7 Exercise session 7, July 26

7.1 Exercises for Maxim Zabzine's lecture

Exercise 7.1.

Exercise 7.2.

Exercise 7.3.

Exercise 7.4.

Exercise 7.5.

Exercise 7.6.

7.2 Exercises for Zohar Komargodski's lecture

Exercise 7.7.

Exercise 7.8. Show that an operator that is both chiral and anti-chiral is the identity operator.

Answer. (Provided by Vivek Saxena.) I'm using the conventions of Appendix A.1 of `arXiv:hep-th/0510251`. A superconformal primary operator $\mathcal{O}$ by definition satisfies $[S_{\alpha i}, \mathcal{O}] = 0$ and $[\bar{S}^m_{\dot{\alpha}}, \mathcal{O}] = 0$. A subclass of such operators are chiral (or antichiral) primary operators which satisfy $[\bar{Q}^{\dot{\alpha}}_m, \mathcal{O}] = 0$ (or $[Q^{\alpha m}, \mathcal{O}] = 0$).

Suppose $\mathcal{O}$ is a superconformal primary which is both chiral as well as antichiral. Then, the super-Jacobi identity involving $\mathcal{O}, Q, \bar{Q}$ reads

$$\begin{aligned}[\mathcal{O}, \{Q^{\alpha m}, \bar{Q}^{\dot{\alpha}}_m\}] - \{Q^{\alpha m}, [\mathcal{O}, \bar{Q}^{\dot{\alpha}}_m]\} &= \{[\mathcal{O}, Q^{\alpha m}], \bar{Q}^{\dot{\alpha}}_m\} \\ \implies [\mathcal{O}, P^{\alpha\dot{\alpha}}\delta^m{}_n] &= 0 \end{aligned} \tag{1}$$

from which one infers that $\mathcal{O}$ commutes with the momentum generator. Likewise, replacing Q by S, it is seen that $\mathcal{O}$ commutes also with the superconformal generator K. Using similar super-Jacobi relations with the other generators of the superconformal algebra, one infers that $\mathcal{O}$ commutes with all generators of the superconformal algebra and hence (by Schur's Lemma), it is the identity operator. □

Exercise 7.9.

Exercise 7.10.

Exercise 7.11.