Transitions de phase et équations non locales

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Institut de mathématique Simion Stoilow de l'Académie Roumaine

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On Saturn-ring defects in a nematic liquid crystal

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Co-auteur(s) ALAMA, Stan [2] ; LAMY, Xavier [3]

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We consider energy minimizing configurations of a nematic liquid crystal, as described by the Landau-de Gennes model. We focus on an important model problem concerning a nematic surrounding a spherical colloid particle, with normal anchoring at the surface. For topological reasons, the nematic director must exhibit a defect (singularity), which may take the form of a point or line defect. We consider two physical regimes in which "Saturn-ring" configurations will be energetically favorable: the case of colloids of small radius, and the case of strong applied magnetic fields.

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On a phase-field approximation of variational problems involving 1D-connected sets

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It is nowadays classical that phase transition models such as the Cahn-Hilliard energy can be used to regularize some more delicate functionals of geometric nature such as the Perimeter functional or more generally the $(N-1)$-Hausdorff measure. This procedure is sometimes called a Phase-Field method in numerical analysis and has been used in order to approximate some classical shape optimization problems or free discontinuity problems arising in the calculus of variations. In this talk I will present an elementary way to constraint the connectedness of the unknown set in the phase-field approach. This applies for instance to the so-called Steiner Problem, for which we indeed get a phase-field approximation, but also to other minimizing functionals on which a connectedness constraint is added. This new approach give rise to some interesting mathematical problems, both from the theoretical point of view than from the numerical one.

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Phase transitions and nonlocal problems arising in compressible flows

Auteur(s): BRESCH, Didier[1]
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The aim of this talk is to present quantitative estimates for transport equations with rough, i.e. non-smooth, velocity fields. The final goal is to use those estimates to obtain new global existence results à la Leray on complex systems where the transport equations is coupled to other PDEs for instance as in fluid mechanics. We will explain for instance how it helps to treat phase transitions and nonlocal problems arising naturally in compressible flows.

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The spectrum of a Schrödinger operator in a wire-like domain with a purely imaginary degenerate potential in the semiclassical limit

Auteur(s): HELFFER, Bernard[1]
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Consider a two-dimensional domain shaped like a wire, not necessarily of uniform cross section. Let V denote an electric potential driven by a voltage drop between the conducting surfaces of the wire. We consider the operator $A_h = -h^2\Delta + iV$ in the semi-classical limit $h \to 0$. We obtain both the asymptotic behaviour of the left margin of the spectrum, as well as resolvent estimates on the left side of this margin. We extend here previous results obtained for potentials for which the set where the current (or ∇V) is normal to the boundary is discrete, in contrast with the present case where V is constant along the conducting surfaces.

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Optimal regularity of entropy solutions to the Eikonal equation

Auteur(s): LAMY, Xavier[1]
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The class of entropy solutions to the eikonal equation arises in connection with the asymptotics of the Aviles-Giga energy, a model related to smectic liquid crystals, thin film elasticity and micromagnetism. We prove, using a new simple form of the kinetic formulation, that this class coincides with the class of solutions which enjoy a certain Besov regularity.

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Distances between homotopy classes of $W^{s,p}(S^N, S^N)$

Auteur(s): SHAFRIR, Itai[1]
Co-auteur(s) BREZIS, Haïm [2] ; MIRONESCU, Petru [3]

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When $sp \geq N$ the space $W^{s,p}(S^N, S^N)$ can be decomposed into homotopy classes according to the degree of the maps. We consider two natural distances between different classes. We prove estimates, and in some cases even explicit formulas, for these distances. Most of the work is joint with Haim Brezis (Rutgers and Technion) and Petru Mironescu (Lyon 1).

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Symmetry results for semi-linear and quasilinear nonlocal equations

Auteur(s): SIRE, Yannick[1]
Co-auteur(s) CABRE, X. [2] ; VALDINOCI, E. [3] ; FAZLY, M. [4]

[1] *U. Johns Hopkins, États-Unis*

[2] *UPC Barcelone, Espagne*

[3] *U. Milan, Italie*

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Motivated by a conjecture of De Giorgi on the Allen-Cahn Equation and classification results for some its solutions, we will describe recent results related to one-dimensional symmetry for solutions of nonlocal equations involving possibly nonlinear nonlocal operators. We will concentrate mainly

in low dimensions and present several ways to attack this problem. We will then describe open problems and links with nonlocal minimal surfaces. This is based on joint works with X. Cabre (UPC, Barcelona), E. Valdinoci (Universita di Milano) and M. Fazly (UT San Antonio, USA).

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Existence and approximation for variational models of brittle fracture

Auteur(s): CHAMBOLLE, Antonin[1]
Co-auteur(s) CRISMALE, V.

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We will present recent results obtained in collaboration with S. Conti (U. Bonn), G. Francfort (U. Paris-Nord), V. Crismale (E. Polytechnique, Palaiseau) and F. Iurlano (U. Pierre et Marie Curie, Paris) on the brittle fracture model of Francfort and Marigo (1998), which is a variational version of Griffith's classical model to predict crack growth. We will discuss existence of minimizers for the static problem, phase-field approximation and the issue of non-interpenetration.

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Integrability of the Brouwer degree and chain rules for distributional Jacobians

Auteur(s): OLBERMANN, Heiner[1]
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We reconsider the proof of uniqueness of isometric immersions of two-dimensional spheres with positive Gauss curvature, with derivatives in a certain Hölder class. We observe that an understanding of the integrability properties of the Brouwer degree is crucial to extend the range of validity for the uniqueness statement. We take this as a motivation to state and prove a theorem about the integrability of the Brouwer degree with irregular arguments. Furthermore, we show how these questions are linked to the validity of the chain rule for distributional Jacobian determinants $[Ju]$ of maps $u : \Omega \to \mathbf{R}^n$ in certain fractional Sobolev spaces. We prove the so-called weak chain rule for $u \in W^{s,n}(\Omega, \mathbf{R}^n)$, where $\Omega \subset \mathbf{R}^n$ and $s > (n-1)/n$, and the so-called strong chain rule for $u \in W^{s,n+1}(\Omega, \mathbf{R}^n)$ where $s > n/(n+1)$.