2 Exercise session 2, July 17

2.1 Exercises about Guido Festuccia's course

Exercise 2.1. Solve the old-minimal generalized Killing spinor equations on S^4.

$$\nabla_\mu \zeta_\alpha = \frac{i}{6} M \sigma_\mu \bar{\zeta} + \frac{i}{3} b_\mu \zeta + \frac{i}{3} b^\nu \sigma_{\mu\nu} \zeta \tag{1}$$

$$\nabla_\mu \bar{\zeta}_{\dot{\alpha}} = \frac{i}{6} \bar{M} \sigma_\mu \zeta - \frac{i}{3} b_\mu \bar{\zeta} - \frac{i}{3} b^\nu \bar{\sigma}_{\mu\nu} \bar{\zeta} \tag{2}$$

Hint: you'll find $b^\mu = 0$.

2.2 Exercises about Francesco Benini's course

Exercise 2.2. Check that the Wilson line W_R in some representation R of some gauge group G is gauge-invariant. Show that this operator is equivalent to a 1d defect operator with: a 1d gauge field $\widetilde{A}$, the pull-back A_τ of the bulk gauge field, some 1d fermions ψ in representation R of the bulk gauge group G and charge 1 under $\widetilde{A}$, namely the 1d Lagrangian is $\mathcal{L}_D = \bar{\psi}(\partial_\tau - iA_\tau - i\widetilde{A}_\tau)\psi + i\widetilde{A}_\tau$.

2.3 Exercises about Wolfger Peelaers' course

Exercise 2.3 (About yesterday's lecture)**.** Check that the localization locus written in the lecture solves the complex BPS equations, namely the variation of the gaugino vanishes. See `https://arxiv.org/abs/1206.6359` for notations. Hint: there is a Fierz identity to rewrite $w_{IJ}\xi^J$.

Exercise 2.4. Verify that the gauge field configuration for the $SU(2)$ $k = 1$ instanton is indeed self-dual.

$$A_\mu^{\text{singular}}(x) = \frac{\rho^2 (x-X)_\nu}{(x-X)^2\big((x-X)^2+\rho^2\big)} \bar{\eta}^i_{\mu\nu} (g \sigma^i g^{-1}) \tag{3}$$

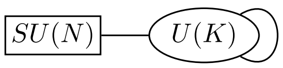
where $\bar{\eta}$ is defined by $\sigma_{\mu\nu} = \frac{1}{2}(\sigma_\mu \bar{\sigma}_\nu - \sigma_\nu \bar{\sigma}_\mu) = \bar{\eta}^i_{\mu\nu}\tau_i$.

Exercise 2.5. Argue that the one-instanton moduli space for $SU(2)$ instantons is $\mathbb{R}^4 \times \mathbb{R}^4/\mathbb{Z}_2$. Compute the moduli space metric explicitly.

$$g_{\alpha\beta} = \int \mathrm{d}^4x \, \mathrm{Tr}\big(\delta_\alpha A_\mu \delta_\beta A^\mu\big) \tag{4}$$

Exercise 2.6. The goal is to derive the ADHM constraints as describing the Higgs branch of the worldvolume theory of instantons in 4d $\mathcal{N} = 4$ SYM. A reference is Tong's lectures `http://www.damtp.cam.ac.uk/user/tong/tasi/instanton.pdf` around equation (1.37).

1. Instantons preserve half of the supersymmetry, namely their world-volume theory is a 0d theory (matrix model) with 8 supercharges. From the brane picture described by a stack of N D3 branes in the presence of a stack of k D(-1) branes argue that the worldvolume theory on the D(-1) branes is the dimensional reduction to 0d of a 4d $\mathcal{N} = 2$ theory with gauge group $U(k)$ with an adjoint hypermultiplet and a collection of N fundamental hypermultiplets, described by the following quiver. Write down its bosonic action explicitly.



2. Perform the Gaussian integral over the auxiliary fields D_{IJ}.

3. Write down the vacuum equations.

4. These equations admit in particular a Higgs branch of solutions where scalar fields originating from the $\mathcal{N} = 2$ vector multiplet vanish. Recover in this way the ADHM equations of `https://en.wikipedia.org/wiki/ADHM_construction`

5. Compute the one-instanton partition function for $SU(N)$ instantons explicitly using the integral representation provided in the lecture.