

Proximal-type algorithm for structured nonsmooth nonconvex problem involving linear operator

Wednesday, July 11, 2018 12:00 PM (30 minutes)

Proximal operator, since introduced by French mathematician Jean-Jacques Moreau in 1962 [Moreau], became a fundamental object used to design algorithms for nonsmooth optimization.

With reference [Attouch-Bolte] as a starting point, proximal-type algorithm for nonconvex model attracts huge interest due to its increasing applications in real world. However, until recently such model cannot contain complexly structure such as the composition of nonsmooth function with linear operator [Bolte-Sabach-Teboulle], otherwise, we have to sacrifice the proximity step [Li-Pong].

We propose a proximal algorithm for minimizing a nonsmooth nonconvex complexly structured. Our algorithm relies on the augmented Lagrange [Gabay-Meicer] and is formulated in a full splitting spirit in the sense of Lions and Mercier [Lions-Mercier]: the nonsmooth functions are processed via their proximal operators, the smooth function via gradient steps, and the linear operator via matrix times vector multiplication. In the setting of the Kurdyka-Lojasiewicz property [Kurdyka], [Lojasiewicz], we show global convergence and derive convergence rates for the iterates regarding the Lojasiewicz exponent.

Finally, we show that the general difference of convex programming can be written in our model. As a theoretical by-product, we deduce a scheme for this problem.

This talk relies on the joint works with Radu Ioan Bot and Erno Robert Csetnek.

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