7 Exercise session 7, July 26

7.1 Exercises for Maxim Zabzine's lecture

Exercise 7.1. On S^1 consider $D = \mathrm{d}/\mathrm{d}x + \lambda$ with $\lambda \in \mathbb{C}$. Check that $\operatorname{ind} D$ does not depend on λ (it is topological), in contrast to the dimensions of the kernel and cokernel of D separately.

Exercise 7.2. Show that the following complex is elliptic

$$0 \to \Omega^0(M_4) \xrightarrow{\mathrm{d}} \Omega^1(M_4) \xrightarrow{\mathrm{d}^+} \Omega^{2+}(M_4) \to 0 \tag{1}$$

where $\mathrm{d}^+ = \frac{1}{2}(1+\star)\mathrm{d}$.

Exercise 7.3. The sphere $S^5\colon \{|z_1|^2 + |z_2|^2 + |z_3|^2 = 1\}$ has a T^3 action multiplying each coordinate by a phase; let e_i be the vector field rotating z_i. Choose the Reeb vector field to be a generic linear combination $v = \omega_1 e_1 + \omega_2 e_2 + \omega_3 e_3$. Compute κ.

Exercise 7.4. Using the fibration $S^5 \to \mathbb{CP}^2$ mapping $(z_1, z_2, z_3) \mapsto (z_1 : z_2 : z_3)$ show the decomposition

$$\Omega_H^{0,p}(S^5) = \oplus_n \Omega^{0,p}\big(\mathbb{CP}^2, \mathcal{O}(n)\big). \tag{2}$$

Exercise 7.5. On a 5d contact manifold, show that there is an orthogonal decomposition of the space of two-forms:

$$\Omega^2 = \Omega_V^2 + \Omega_H^{2+} + \Omega_H^{2+} \tag{3}$$

where some projectors are $\frac{1}{2}(1 \pm i_V\star)$.

Exercise 7.6. On a contact manifold show

$$F_H^+ = 0 \quad \text{and} \quad F_V = 0 \quad \Longleftrightarrow \quad \star F = -\kappa \wedge F. \tag{4}$$

7.2 Exercises for Zohar Komargodski's lecture

Exercise 7.7. Using the propagator $\langle \phi_a^i(x) \phi_b^j(y)\rangle = \frac{g_{\mathrm{YM}}^2}{(2\pi)^2}\delta^{ij}\delta_{ab}\frac{1}{|x-y|^2}$ and Wick contractions, show that

$$g(x,y) = \big\langle \operatorname{Tr}\big(\phi^{i_1}\dots\phi^{i_k}\big)(x)\operatorname{Tr}\big(\phi^{j_1}\dots\phi^{j_k}\big)\big\rangle = \frac{N^k g_{\mathrm{YM}}^{2k}}{(2\pi)^{2k}}\Big(\delta^{i_1 j_1}\delta^{i_2 j_2}\dots\delta^{i_k j_k} + \text{cyclic}\Big)\frac{1}{|x-y|^{2k}} \tag{5}$$

in the planar limit.

Exercise 7.8. Show that an operator that is both chiral and anti-chiral is the identity operator.

Exercise 7.9. Using the appendix of Minwalla et al on Superconformal Index, show that Coulomb branch operators (namely those obeying $[S,\mathcal{O}] = [\overline{S},\mathcal{O}] = [\overline{Q},\mathcal{O}] = 0$) must obey $j_r = 0$ and $S = 0$.

Exercise 7.10. Let $\mathcal{O}_i$ be the marginal operators in a CFT, and λ^i the corresponding couplings. Consider the $O(\lambda^2)$ term $C_{ab}^i\lambda^a\lambda^b$ in the beta function β_i. Show that C_{ab}^i is proportional to the coefficient of $\mathcal{O}_i$ in the OPE of $\mathcal{O}_a$ and $\mathcal{O}_b$.

Exercise 7.11. Check that

$$\int e^{ipx} x^{-2d}\mathrm{d}^d x = \begin{cases} p^d \log(p^2) & \text{if } d \text{ is even;} \\ p^d & \text{if } d \text{ is odd.} \end{cases} \tag{6}$$