittle fracture

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We will present recent results obtained in collaboration with S. Conti (U. Bonn), G. Francfort (U. Paris-Nord), V. Crismale (E. Polytechnique, Palaiseau) and F. Iurlano (U. Pierre et Marie Curie, Paris) on the brittle fracture model of Francfort and Marigo (1998), which is a variational version of Griffith's classical model to predict crack growth. We will discuss existence of minimizers for the static problem, phase-field approximation and the issue of non-interpenetration.

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Le spectre d'un opérateur de Schrödinger dans un domaine de type fil, avec potentiel dégénéré purement imaginaire, dans la limite semi-classique/The spectrum of a Schrödinger operator in a wire-like domain with a purely imaginary degenerate potential in the semiclassical limit

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Consider a two-dimensional domain shaped like a wire, not necessarily of uniform cross section. Let V denote an electric potential driven by a voltage drop between the conducting surfaces of the wire. We consider the operator $A_h = -h^2\Delta + iV$ in the semi-classical limit $h \to 0$. We obtain both the asymptotic behaviour of the left margin of the spectrum, as well as resolvent estimates on the left side of this margin. We extend here previous results obtained for potentials for which the set where the current (or ∇V) is normal to the boundary is discrete, in contrast with the present case where V is constant along the conducting surfaces.

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Régularité optimale des solutions d'entropie de l'équation eikonale/Optimal regularity of entropy solutions to the Eikonal equation

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The class of entropy solutions to the eikonal equation arises in connection with the asymptotics of the Aviles-Giga energy, a model related to smectic liquid crystals, thin film elasticity and micromagnetism. We prove, using a new simple form of the kinetic formulation, that this class coincides with the class of solutions which enjoy a certain Besov regularity.

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Sur l'approximation champ de phase des problèmes variationnels faisant intervenir des ensembles connexes 1D/On a phase-field approximation of variational problems involving 1D-connected sets

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It is nowadays classical that phase transition models such as the Cahn-Hilliard energy can be used to regularize some more delicate functionals of geometric nature such as the Perimeter functional or more generally the $(N-1)$-Hausdorff measure. This procedure is sometimes called a Phase-Field method in numerical analysis and has been used in order to approximate some classical shape optimization problems or free discontinuity problems arising in the calculus of variations. In this talk I will present an elementary way to constraint the connectedness of the unknown set in the phase-field approach. This applies for instance to the so-called Steiner Problem, for which we indeed get a phase-field approximation, but also to other minimizing functionals on which a connectedness constraint is added. This new approach give rise to some interesting mathematical problems, both from the theoretical point of view than from the numerical one.

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Deux problèmes variationnels liés aux opérateurs en forme divergence avec symbole à croissance rapide

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Dans cette lecture nous présentons des résultats concernant deux problèmes distincts, obtenus en collaboration avec Marian Bocea.

Premièrement, nous étudions la famille d'équations aux dérivées partielles $-\varepsilon\Delta u - 2\Delta_\infty u = 0$ ($\varepsilon > 0$) dans un domaine Ω avec une condition aux limites de Dirichlet. Dans le cas où $\varepsilon = 1$, qui est étroitement lié à l'étude des fonctions harmoniques exponentielles, on établit l'existence et l'unicité d'une solution classique. Celle-ci est l'unique minimiseur de la fonctionnelle d'énergie convenable associée à ce problème dans un sous-ensemble fermé d'un espace d'Orlicz-Sobolev. Plus spécifiquement, cette énergie est l'intégrale sur Ω de la densité exponentielle d'énergie $u \mapsto \frac{1}{2}\exp(|\nabla u|^2)$. On explore aussi les connections entre les solutions classiques de ces problèmes et des fonctions ∞-harmoniques et harmoniques par l'étude du comportement aux limites des solutions quand $\varepsilon \to 0^+$ et, respectivement, $\varepsilon \to \infty$. Dans le premier cas, on retrouve un résultat de L. C. Evans & Y. Yu (CPDE, 2007).

Puis, nous étudions le problème de minimisation

$$\Lambda_1(p) := \inf_{u \in X_0 \setminus \{0\}} \frac{\displaystyle\int_\Omega (\exp(|\nabla u|^p) - 1)\, dx}{\displaystyle\int_\Omega (\exp(|u|^p) - 1)\, dx},$$

où $X_0 = W^{1,\infty}(\Omega) \cap (\cap_{q>1} W_0^{1,q}(\Omega))$, quand $\Omega \subset \mathbb{R}^D$ ($D \geq 1$) est un domaine ouvert, borné, convexe avec la frontière régulière et $p \in (1, \infty)$. On montre que $\Lambda_1(p)$ est soit zéro, lorsque le maximum de la fonction distance à la frontière de Ω est strictement supérieur à 1, soit un numéro réel strictement positif, lorsque le maximum de la fonction distance vers la frontière de Ω se trouve dans l'intervalle $(0, 1]$. Dans le dernier cas nous donnons des estimations pour $\Lambda_1(p)$ et nous montrons que pour $p \in (1, \infty)$ suffisamment large $\Lambda_1(p)$ coïncide avec la fréquence principale du p-Laplacien dans Ω. Nous discutons aussi des cas particuliers et des problèmes connexes.

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Applications harmoniques fractionnaires et surfaces minimales locales ou non locales

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Dans cet exposé, je présenterai des résultats de régularité partielle pour les applications harmoniques fractionnaires. L'équation sous-jacente est l'analogue du système des applications harmoniques à valeurs dans une variété où le Laplacien est ici remplacé par le Laplacien fractionnaire. J'expliquerai également leur lien avec les surfaces minimales à frontière libre et les surfaces minimales non locales de L. Caffarelli, J.M. Roquejoffre, et O. Savin.

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Intégrabilité du degré de Brouwer et règle de la chaîne pour les Jacobiens au sens des distributions/Integrability of the Brouwer degree and chain rules for distributional Jacobians

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We reconsider the proof of uniqueness of isometric immersions of two-dimensional spheres with positive Gauss curvature, with derivatives in a certain Hölder class. We observe that an understanding of the integrability properties of the Brouwer degree is crucial to extend the range of validity for the uniqueness statement. We take this as a motivation to state and prove a theorem about the integrability of the Brouwer degree with irregular arguments. Furthermore, we show how these questions are linked to the validity of the chain rule for distributional Jacobian determinants $[Ju]$ of maps $u : \Omega \to \mathbf{R}^n$ in certain fractional Sobolev spaces. We prove the so-called weak chain rule for $u \in W^{s,n}(\Omega, \mathbf{R}^n)$, where $\Omega \subset \mathbf{R}^n$ and $s > (n-1)/n$, and the so-called strong chain rule for $u \in W^{s,n+1}(\Omega, \mathbf{R}^n)$ where $s > n/(n+1)$.

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Les hamiltoniens effectifs de Peierls-Onsager en tant que OPD magnétiques

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On presente des résultats obtenus en collaboration avec Horia Cornean, Bernard Helffer et Viorel Iftimie concernant l'utilisation du calcul pseudodifférentiel magnétique pour la construction des hamiltoniens effectifs de Peierls - Onsager pour l'étude des électrons dans un potentiel périodique et un champ magnétique faible et lisse.

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Topologie de Hölder sur le groupe de Heisenberg/Hölder Topology of the Heisenberg group

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The Heisenberg groups are examples of sub-Riemannian manifolds homeomorphic, but not diffeomorphic to the Euclidean space. Their metric is derived from curves which are only allowed to move in so-called horizontal directions.
When one considers approximation or extension problems for Sobolev maps into the Riemannian manifolds it is known that topological properties of the target manifold play a role. However, due to the homeomorphism, the topology of the Heisenberg group is the same as the Euclidean space. A notion of Hölder topology is needed. I will report on some progress (with Hajlasz) on some topological features of the Heisenberg group, in particular on an embedding question due to Gromov.

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Distances entre classes d'homotopie de $W^{s,p}(S^N, S^N)$/Distances between homotopy classes of $W^{s,p}(S^N, S^N)$

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When $sp \geq N$ the space $W^{s,p}(S^N, S^N)$ can be decomposed into homotopy classes according to the degree of the maps. We consider two natural distances between different classes.
We prove estimates, and in some cases even explicit formulas, for these distances. Most of the work is joint with Haim Brezis (Rutgers and Technion) and Petru Mironescu (Lyon 1).

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Résultats de symétrie pour des équations non locales semi-linéaires et quasi-linéaires/Symmetry results for semi-linear and quasilinear nonlocal equations

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Motivated by a conjecture of De Giorgi on the Allen-Cahn Equation and classification results for some its solutions, we will describe recent results related to one-dimensional symmetry for solutions of nonlocal equations involving possibly nonlinear nonlocal operators. We will concentrate mainly in low dimensions and present several ways to attack this problem. We will then describe open problems and links with nonlocal minimal surfaces. This is based on joint works with X. Cabre (UPC, Barcelona), E. Valdinoci (Universita di Milano) and M. Fazly (UT San Antonio, USA).

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Ouverture